2006-626: REPLACE MATH TAUGHT DIFFERENTIAL EQUATIONS COURSE WITH A BME TAUGHT PHYSIOLOGICAL MODELING COURSE

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Replace Math Taught Differential Equations Course with a BME Taught Physiological Modeling Course

ABSTRACT

Traditionally, a course in differential equation taught by the Math department is required in most engineering programs. With the less prescriptive requirements by ABET and Criterion 8 for Biomedical Engineering programs, programs can be more creative than ever before in offering differential equations. The only ABET requirement dealing with differential equations in BME is Criterion 8 "the capability to apply advanced mathematics (including differential equations and statistics), science, and engineering to solve the problems at the interface of engineering and biology". A physiological modeling course conveniently satisfies both requirements. Here it is proposed that a differential equations based physiological modeling course replace the more traditional differential equations course taught in the math department. The motivation for exploring this possibility is that: (1) the differential equation course can be replaced by a math/science elective and (2) students see differential equations applied in modeling physiological systems. We have been teaching a physiological modeling course at the University of Connecticut's BME program over the last four years developing the curriculum, and after the proof of concept is approved, will implement the replacement.

INTRODUCTION

Designing and updating a biomedical engineering (BME) curriculum is a daunting challenge. BME is unlike most engineering programs with the need for more life science courses as a core component of the curriculum. Described here is the rationale for using a differential equation based physiological modeling course as a substitute for a math taught differential equations course, and the subsequent flexibility it allows in the curriculum.

In addition, many BME programs prepare their students for more options than the usual engineering program. Approximately one-third of BME students in our program plan to attend medical or dental school and expect that the medical and dental school requirements be a part of the basic program. We have accomplished this challenge with basic medical/dental school requirements of one year of biology and organic chemistry in our curriculum. An additional course in the curriculum like genetics, molecular biology or biochemistry would be an asset.

Many universities are requiring engineering programs to reduce the total number of credit hours, while increasing the number of general education credit requirements. In the past year at the University of Connecticut (UConn), we were forced to reduce the number of semester credit hours in BME from 133 to 127 . This has caused us to rethink our curriculum and to optimize our course offerings.

Another consideration in a BME curriculum is ABET, the organization that accredits all engineering programs. ABET's Engineering Criteria 2000 allows programs to define themselves with great flexibility. With this flexibility in mind, BME programs can be more creative than ever and maximize course double counting to construct a more expansive curriculum. Details on ABET requirements and how BME programs can leverage courses are described in the next section.

Engineering programs have typically included four semesters of math courses in the curriculum. These are usually Calculus I, II and III, and a differential equations course. The math department teaches these courses to all university students and do not provide any specific information to any one major. I have heard it said many times by engineering faculty that engineering students need math courses taught by a math instructor with an engineering or applied background that draws examples from the discipline. In some peoples' minds, part of the retention problem for engineering students' lies in the quality of math instruction and the uninspiring material covered.

In this paper, it is proposed that the required course in differential equations be replaced by a differential equation based physiological modeling course. The physiological modeling course will serve two purposes. One is to introduce differential equations into the curriculum with a course with applications of interest to the BME students. In place of the differential equations course, we will offer a math/science elective. The math/science elective will include a number of life science electives, and those students who feel uneasy about their mastery of differential equations after taking the physiological modeling course, have he opportunity to take the differential equations course.

ABET REQUIREMENTS

With ABET's less prescriptive Engineering Criteria 2000, engineering programs were given the opportunity to be more creative to server the needs of their consistencies. There are two areas that ABET is rigid [1]. Under Criterion 4 (Professional Component), ABET requires all engineering programs have a minimum number of engineering topics (1 ½ years), math and science (1 year), a general education component and a capstone design experience. In addition, under Criterion 8 (Program Specific Criteria), ABET requires BME programs to satisfy

> "The structure of the curriculum must provide both breadth and depth across the range of engineering topics implied by the title of the program.
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> The program must demonstrate that graduates have: an understanding of biology and physiology, and the capability to apply advanced mathematics (including differential equations and statistics), science, and engineering to solve the problems at the interface of engineering and biology; the ability to make measurements on and interpret data from living systems, addressing the problems associated with the interaction between living and non-living materials and systems."

