Elements of Good Problem-Solving Tasks in Thinking Classrooms

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Introduction

Engineering is an interdisciplinary field that requires extensive knowledge of STEM topics. The ability to apply mathematical concepts in engineering applications is no exception. Some undergraduate engineering students struggle with early course work typically entrenched in learning underlying mathematics. Students are often able to understand engineering principles, but are unable to understand the mathematics behind the principles. This is due to students finding it difficult to make connections and apply mathematics outside of routine computational calculations. [1]

Traditional instruction of mathematics has relied predominantly on teacher-centered pedagogies or passive learning (e.g lecture). [2] Active learning differs in that it includes student-centered approaches that are "any instructional method that engages students in the learning process. In short, active learning requires students to do meaningful learning activities and think about what they are doing." [3] Active learning has been shown to increase student understanding and reduce class failure rates in STEM courses. [4]

An active learning environment with student-centered learning is better suited to practice problem-solving and develop conceptual understanding. [5] Standards are shifting to become more focused on developing student's ability to problem solve rather than complete computational calculations. For example, the Common Core Standards for Mathematical Practice includes the following: "reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, and model with mathematics." [6] The ability to reason and apply the tools of mathematics to complex systems is also a crucial aspect of engineering as demonstrated by the Accreditation Board for Engineering and Technology (ABET) criteria: "An ability to apply a knowledge of mathematics, science, engineering, and technology to engineering technology problems that require limited application of principles but extensive practical knowledge." [7] These standards demonstrate the interdisciplinary nature of engineering and the need to conceptually understand mathematics through problem-solving.

The purpose of this study is to explore and discover what elements lead to good problem-solving tasks in an active learning mathematics focused classroom. Elements were determined using interviews with mathematics instructors that currently use active learning techniques and problem-solving tasks in their classrooms. Instructors were asked to describe the process they use for creating tasks. The strategies described by the instructors were analyzed and grouped into emergent themes. These themes are discussed in this paper and will ultimately be compiled into a guide made for instructors on how to create good problem-solving tasks for mathematics and engineering courses that heavily use mathematics. The goal is to enhance mathematics education throughout an entire post-secondary program to better prepare all students for their degree programs and careers, particularly in engineering.

Background

There are a variety of strategies for incorporating active learning into the classroom. This study will focus on the strategies outlined by Peter Liljedahl, one of the leading researchers in the field of active learning techniques used in mathematics. [8] His research has identified elements that are critical to creating a classroom environment referred to in his research as "thinking classrooms." The three most important elements identified include: 1) good problem-solving tasks, 2) visibly random groups (randomized groups with the randomization process clearly visible to the students), and 3) use of vertical non-permanent surfaces to solve problems (e.g. whiteboards mounted on a wall). The combination of these elements have been found to increase student engagement and ability to problem-solve. [8]

Liljedahl's research indicates having good problem-solving tasks is critical for students to develop good problem-solving habits. One area not clearly defined are the elements that make a problem-solving task "good." Identifying and compiling these elements into an instructor's guide for developing problem-solving tasks will provide a valuable resource in enacting active learning in mathematics focused classrooms.

Research Methods

Mathematics instructors were recruited using a targeted snowball approach. An email solicitation was sent to specific instructors known by one of the co-authors to be using active learning techniques. Many were familiar with and employed Liljedahl's strategies, but this was not required for participation in the study in an effort to be inclusive of all active learning approaches. Instructors willing to be interviewed after the first wave of recruitment were asked to make additional recommendations to increase the pool of potential interview candidates.

The sample used for this paper includes five mathematics instructors that use active learning techniques and problem-solving tasks in their classroom. The instructors participated in semi-structured interviews that lasted approximately 20 to 30 minutes. The instructors all taught in the United States at the post-secondary school level, either at a university or community college. The courses the instructors currently taught ranged from college algebra to advanced calculus. Three of the five instructors had previously taught at the secondary school level before teaching at the post-secondary level.

The interviews were conducted in person or via the online video conference software Zoom. The instructors were asked questions pertaining to their use of active learning in their classrooms, student reactions, strategies for developing classroom problems, and content or feedback provided to students (see Appendix for interview protocol). Additional questions were kept to a minimum and primarily used to clarify participant responses.

Two members of the research team reviewed interview transcripts using an open coding approach. [9] The first round of coding involved the first reader coding all of the transcripts and creating an initial codebook. The second reader then combined repetitive codes to make the initial codebook more concise.

The second round of coding involved the first and second reader independently using the initial codebook on a single transcript. The readers then discussed the appropriateness of the codebook and made further refinements to better represent the utterances made by the participants. Both readers then independently coded a second transcript. Further adjustments were made to the codebook, requiring that the readers revisit the first transcript using the most recent codebook. This process was repeated for the third transcript. High overlap, i.e., interrater reliability, in coding by the two readers on the third transcript with no new codes suggested a stabilization of the codebook. [10] The fourth and fifth transcripts were then reviewed only by the first reader using the finalized codebook.

