Best in 5 Minutes: Improving students’ conceptual understanding of arch construction and behavior using physical models of masonry arches in a classroom exercise

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Introduction

The best works of structural engineering strive to find balance among efficiency (the best use of natural resources), economy (the best use of public funds), and elegance (a measure of aesthetic quality or cultural significance) – all under the umbrella of safety and serviceability [1]. David Billington, a long-time Professor at Princeton University, bestowed an honorarium on structures that demonstrated this balance, structures that embodied what he called the three ideals of Structural Art: Scientific, Social, and Symbolic [1]. In *Perspectives on the Evolution of Structures*, a course offered in the Department of Civil and Systems Engineering at Johns Hopkins University, and based on Billington’s *Structures and the Urban Environment* course at Princeton, instructors take students on a semester-long tour of exemplary works of structural engineering, helping them learn how to critically evaluate structures not only on the basis of safety and serviceability, which they will learn later, but on the ideals of structural art.

Of these three ideals, critiquing the Scientific, which relies on a basic understanding of structural behavior and an ability to perform simple structural analyses, presents the greatest challenge to students. The course, while offered by the Department of Civil and Systems Engineering, draws students from a variety of majors across the School of Arts and Sciences and the School of Engineering, and even the engineering students are typically in their first-year and so have not yet taken a course in Statics. Thus, in an attempt to improve students’ conceptual understanding of structural behavior, the instructors regularly incorporate active learning exercises using physical models of structures into the classroom.

This paper will briefly provide some background on the class and its objectives, and discuss the pedagogy of teaching structural behavior using physical models. Its primary purpose, however, is to describe a specific hands-on activity used to improve students’ conceptual understanding of masonry arch behavior, providing them with the necessary tools to critique the Scientific ideal (measured by the efficient use of material) and the Social ideal (measured by the economy) of different traditional arch forms.

Background

Throughout the course of the semester, *Perspectives on the Evolution of Structures* takes students on a photographic tour of real structures around the world to tell the stories of how a particular structural form (e.g. the suspension bridge), or structure built of a particular material (e.g. reinforced concrete), or structural engineer’s design philosophy (e.g. Gustave Eiffel) evolved over time to create a work of structural art. As an example, the Eiffel Tower – the tallest
structure in the world when completed in 1889 – is worthy of the title Structural Art as it was efficiently shaped to resist the wind load, was economical in that it paid for itself in tourist revenue within a short period of time, and met the symbolic ideal in its light aesthetic.

In one of the first lectures of the semester, *Wood and Stone: The Foundations of Structural Engineering*, instructors present the evolution of the masonry arch form in Europe and of the wooden truss in the United States. This lecture provides an important introduction to engineering concepts that students will see repeated throughout the semester as trusses and arches are truly the building blocks of many structures. It also provides students with an opportunity to play with physical models of these structures, and in the case of masonry arches, to feel for themselves both the stabilizing and destabilizing loading conditions.

Masonry arches are classically described in the classroom using Robert Hooke’s hanging chain analogy, published in 1676 as the anagram [2]:

abcccddeeeefggiiiiiiilmmmmnnnnooprrsssttttttuuuuuuuux, whose letters, when rearranged, form the Latin phrase:

*Ut pendet continuum flexile, sic stabit contiguum rigidum inversum*

Translating to English, we explain that Hooke was describing how one might understand the flow of forces through an arch – that is, by making a comparison with the flow of forces through an inverted flexible chain:

*As hangs the flexible line, so but inverted will stand the rigid arch*

While we begin with this useful analogy, it can still be a challenge for students to take it on faith that the funicular shape of a free-standing arch is a parabola1, especially when we see many more semicircular arches than parabolic arches in our built environment. When we introduce the hanging chain – the tension structure – students can put their hands on it, feel how it moves under the load they apply to it, feel for themselves that it cannot withstand any compression, any bending. To provide students with the opportunity to have the same level of hands-on interaction with a masonry arch, we developed three masonry arch models that the students build and test themselves during class.

**Physical Models in the Engineering Classroom**

Physical models have long been used in structural analysis and design to evaluate the stability of existing structures or to determine the funicular shape of a structure during design. Though computer models are now able to reliably predict the behavior of many structures subject to a variety of loads, physical models “give an understanding of structural behavior that no computer program can provide” [3]. Of course it’s not only practicing engineers who benefit from an

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1 The funicular shape is actually a catenary under self-weight alone, but it is simpler to explain (and illustrate) the self-weight as nearly uniformly-distributed, in which case the funicular shape is a parabola.
improved understanding of structural behavior; if we can provide this to our students, they will enter the workforce that much more prepared.

These days, active learning - defined as “any instructional method that engages students in the learning process” – is synonymous with good pedagogy [4]. Over the last several years, researchers have provided ample evidence of its ability to increase students’ long-term retention and conceptual understanding of lecture material [4]. Giving students the opportunity to work with physical models in the classroom is just one of several active learning exercises[2] we use in Perspectives to supplement the slide lectures, which while critically important to the delivery of the course, rely on students’ abilities to passively absorb lecture material.