It should be noted that Criterion 4 does not specify that a differential equations course be taught, and certainly does not require it be taught in the math department. The only ABET requirement dealing with differential equations in a BME program is in Criterion 8. Criterion 8 does not require a differential equations course be taught by the math department, or even require a course in differential equations. What it does require is that graduates have the capability of applying differential equations at the interface of engineering and biology. So, even if a BME program did have a differential equations course in the curriculum, it would not be compliant unless it demonstrated that graduates could apply this knowledge to problems in engineering and biology.

A single physiological modeling course based on differential equations with engineering and biology content would satisfy this requirement. This capability can be demonstrated by examination of the student work on an exam or homework by a group of faculty.

Further, it is a simple matter to demonstrate that graduates have this capability by collecting all student work on an exam or homework assignment, and having a group of BME faculty examine it.

PHYSIOLOGICAL MODELING COURSE

The author has been teaching a differential equations physiological modeling course to sophomores for the past four years. The textbook used in the course is *Introduction to Biomedical Engineering* [2],[3]. In addition, the author has been teaching a freshman BME engineering fundamentals course using [2],[3] that focuses on statics[i]. As a supplement to both courses, he has written course notes that are being developed into a book in physiological modeling.

The physiological modeling course starts with the solution of differential equations using the classical method, MATLAB and SIMULINK. First-order differential equations are solved by the integrating factor, with and without an input. The notion of the time constant is described next. The second-order differential equation is then considered with and without inputs. The traditional three cases (overdamped, critically damped and underdamped) are presented with a variety of inputs: impulse, step, ramp, exponential and sinusoidal. Examples are presented in handling situations when the form of the forced response is the same as the free response. The second-order case is extended to solutions of differential equations of any order. MATLAB makes the extension quite straightforward computationally. In all cases, initial conditions are varied from zero initial conditions to any arbitrary set. Additionally, the D-operator and state-variables for solutions of a set of differential equations are introduced. MATLAB symbolic toolbox and SIMULINK are then used to solve differential equations. Sections on solving nonlinear differential equations using linearization techniques using the classical technique, solving a difference equation, and system identification for first and second-order systems are planned for future classes.

Presented next is compartmental analysis, starting with a cell and increasingly larger systems, such as the circulatory and respiratory systems, and infectious diseases using a small family model and the spread of an infection in a community. In addition, biochemical reactions are also covered. A thorough treatment of transient electric circuits is presented involving resistors (R), capacitors (C) and inductors (L). Here, cases more advanced than the typical series and parallel RLC circuit are considered, cases including any configuration of R, C and L with orders greater than the second-order case. This section provides an excellent treatment on determining initial conditions for differential equation solutions using capacitors and inductors.

The dynamic model of the neuron is then presented from a biochemical and electric circuit basis. In the future, the Hodgkin-Huxley model will be presented with a complete SIMULINK neuron model from the dendrite to axon hillock and networks of neurons.

The course finishes with the Laplace transforms and solving differential equations with the Laplace transform. Here we cover calculating the Laplace transforms of a variety of functions, properties of a Laplace transform and using the partial fraction expansion method. Use of MATLAB for solving Laplace transforms is presented.

Additional chapters cover the basics for analyzing mechanical systems similar to the treatment of electric circuits. A section on muscle mechanics that includes cross-bridge models and large scale muscle models follows. The eye movement system using both the cross bridge and large scale muscle model for one dimension (horizontal) and three dimensions is also planned with a head-neck movement system. Fluid and thermal systems will also be included with appropriate physiological models. The inclusion of extra sections allow the course to vary from year to year, focusing on electrical one year, mechanical another year, and fluids and thermal other years.

Other chapters in the book are a brief treatment of control theory and the Z-transform. Also included is the neural control of the fast eye movement system, using a SIMULINK model of a parallel network of realistic whole neurons from the superior colliculus to the paramedian pontine reticular formation, cerebellum and motor neurons innervating a cross bridge and large scale muscle model. The neuron and muscle model are viewable via simulation from the molecular to the large system level. Finally a section on parameter estimation in both the frequency and time domains is presented.

