



## Understanding Design, Tolerating Ambiguity, and Developing Middle School Design Based Lessons

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## **Abstract**

We have, over three years, developed a set of practices that helped move middle school mathematics, science, and special education teachers away from trepidation with engineering design and toward comfort with ambiguity, confronting and reducing content knowledge gaps for themselves and their students, and engaging a professional support network. Teachers need deep understanding of the mathematics and science they will teach and knowledge of how students develop understanding of content, how to set significant learning goals, how to select and implement appropriate instructional tasks, and how to assess learning. Common Core middle grades standards include the design process in the science framework, but the design process is not easy to learn and then integrate into broader pedagogical content knowledge teachers must deploy to be successful. Teacher preparation and scaffolding are key to implementation of design based learning to support student learning gains. Well-designed professional development experiences are integral to developing such knowledge and skills.

Teachers Engaged in STEM and Literacy (Project TESAL) supported middle school teachers utilizing design based learning with the ultimate goal of increasing student achievement and engagement in STEM disciplines. We focus here on how Project TESAL participating teachers shifted their stance toward ambiguity, developed comfort with the design process for integrating mathematics and science instruction, and how their lesson plans and focus group interviews revealed such change over time. We discuss findings from analyses of data across three years from content knowledge tests (Diagnostic Mathematics Assessments for Middle School Teachers [DTAMS]), surveys (Teacher Efficacy and Attitudes Toward STEM [T-STEM]), individual interviews and focus groups, teacher generated design lesson plans, and observations as participating teachers implemented lessons in their classrooms.

Teachers who participated all three years discussed the integration of engineering design, complex instruction and group worthy tasks, productive struggle, mathematics-science integration, mathematical modeling, and literacy foci as fitting together in a seamless whole that allowed instruction guided by this perspective to naturally incorporate these effective practices. Connected to this was the challenge of acquiring and implementing that complex perspective.

## **Introduction**

Rising Above the Gathering Storm identified the need to “encourage more US citizens to pursue careers in mathematics, science, and engineering” [1]. Teachers Engaged in STEM and Literacy (Project TESAL) was designed to strengthen the STEM educational and career pipeline in Appalachia. Difficulties attracting students to STEM careers are enhanced in Appalachia and West Virginia (WV) [2], [3]. WV is far below the national average in percentage of STEM degrees (21% vs. 30%). 17% of adults over 25 in WV have a Bachelor’s degree (lowest nationally), and many communities have much lower rates [4]. Thirteen of 55 WV counties are “low education counties” where “25 percent or more of residents 25-64 years old had neither a high school diploma nor GED” [5]. Declining population with out-migration of college graduates and in-migration of less-than-high-school graduates characterizes this region. Given that many Appalachians desire to live, work, and make a difference in their home communities [6], the vision underlying this project was to leverage engineering design of appropriate technologies applicable to societal challenges in both developing nations and resource-poor rural areas. Doing

so is a powerful context for teaching and learning, and for motivating and preparing students in WV to pursue STEM educational and career paths that enable them to contribute to their home community.

We build on existing approaches to eliminate gaps between classroom mathematics/science and real world problem solving in engineering [7]. These approaches include developing special skills of modeling more abstract concepts and utilizing a greater number of hands-on activities in the classroom [8]. These approaches benefit all students including those in lower achieving brackets [9]. The benefits of folding authentic contexts into classroom tasks provide an opportunity for greater engagement of students in their own understanding of realistic situations, as well as developing their own scientific reasoning for those situations [10], [11]. Within engineering design based approaches, problems presented to students as contexts for teaching concepts are ill-defined and do not require a specific order of steps to be followed [11]; this in contrast to more traditional methods that necessitate student responses in terms of a single correct solution.

One useful definition of design is, “Engineering design is a systematic, intelligent process in which designers generate, evaluate, and specify concepts for devices, systems or processes whose form and function achieve clients’ objectives or users’ needs while satisfying a specified set of constraints” [12]. Once the design process is mastered students are able to: a) tolerate ambiguity and cycle from divergent to convergent thinking processes in an iterative loop to find a design solution, b) maintain sight of the big picture, c) handle uncertainty, d) justify and make decisions, e) think as part of a team in social processes, and f) think and communicate in several languages of design [12]. Further, engineering design solutions most often relate to real problems in our environment that require manual manipulation of physical elements and materials [13]. Such manipulation gives an opportunity to expose students to authentic problems and guide them through their experience to improved content knowledge [14].