The Lesson: Evolution of Masonry Arch Forms

The oldest evidence of a masonry arch comes from Mesopotamia in 4000 BCE. The Egyptians began cutting stone in 3000 BCE and by 500 BCE the Etruscans, across the Mediterranean from Egypt, were cutting wedge-shaped stones called voussoirs that could be pieced together to form a semicircular arch. The Roman Empire eventually absorbed the region where the Etruscan society lived, and with it, their arch technology [5]. The Roman arch was a successful structural form and many examples still survive from the Roman Empire, among them the impressive Pont du Gard and Segovia aqueducts. However, using Hooke’s hanging chain analogy, the students immediately observe that the semicircular Roman arch is very different from the funicular catenary form of the inverted chain. We can explain how it works simply by asking one student to hold the chain and another student to apply a tensile force to the chain to turn it into a semicircle. We note that the tensile force applied to turn the chain into a semicircle is analogous to the compressive force applied to the arch by the fill above the arch, which changes the flow of forces into one that fits within the geometric confines of the semicircular arch (see Figure 1).

![Figure 1. Line of action of a uniformly distributed load through a freestanding semicircular arch (left) and through a semicircular arch with lateral forces due to fill (right)](image)

2 Others include Think-Pair-Share, one-minute papers, and peer editing of draft writing assignments.
Eventually the Roman arch made its way into building structures, most notably with the Romanesque churches of the early Middle Ages. These churches were characterized by Roman arches and heavy masonry walls with relatively small windows. But the loads on a church roof are different from the loads on a bridge (i.e. there is no fill over the arches) and so the line of action of the force is closer to a parabola than to a semicircle. Thus, like the arch on the left in Figure 1, the rings in the Romanesque churches needed to be thick enough to safely accommodate the parabolic line of action of the relatively free-standing arch. This extra thickness meant that both additional weight and additional horizontal thrust existed at the ends of the arches, which were ultimately supported by the columns and walls of the church. Thus, the use of the semicircular arch limited the size of the church and the size of the openings in the walls, which needed to be heavy to support the large thrust of the arches [6] [7].

By the middle to late Middle Ages, church architecture had evolved from Romanesque to Gothic, characterized by its skeletal masonry framing which accommodated large windows, and of course, its Gothic arches. Side-by-side images clearly demonstrate to the students the benefit of Gothic architecture over Romanesque when the goal was to provide a structure that exhibited an aesthetic so light, that it appeared to reach to the heavens (Figure 2).

![Figure 2. Comparison of a Romanesque church [8] and a Gothic Cathedral [9] (left) and an illustration of the nearly-parabolic line of action in a Gothic arch (right)](image)

Though the Gothic arch is not perfectly funicular with regards to the loads on the roof, it is significantly more so than a semicircular arch. This allowed for a thinner ring and thus a lighter arch, which in turn meant that the arch could span a longer distance and produce less thrust on the supporting structure than could a semicircular arch.3 The thrust, of course, was supported by another feature of Gothic Cathedrals: the flying buttress.

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3 Note that this discussion is a simplification of actual structural behavior of Romanesque and Gothic architectures which used three-dimensional vaults.
The Activity: Construction and Testing of a Parabolic and Semicircular Arch

The physical demonstration consists of three arch models: a parabolic arch with a thin ring, a semicircular arch with a thin ring, and a semicircular arch with a thick ring. All three models have the same span length, the same number of voussoirs, and are made of the same wood species (see Figure 3). The class is typically small enough that the instructors can divide students into three groups and give each group responsibility for constructing one arch as a team.

Each team is provided with abutments – something physical to resist the thrust of the arch at each end, a pile of arch voussoirs, and centering. Beginning with the thin parabolic arch and working one at a time in the order shown in Figure 3, each team constructs its assigned arch while being timed. When an arch is complete, the centering may be removed, creating a self-supporting arch. Once all three arches are complete, the students take turns applying load to each using their hands so they can feel for themselves the arches’ relative stability under load.

While performing the exercise, students learn how one might critique a simple structure on the basis of its scientific and social ideals. They also learn how design is often a compromise between those ideals. By comparing the behavior of the thin semicircular arch and the thin parabolic arch, students feel for themselves that the thin parabolic arch provides a more stable support for a uniform load than does the thin semicircular arch, which can barely support its own self-weight. Students then see that the stability of the semicircular arch form can be improved if the ring is made thicker, thus containing the flow of forces within the ring, but subsequently reducing the efficiency of the structure by using more material. Finally, using the time required for construction as a metric for critiquing economy, students learn through first-hand experience that it takes significantly less time (resulting in greater economy) to construct the less efficient semicircular arches than it does to construct the parabolic arch. This conflict is one that fluctuates over time and geography with the changing costs of materials and labor, and is also one that has enabled the myriad of creative design solutions throughout history that are presented to the students throughout the semester.

Conclusion

Though constructing the arch models just right – in a way that reinforces their stability under load – was challenging, it was worth it in the end to have a physical analog to the hanging chain that we regularly use in the classroom to remind students of funicular shapes. Do you also use Robert Hooke’s chain analogy in class to teach students about funicular structures? Try completing this lesson and having a bit of fun by giving students the opportunity to build some arches! For anyone interested in constructing a set of arches for themselves, please do not struggle with the design: contact the author for a set of scale drawings. You will have to fabricate the voussoirs yourself.

4 The thick semicircular arch does not require abutments on most surfaces because its weight creates enough friction to resist the thrust.
Figure 3. Top: students constructing the thin parabolic arch (left) and the completed thin parabolic arch (right); Middle: students constructing the thin semicircular arch (left) and the completed thin semicircular arch (right); Bottom: students constructing the thick semicircular arch (left) and the completed thick semicircular arch (right)
References