Results and Discussion

The final list of emergent themes resulting from interviews with mathematics instructors currently using active learning were categorized into three areas: 1) task structure, 2) task development, and 3) problem-solving environment. The terms used for emergent themes may have overlap with terms used in existing literature. A description for each emergent theme is provided in the category tables to clarify the definition of the theme for this study to avoid confusion. Each category is broken down in the following subsections.

Task Structure:

The category of task structure revealed seven codes (Table 1). The most frequent emergent themes within and across participants were “open-ended task,” “problem seeking,” and “critical thinking.” These themes underline the importance of creating a task that incentivizes students to collaborate. Students bring their own unique perspective to the classroom, so problems should encourage each student to present their own perspective. Tasks should avoid solutions with set, unaltered paths, which creates conversation and debate as to which method is preferred. The difficulty of the problem plays an additional role in student engagement. Students should not read a problem and immediately know the answer because easy problems tend to encourage students to work alone. The urge to ask further questions should be encouraged by increasing the problem difficulty.

Instructors also stressed the importance of making problems interesting for the class they currently taught. This usually took the form of a context that emulated the real world such as the amount of tile needed to floor a room or finding out how fast Usain Bolt runs. It is impossible for every context to appeal to every student, but having different contexts is more likely to engage students in a problem they would have otherwise had no interest in. This means the curriculum for each semester should change from term-to-term to tailor problems to the interests of that particular class of students. The task itself can also generate interest by being a physical activity like flashcards or a matching game.

Table 1: Emergent codes focusing on task structure.

Code	Description	Sample Quote
Open-ended task	Task has multiple points of entry, different ways of solving, and/or unique solutions. Students often apply mathematical tools they have previously learned.	“where there's multiple entry points, there could be entry points for those who are really grasping the mathematics but also those where maybe they're struggling a bit and there's at least a place where they can engage and, and do some intuitive bring up some intuitive ideas.” ~Professor Hansen
Problem seeking	Set of data or a scenario is presented with no concrete problem statement. Students ask questions about the scenario to determine the problem. Students often derive mathematical tools during analysis of data.	“So the example today in my math class [course code], the college mathematics it was using the data on male and females acceptance rate into a University. The only information I said was that, here's the data, so I didn't tell them what university to use, I didn't tell him anything. I said, here's the data, they went to the boards with a partner and I just basically said, what do you see? Go.” ~Professor Warner
Critical thinking	Students critically think about a task rather than solving the task from memory. Students should choose mathematical tools after analyzing the problem. Avoid problems where students can simply input values into an equation and solve for the unknown, i.e. plug and chug	“Because you don't want the the work of the problem to necessarily just be all of the tedious calculations. What you want the work of the problem to be in the thinking that goes into solving it.” ~Professor Makinson
Interesting problems for students	Problems are personally interesting to the students in the class.	“I definitely see an increase with some students when the problem is about something that they're interested in. And I see a decrease. The problem is about something they're not interested in.” ~Professor Warner

Difficult task	Tasks must be appropriately difficult to encourage students to collaborate, rather than work alone.	“I want them to talk to each other about problems that are hard to do individually. I find that when students are given problems that they already know how to do, they're not going to talk to each other because they already know how to do that.” ~Professor Perry
Physical activity	Task involves students physically conducting an activity or lab.	“...I have had an activity that I fully developed with to measure light intensity. I'm using various sheets of window tint, cut up squares of window tint and a light sensor. And we actually physically go outside and measure the light intensity as we add layers of this window tint on and it produces a wonderful exponential decay function...” ~Professor Hansen
Emulate real world	Tasks are grounded in the real-world.	“And so the way the SIR model works, so you have these three populations that doesn't require too much background, people know what it's like to get sick and people know that if you're around sick people, you get sick.” ~Professor Perry

Task Development:

The category of task development revealed four codes (Table 2). The process for developing a task for the class took one of two primary routes: 1) create an original problem, or 2) modify an existing problem. The difficulty in developing such tasks was alleviated by some instructors through conversations with colleagues as part of a “community of practice.” The reason why most instructors simply did not directly source problems from a textbook relates to the code “interesting problems for students.” Creating problems tailored to students required that problems from sources (e.g., textbooks, online, or research) be changed to make the context more interesting to that particular class of students.

A common modification to sourced problems was removing information. This made the task more open-ended and better suited for a thinking-classroom. When determining the amount of information to remove, one instructor had the following comment: “I probably err on the side of removing too much. But here's why, I would rather my students ask me for some info like, hey, we’re going to need to know this thing. And if they really do that, I can provide that information.” This suggests that it is hard not to go wrong in terms of removing too much information.