Another important advantage offered by engineering activities is closely connected to students designing their own artifacts [11], and thus further improving their ability to manipulate and navigate changing circumstances and perspectives including actively taking ideas apart and putting them back together based on data driven speculation [7]. Students are actively involved as they create explanations, make predictions, and argue their positions based on evidence they collect [14]. These student proficiencies go beyond low-level skills that are fostered in test-driven curricula and expand to multi-leveled solutions and organized collections of facts and relations among concepts [7], [13].

Our engineering design based approach to teaching content and developing problem solving skill dictates a new role for the teacher. Teachers must shift from an evaluative to interpretive perspective as they move away from guiding students to correct answers and toward emphasizing student exploration and engagement [15]. The teachers’ focus should target encouragement of students’ reflections on their reasoning and interpretations of problem situations [7]. Contrary to current practices of warning students when they take a wrong step in their solution efforts, teachers need to encourage students to focus on their interpretation specific ideas and their connections to the problem at hand [13].

National standards documents have made clear that mathematics is an essential tool for scientific inquiry, and science is a critical context for developing mathematics competence [15], [16]. Mutually reinforcing science and mathematics understandings while teaching either discipline is

a pragmatic and readily available interdisciplinary opportunity [17], [18]. *A Framework for Science Education* gives engineering and technology a greater focus [19]. In our approach, *Common Core State Standards for Mathematics* content domains (e.g., ratios and proportional relationships, statistics and probability), and standards for mathematical practice (e.g., making sense of problems and persevering in solving them, modeling mathematics, choosing appropriate tools) are integrated with science and engineering practices from next generation standards (e.g., “asking questions/defining problems”, “using mathematics/computational thinking”), as well as crosscutting concepts focused on “systems/system models” [20], [21]. Engineering design projects provide extensive opportunities for engaging in practices common to both the *CSSM* and *Framework*: defining problems, constructing explanations, developing models, and attending to precision.

Middle grade CSOs include engineering design in the science framework, but the design process is not easy to learn. This is at least partially because design is a dynamic iterative process rather than a specific skill or piece of content knowledge. Such processes are less often part of traditional teacher training. Therefore, teacher preparation and scaffolding are key to implementation of design based learning and related student learning gains [22]. Project TESAL addressed teachers’ knowledge of pedagogy and their content knowledge [23]. Teachers need to know how students develop understanding of content, how to set significant learning goals, how to select/implement appropriate instructional tasks, and how to assess learning [24] - [27]. In order to successfully impact student learning, teachers must have deep understanding of mathematics and science they teach. Well-designed professional development experiences are integral to developing such knowledge and skills [28]. Project TESAL targets improved mathematics and science content knowledge in an engineering design based approach. We strive to shift students and teachers from being processors of information toward becoming creators of mathematics and science models as tools to help solve societally relevant scientific challenges through design/development of appropriate technologies.

Project TESAL incorporates characteristics of effective professional development in mathematics and science [28] - [33]. Teachers engage in significant mathematics and science content related to the work of teaching as they develop, design, implement, and refine modules to address middle grade content standards and objectives (CSOs) in mathematics, science, literacy, and engineering design. Teachers collaborate with peers and experts in engineering design, literacy, science, and mathematics education as part of a team moving through learning, development, and implementation cycles. This work is aligned with research in that is ongoing, content-focused, embedded in the work of teaching, and aligned with WV CSOs.

Engineering design done well requires an unfamiliar role for many teachers. Teachers must shift from evaluative to interpretive perspectives while moving away from guiding students to correct answers and toward emphasizing exploration and engagement [34]. Teaching practices must foster student reflection on their own reasoning and interpretation of problems [35]. Rather than warning students when they take a wrong step, teachers must use student errors as opportunities to focus on interpreting specific ideas and connections to the problem at hand [36]. For many teachers, this requires different work from that which they have likely experienced before as professionals and as learners. They must move away from transmission models of teaching and, rather, focus more on creating opportunities for students to explore, make sense of ideas, and support students in making connections.

Project TESAL targets development of these new roles for teachers as well as improved mathematics and science content integrated in an engineering design based method [34]. We strive to shift students and teachers from being processors of information toward becoming creators of mathematics and science models [35] as tools to help solve societally relevant scientific challenges through the design and development of appropriate technologies.