MATH DIFFERENTIAL EQUATIONS COURSE

The following comments are based on answers from students on their experiences in a differential equations course taught by the math department. All too often, a differential equation course taught by the math department focuses on proofs of 1st order differential equations and special first-order models such as the prey-predator model. Little time is spent on second-order models. Never are higher order differential equations with realistic engineering applications covered or initial conditions derived from a system. Coverage on Laplace transforms varies greatly as it is the last topic in the course. Some instructors do not cover it, and when covered, only the definition and a few Laplace transforms are covered. Material covered in the math differential equation course that is not included in the physiological modeling course includes eigenvectors, saddle points and spiral sinks.

CURRICULUM CHANGES

The motivation for the proposal in this paper is that:

(1) A Math offered differential equation course be replaced by another math/science elective offering greater flexibility and
(2) Students can learn differential equations and learn about modeling physiological systems at the same time. Many students learn abstract material better when they see practical applications. This helps satisfy two ABET requirements, and it is my feeling, this will enhance the learning process and have a positive impact on later courses.

The differential equations based physiological modeling course is offered in the fall semester of the sophomore year. Rather than requiring the traditional Math taught differential equations course in the spring semester of the sophomore year, a math-science elective is substituted with a number of choices including: life science courses, Math taught differential equations and a partial differential course. Thus students interested in medical or dental school can use a course like genetics, molecular biology or biochemistry to satisfy this requirement. ABET is completely satisfied with regard to Criterion 8, and the one year of math and science is still satisfied as all the optional courses are math or science electives.

FEEDBACK

I have recently obtained informal feedback from UConn students who have taken the physiological modeling course at the same time as taking their differential equations course, those who have taken it after, and those who have taken it before. More data needs to be taken as the course achieves steady state, as changes are still occurring in the content of the course. This is not a formal survey and only seventeen students took part in the survey. The following questions were asked (BME 211 is the physiological modeling course and Math 211 is the differential equations course).

1. Compare the coverage of differential equations in Math 211 and BME 211 (i.e., 1st order, 2nd order and higher order solutions of differential equations; Solutions of differential equations with inputs of step, cosine/sine, ramp, exponential, repeated roots; Simulink; MatLab solutions of differential equations; Block Diagrams; D-Operator; Laplace transforms.
2. Solving differential equations with engineering applications was helpful in understanding how differential equations are used (i.e., compartmental analysis and electric circuits).
3. Describe the material covered in Math 211 not covered in BME 211.
4. Compare the coverage of Laplace transforms in BME 211 and Math 211.
5. Would you be supportive of replacing a required Math 211 with a Math & Science elective (i.e., could select from a list of courses that would include Math 211, Biol 108, Genetics, molecular biology, microbiology, biochemistry, ...)?

Most students indicated that

1. Coverage of differential equations in BME 211 exceeds that of Math 211.
2. Seeing applications in BME 211 reinforced learning.
3. Great variety of coverage in Math 211 as indicated in the previous section.
4. Coverage of Laplace was uneven in Math 211. None covered partial fraction expansion or using Laplace transforms to solve differential equations or using MATLAB.
5. Most indicated that they would prefer not taking Math 211, but rather take another math science elective.

CONCLUSION

This paper proposed that the required course in differential equations be replaced by a differential equation based physiological modeling course. The physiological modeling course will serve two purposes. One is to introduce differential equations into the curriculum, a course with applications of interest to the BME students. The second is to replace the math taught differential equations course with a math/science elective. It is our plan to investigate replacing the math taught differential equations course with a math/science elective after careful consideration. The math/science elective would include a number of life science electives, and for students who feel uneasy about their mastery of differential equations after taking the physiological modeling course, the opportunity to take the differential equations or a partial differential equations course.

REFERENCES

1. 2005-2006 CRITERIA FOR ACCREDITING ENGINEERING PROGRAMS, ABET, Inc., www.abet.org.
2. Enderle, J.D., Blanchard, S.M. and Bronzino, J.D., *Introduction to Biomedical Engineering*, Elsevier, Amsterdam, 2000.
3. Enderle, J.D., Blanchard, S.M. and Bronzino, J.D., *Introduction to Biomedical Engineering (Second Edition)*, Elsevier, Amsterdam, 2005.

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[i] The freshman course introduces continuous and discrete-time systems, basic mathematics (vectors and scalars, complex algebra), static biomechanics (forces, torques and moments), biological signals, dc electric circuits, static chemical and electrical properties of the membrane of a neuron, and the basics of MATLAB.