Table 2: Emergent codes focusing on task development.

Code	**Description**	**Sample Quote**
Original task	Instructor creates an original task for the class.	“...I just try to make up on my own, which maybe the first draft is not very good. And after implementing a few times, I kind of you know, dial in details and it becomes better.” ~Professor Hansen

Re-imagined problem	Instructors start with a problem from a source (e.g. textbook) as a base, then modify the context and/or content to make the problem more interactive and/or interesting.	"...we're changing the context like I found this one problem where I really liked the math, but I had a problem with the context." ~ Professor Globe
Remove information	Assumptions and information is removed from a problem to prevent students from directly arriving at the answer.	"I probably err on the side of removing too much. But here's why, I would rather my students ask me for some info like, hey, we're going to need to know this thing. And if they really do that, I can provide that information" ~Professor Warner
Community of practice	Class instructors collaborate and/or discuss with other instructors employing similar methods.	"Like I needed my own group where I needed to go and say this is what I tried and this is what didn't work. And, you know, maybe people had a suggestion. So in a sense, I needed to learn about active learning, actively" ~Professor Perry

Problem-Solving Environment

The category of problem-solving environment revealed fourteen codes (Table 3). Instructors noted that designing good problem-solving tasks was only half the battle in terms of creating a thinking classroom. The effectiveness of the problems was closely linked to the environment in which the problem was presented.

One theme that was common between all instructors was providing "minimal background." Instructors wanted to provide as much time as possible for class discussions rather than lectures. Some instructors achieved minimal lecture in class by using a "flipped classroom" approach. Instructors viewed in class discussions and "contextualizing mathematics" as a better use of class time.

Instructors described a need to be flexible in terms of the amount of time they spent on each presented task. The code "dynamic lesson plan" captures the need to recognize that students will find certain tasks more or less interesting than originally anticipated. This can be difficult to predict during the planning stage. Greater focus should be placed on tasks that instructors find engaging to students. This relates to the code "provide ample time." If there's a rich discussion occurring, the instructor should not end it early for the sake of moving onto the next task.

"Classroom management" techniques varied depending on the instructor. One recurring aspect was creating an expectation for collaboration. Explaining to students why collaborative learning was beneficial made students more willing to work with one another. Some instructors also commented on how group composition changed collaboration levels. This prompted the use of different techniques for "group formation." "Managing student emotions" was another key component to consider when presenting students with a likely unfamiliar environment.

Instructors need to be cognizant of student emotions and the potential that their initial confidence in such environments may be low.

Table 3: Emergent codes focusing on the problem-solving environment.

Code	**Description**	**Sample Quote**
Collaborative learning	Students working collaboratively, i.e., group work, performing tasks such as think-pair-share and asking each other questions. Student collaboration is designed to level student knowledge and elicit communication between students.	“I use many, I have students working together in mostly pairs, at whiteboards to solve problems. I also sometimes depending on the day, they will work together in small groups to solve problems.” ~Professor Warner
Group formation	There is a formal process for selecting student groups. This can include visibly random groups, grouping based on student ability, or more.	“And it's kind of random, so yeah I assign them to their first groups and then every time we start a new unit they get new groups. Sometimes I assign them, sometimes they choose.” ~Professor Globe
Non-permanent surfaces	In-class work is done on non-permanent surfaces like whiteboards or windows. The non-permanent surface can be mounted on the wall or be portable to use at a table.	“I get students up to the whiteboards a lot” ~Professor Globe
Minimal background	Minimal to zero background is given prior to the start of a task or activity.	“Oh, it'd be no more, no more than a couple of minutes, right.” ~Professor Makinson
Contextualize mathematics	Discuss the nature and use of mathematics rather than lecturing about operations.	“Just the skills so that we can use the class time to just discuss calculus, what do we use it for, why do we do it this way, where do these formulas come from.” ~Professor Perry
Dynamic lesson plan	The amount of time spent on certain topics varies based on student engagement. The process of engaging students to think about mathematics is prioritized over learning the operations.	“... the plan is not to have a plan. Because, you know, you want to be open minded to what the students bring to the classroom, even when it's their struggles that they bring to the classroom.” ~Professor Hansen
Provide ample time	Students often need more time than expected to solve tasks. Try not to end good tasks short. Let tasks go on if the students are engaging with the content.	“Yeah, but for the most part I want students to talk to each other and have time to process” ~Professor Perry
Technological aids	Technology is used to help enhance the classroom experience. Examples include presenting a task in the form of a video or using In-class polling technology.	“Occasionally, we use the document camera to have a student come up and display their work” ~Professor Hansen
Classroom management	Set classroom expectations for an active learning environment. Make clear what students need to be doing and why it	“They're used to being in a kind of, what we call in [US State], “a sit and get” to add setting right and so you need to give guidelines on how to interact if