### **Description of the Program and Model**

We present a model for professional development followed by data related to its impact. This model emerged from our work with middle school mathematics and science teachers in rural Appalachia. The model utilizes iterative design/redesign to address “the engineering problem” of building teacher content knowledge for teaching mathematics and science. Project TESAL (Teachers Engaged in Science And Literacy) is a three year Math Science Partnership providing proximal context for developing this model. Project TESAL involved two weeks professional development each summer, two days each semester, and classroom observations/support. Teachers were expected to participate all three years and create then implement and refine two lesson plans per year. Project TESAL involved 25 participating teachers and 22 comparison non-participating teachers from four counties with 41% to 67% low-income students, less than 80% highly qualified mathematics or science teachers, and below average mathematics and science test scores in a state well below the national average.

Our model includes the following steps:

Step 1: Identify mathematics and science knowledge gaps utilizing established standardized assessments.

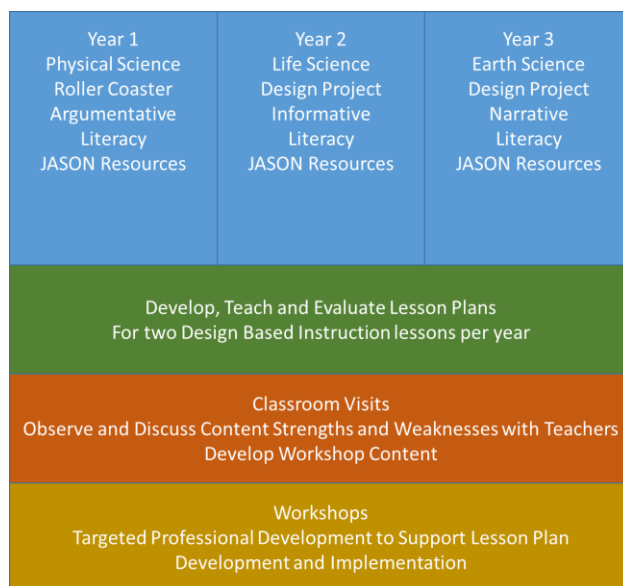
Step 2: Engage teachers in productive struggle as learners in design tasks requiring that knowledge and including mathematical modeling for prediction prior to building and testing designs.

Step 3: Evaluate teacher learning through observations, focus group interviews, and pre-post testing with standardized assessments.

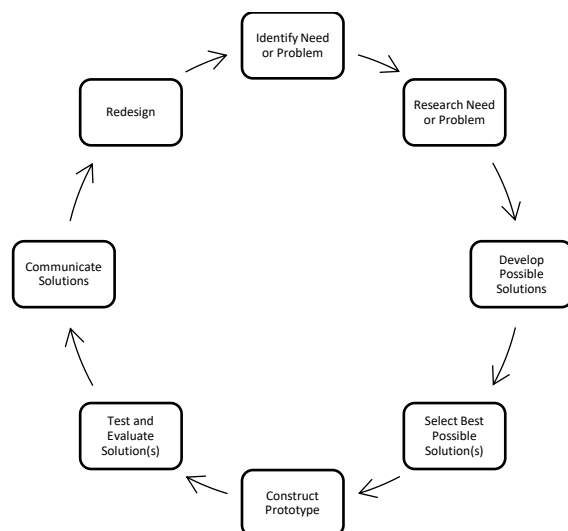
Step 4: Redesign design tasks to further support teacher learning, and require participating teachers to implement design lessons addressing related knowledge gaps with their students.

Project TESAL was a three year program including two weeks professional development each summer, two days each semester, and classroom observations/support (see Figure 1). Each year was themed around a science and literacy foci (Year 1: Physical Science / Argumentation; Year 2: Life Science / Informational; Year 3: Earth Science / Narrative) integrated with grade appropriate mathematics. Participating teachers remained in the program all three years whenever possible and were responsible for creating, implementing, and refining two lesson plans per year. A brief example of our approach: Teachers experienced an engineering design lesson as learners in groups designing and building a paper roller coaster where a marble should take 45 seconds to traverse the track. Mathematical modeling was used to predict time based on coaster design components. We introduced a design process (see Figure 2) and emphasized redesign in this context. Redesign led to a literacy assignment to write an instruction manual on how to build the redesigned coaster. Groups had to build each other’s coaster from that instruction manual. Conversations during the coaster project, content knowledge tests, and later classroom observations highlighted specific content knowledge gaps for teachers. Teachers had

misconceptions about how the marble's mass influences travel on the track, confusing how potential energy, kinetic energy, force, and speed differentiate. We developed new design modules for teachers requiring them to build and test ramps at various heights to launch small and large marbles first to hit a target and later to hit a target with enough force to break a napkin. Measurements from designs with small marbles were used to build mathematical models predicting mechanics with large marbles. Scaffolding for mathematical modelling was provided in an Excel file with embedded equations and dynamic trajectory graph. Models were tested against observations. We knew scientific and mathematical content in the modules would challenge teachers. Teachers individually completed web-based versions and experienced struggle similar to that experienced by their students prior to face-to-face professional development. Teachers completed modules a second time in groups during professional development where peers and content experts provided scaffolding as needed and worked to adapt portions of modules to middle grade students.



**Figure 1: TESAL Program Design and Participating Teachers Building Roller Coaster**



**Figure 2: Design Process Model Utilized with Participating Teachers**

## Data Collection

We focus this evaluation on analysis of surveys (T-STEM), content knowledge tests (DTAMS), and focus groups completed both before and after professional development, as well as teacher-generated engineering design lesson plans and observations as teachers implemented lessons in their classrooms. We conducted individual and focus group interviews during the first and second year of implementation and individual interviews only during the final year. Individual and focus group interviews were audio recorded, transcribed, and analyzed utilizing NVIVO qualitative data analysis software. Teachers submitted their lesson plans, and a member of the project team observed and provided feedback on their teaching of at least one lesson per year.

**Table 1: T- STEM subscales and representative items**

Subscale	Sample Item
Eng. Design Teaching Efficacy/Beliefs (11 items)	I know the steps necessary to teach engineering design effectively.
STEM Outcome Expectancy (9 items)	When a student does better than usual in STEM, it is often because the teacher exerted a little extra effort.
Student Technology Use (8 items)	During STEM instructional meetings (e.g. science class, mathematics class, STEM-related clubs or organizations, etc.), how often do your students...Use a variety of technologies, e.g. productivity, data visualization, research, and communication tools.
STEM Instruction (14 items)	During STEM instructional meetings (e.g. science class, mathematics class, STEM-related clubs or organizations, etc.), how often do your students...Develop problem-solving skills through investigations (e.g. scientific, design or theoretical investigations).
21st Century Learning (11 items)	I think it is important that students have learning opportunities to...Lead others to accomplish a goal.
Teacher Leadership Attitudes (6 items)	I think it is important that teachers ...Take responsibility for all students' learning.
STEM Career Awareness (4 items)	I know...About current STEM careers.

The Teacher Efficacy and Attitudes Toward STEM (T-STEM) Survey is intended to measure changes in teachers' confidence and self-efficacy in STEM subject content and teaching, use of technology in the classroom, 21st century learning skills, leadership attitudes, and STEM career awareness [37]. The 63 items across 7 subscales utilize a 5 point Likert-type response format where higher numbers indicate more positive attitudes or higher self-efficacy in the construct being assessed (see Table 1). Strong reliability and validity data have been reported for representative samples of mathematics and science teachers.

The Diagnostic Mathematics and Science Assessments for Middle School Teachers (DTAMS) serve two purposes: (1) to describe the breadth and depth of mathematics and science content knowledge so that researchers and evaluators can determine teacher knowledge growth over time, the effects of particular experiences (courses, professional development) on teachers'

knowledge, or relationships among teacher content knowledge, teaching practice, and student performance and (2) to describe middle school teachers' strengths and weaknesses in mathematics and science knowledge so that teachers can make appropriate decisions with regard to courses or further professional development [38]. The assessments utilize a combination of multiple choice and open response explanation format items to measure mathematics knowledge in four content domains (Number/Computation, Geometry/Masurement, Probability/Statistics, Algebraic Ideas) and science knowledge in three domains (Physical Science, Life Science, Earth/Space Science). Strong reliability and validity data have been reported for these assessments. We focus here on the mathematics section multiple choice items of the DTAMS where a score of 16 indicates all items correct. T-STEM and DTAMS data were collected prior to engaging in professional development and annually throughout the program. In addition to participating teachers, we collected T-STEM and DTAMS data from non-participating teachers to serve as a comparison group.