	benefits them. Develop facilitation skills. Good problems help with classroom management.	you know if they're not used to it. And so I do give them guidelines and remind them about that." ~Professor Makinson
Manage student emotions	Instructors should provide encouragement to positively impact student confidence in solving problems and level of comfort with an active learning environment.	"Yeah, so I have to do a lot of building up, to encourage them to actually engage and to try to do something and to work and to listen to one another and to look around the room and get ideas." ~ Professor Warner
Flipped classroom	Students learn mathematical operations outside the classroom.	"The reason for flipping, especially in calculus courses, students that come from high schools already have taken calculus. If I lecture, they're bored. Students that have never taken calculus, if I lecture, they're lost because I'm going too fast for them. So, what do you? What I usually do is I say, look, there's these videos online on my on my website, watch those videos and then do these problems." ~Professor Perry
Online discussion board	Have an online discussion board or forum where students can discuss tasks outside of class. Students should be able to introduce themselves to one another online.	"...I do like an introduce yourself forum where they have to, you know, talk about their career goals and share like two fun facts about themselves that no one in the class knows, like a few other things and they comment on each other's and all that." ~Professor Globe
Growing pains	Instructors initially dislike using active learning techniques or are unhappy with the initial lack of success with the methods.	"I think learning how to teach actively kind of creates these feelings in teachers too, because sometimes things don't work. And sometimes you don't know what to do." ~Professor Perry
Formative Assessment	There is no direct grade attached to the task itself. Qualitative feedback is provided to students in-class by the instructor.	"And I basically give them, it could be a handout where they have to fill it out and then upload it to Blackboard, or to a course website before class, so I can get a sense of where the class is before I start" ~Professor Makinson

Implications and Future Work

Engineering is an interdisciplinary field. Conceptually understanding mathematics is not only important for engineering, but for STEM subjects in general. Student-centered learning is a great way to incorporate interdisciplinary activities into a mathematics classroom. [11] To take full advantage of the new environment, instructors need to know what elements make good problem-solving tasks. An instructor's guidebook will be created and made available based on the findings and discoveries of this study on how to create problem-solving tasks.

The three main categories of emergent themes were task structure, task development, and problem-solving environment. The emergent themes in task structure are useful for understanding what elements make a good problem-solving task. Not all elements need to be included in the same problem. For example “physical activity” and “problem-seeking” were shown by instructors to result in an effective activity-based mathematics task. Instructors who do not have access to physical items could combine an “open-ended task” with something that attempts to “emulate real world” and end up creating a very rich mathematics task. Instructors should try to mix and match as many elements from the task structure themes when designing new tasks.

It is clear from the interviews that writing good tasks is a challenging process. The emergent themes from task development are useful for ensuring instructors know how to create good tasks throughout their courses. Codes like “re-imagined problem” or “remove information” demonstrate that an instructor does not need to create an entire curriculum from the ground up to create an active-learning classroom. This makes the barrier to entry more appealing for instructors who predominantly use traditional teacher-centered approaches in their current course offerings.

Knowing the particular challenges previous instructors faced in creating an active-learning environment will help instructors avoid common pitfalls. For example, becoming comfortable with a “dynamic lesson plan” is a difficult shift to make if an instructor previously had full-control through lecture. These elements of creating a problem-solving environment will also be included in the guidebook as a class cannot have good problem-solving tasks without an environment conducive to active learning.

In future work, participating instructors could be expanded to see if the emergent themes hold true for a larger, more diverse sample across multiple fields of study employing mathematics. It would also be pertinent to include insights that reveal the student’s perspective on using the emergent themes. These additional steps would greatly advance the findings of this study and the future guidebook to the benefit of any instructor looking to design good problem-solving tasks.

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Appendix

Interview Protocol

1. State name, occupation, course subject, level of students, and active learning methods utilized. How familiar are you with Peter Liljedahl's research?
2. Describe what it was like using active learning methods in your classroom for the first time.
    a. What aspects of the methods were either effective or ineffective at achieving the learning outcomes for the lesson.
    b. How did the students respond to the methods?
3. What strategies have you used for developing classroom problems?
    a. [Ask this question only if the respondent notes they have used textbook problems] Do you have any recommendations or best practices in converting these types of questions into a thinking problem?
    b. [If problem length is mentioned] What impact did the length of problem have on student actions in the classroom?
4. How many times have you taught the class using active learning techniques?
    a. [If multiple times] Have you made changes to the problems you use in the class? If so, as you have refined these problems, what changes made notable positive or negative differences in student engagement within the classroom?
5. Do you provide students with any additional content or background before giving them the problem?
    a. [If yes] Can you describe the type of content or background?
    b. [if yes] How long do you typically spend on the background?
    c. [if yes] Is there any content that doesn't require background?
6. Do you provide students with feedback after they have completed the problem?
    a. [If yes] What sort of feedback do you provide?