## Results and Findings

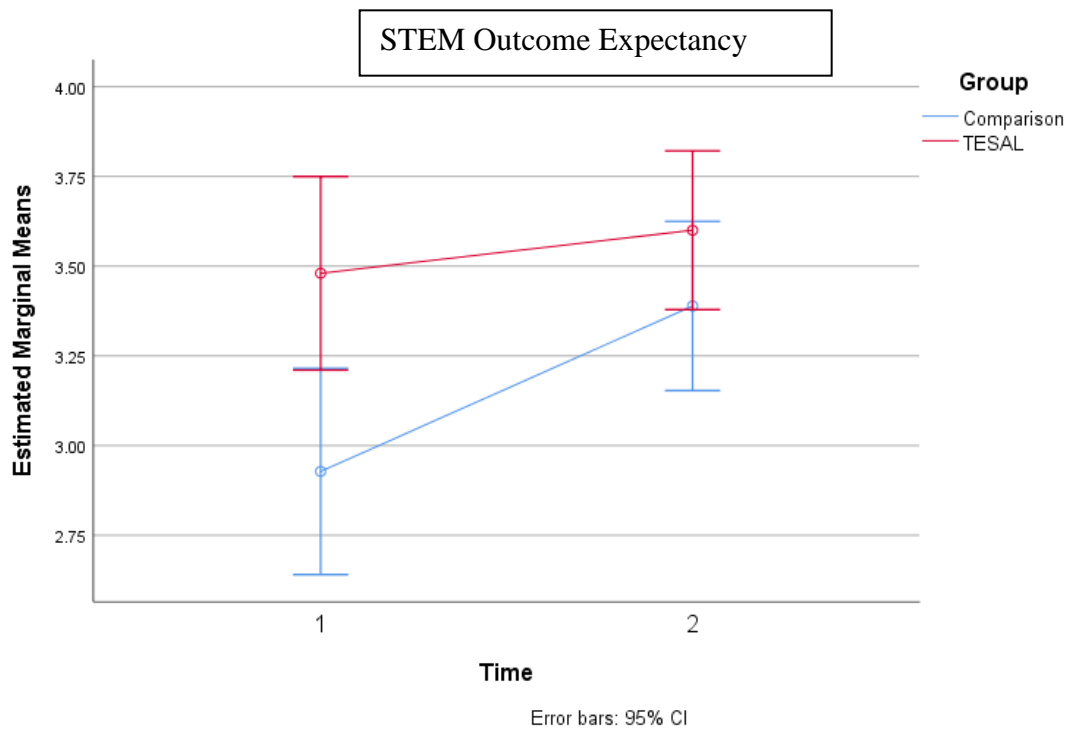
The 25 participating teachers had 1 to 32 years teaching experience (median = 8 years) and considered themselves science educators (n=11), mathematics educators (n=9), special educators teaching math or science (n=4), or technology educators teaching math or science (n=1). All participants had at least a bachelor degree and 17 (68%) were highly qualified based on federal definitions. Mean (standard deviation) T-STEM subscale scores pretest and posttest for participating TESAL and comparison group teachers are provided in Table 2. A 2 (time) by 2 (group) multivariate mixed ANOVA was utilized to examine these scores. There was a significant main effect of time [ $F(7, 39)=7.34$  Pillai's Trace,  $p<.01$ ] and a significant interaction between time and group at the multivariate level [ $F(7, 39)=13.08$  Pillai's Trace,  $p<.01$ ].

**Table 2. T-STEM pre- and post-test descriptive statistics**

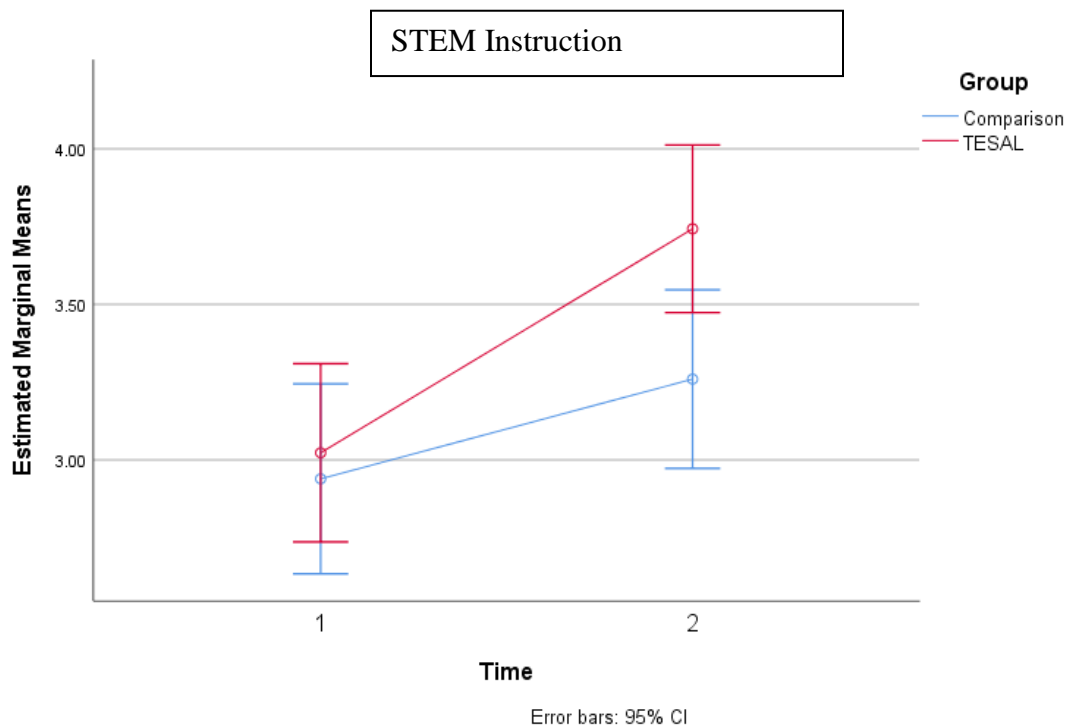
	TESAL pretest	TESAL posttest	Comparison pretest	Comparison posttest
EDTEB	2.91(.71)	3.95(.49)	3.24(1.06)	2.98(.85)
STEMOE	3.48(.50)	3.60(.66)	2.93(.82)	3.39(.38)
STU	2.70(.57)	3.29(.83)	3.22(.57)	2.80(.72)
STEMI	3.02(.56)	3.74(.70)	2.94(.85)	3.26(.63)
CLA	4.61(.35)	4.65(.40)	3.45(.90)	4.55(.41)
TLA	4.58(.38)	4.76(.32)	4.57(.40)	4.54(.34)
STEMCA	3.37(.82)	4.30(.57)	4.30(.86)	3.56(.90)
n	25	25	22	22

*Note: Numbers in parentheses are standard deviations. Subscales: EDTEB=Engineering Design Teaching Efficacy and Beliefs, STEMTOE=STEM Teaching Outcome Expectancy, STU=Student Technology Use, STEMI=STEM Instruction, CLA=21<sup>st</sup> Century Learning Attitudes, TLA=Teacher Leadership Attitudes, STEMCA=STEM Career Awareness.*

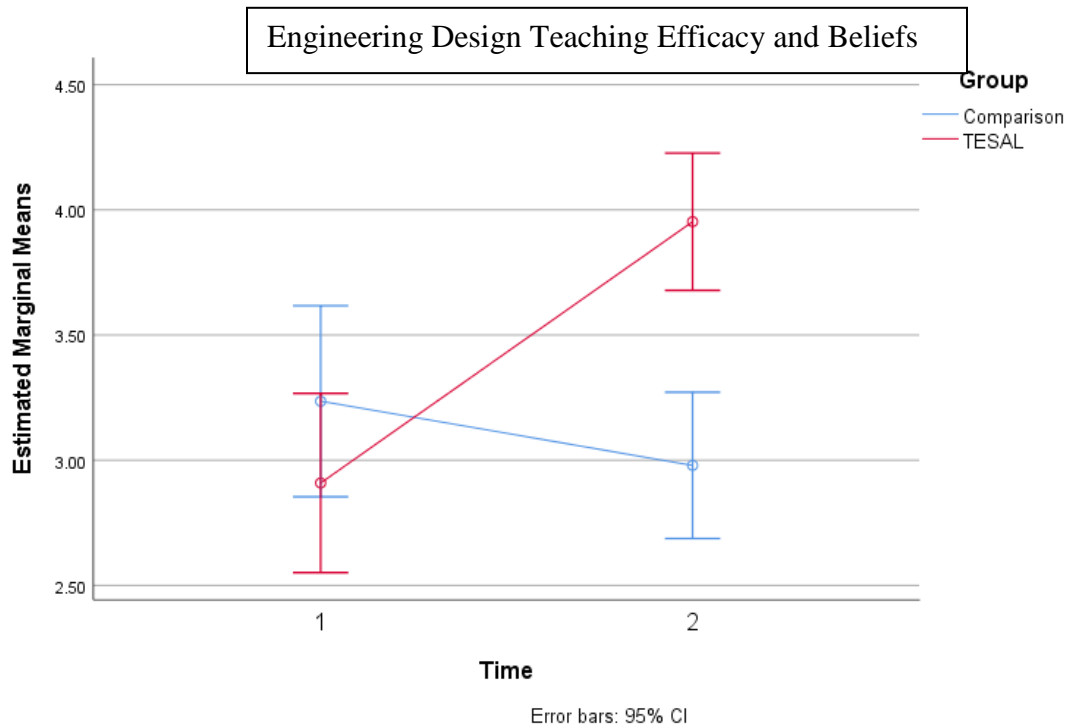




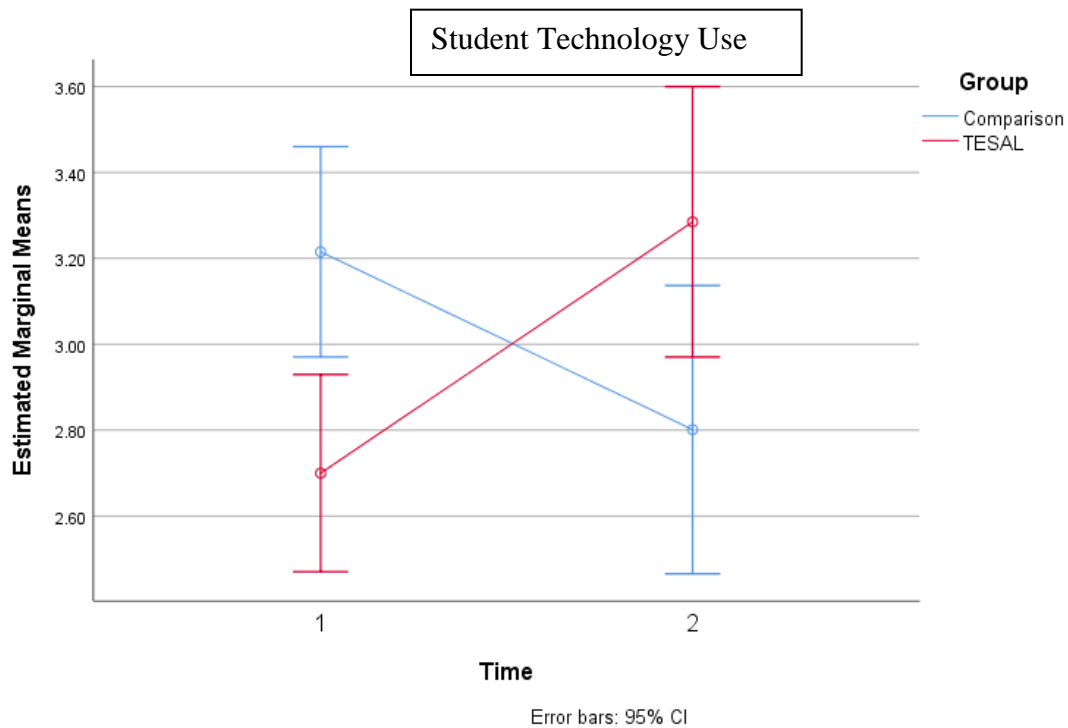
**Figure 3.** *Estimated marginal means on STEM Outcome Expectancy relative to significant change over time with no significant difference between groups or interaction effect.*



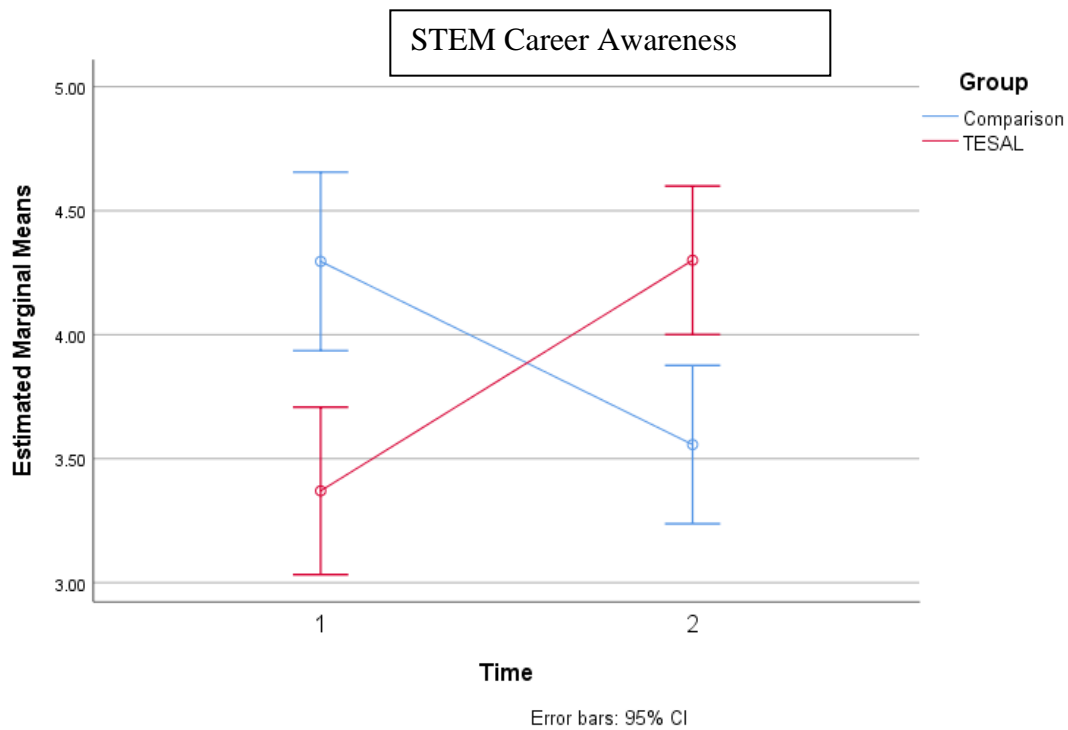
**Figure 4.** *Estimated marginal means on STEM Instruction relative to significant change over time with no significant difference between groups or interaction effect.*



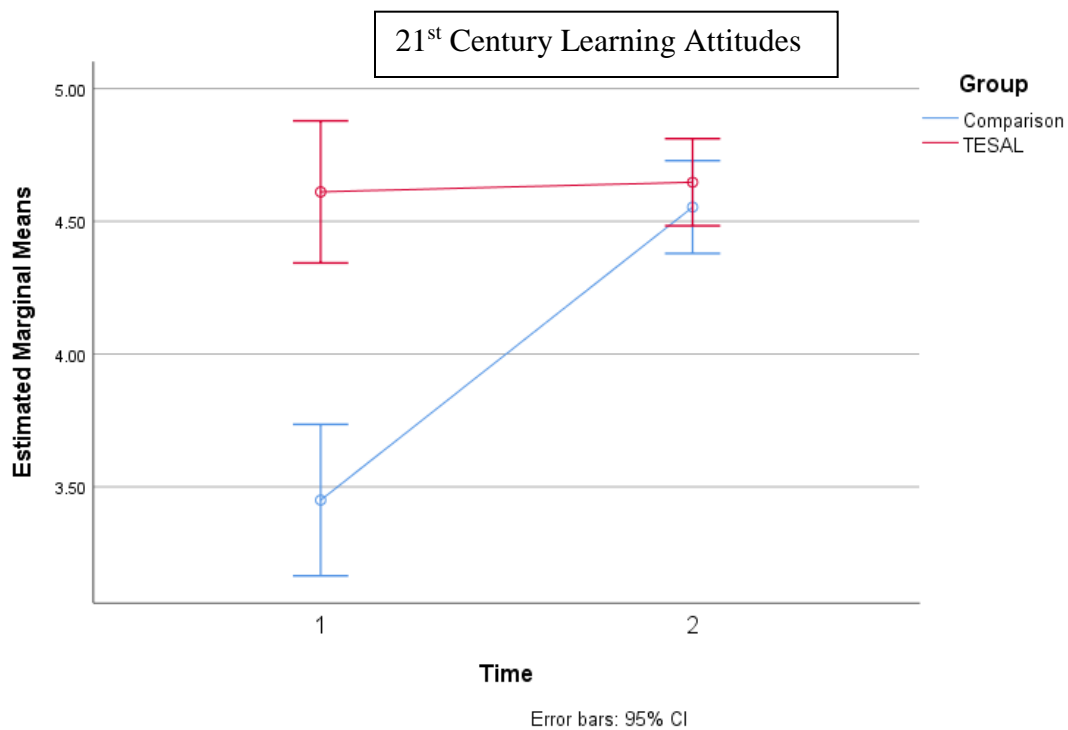
**Figure 5.** Estimated marginal means on Engineering Design Teaching Efficacy and Beliefs relative to significantly different change across time for each group.



**Figure 6.** Estimated marginal means on Student Technology Use relative to significantly different change across time for each group.



**Figure 6.** Estimated marginal means on STEM Career Awareness relative to significantly different change across time for each group.



**Figure 8.** Estimated marginal means on 21<sup>st</sup> Century Learning Attitudes relative to significantly different change across time for each group.

Significant multivariate effects were followed up with univariate analyses of their components. Significant univariate main effects that did not have corresponding interaction effects included STEM Outcome Expectancy [ $F(1, 45)=9.31, p<.01$ ] and STEM Instruction [ $F(1, 45)=24.19, p<.01$ ], which are visualized in Figures 3 and 4, respectively.

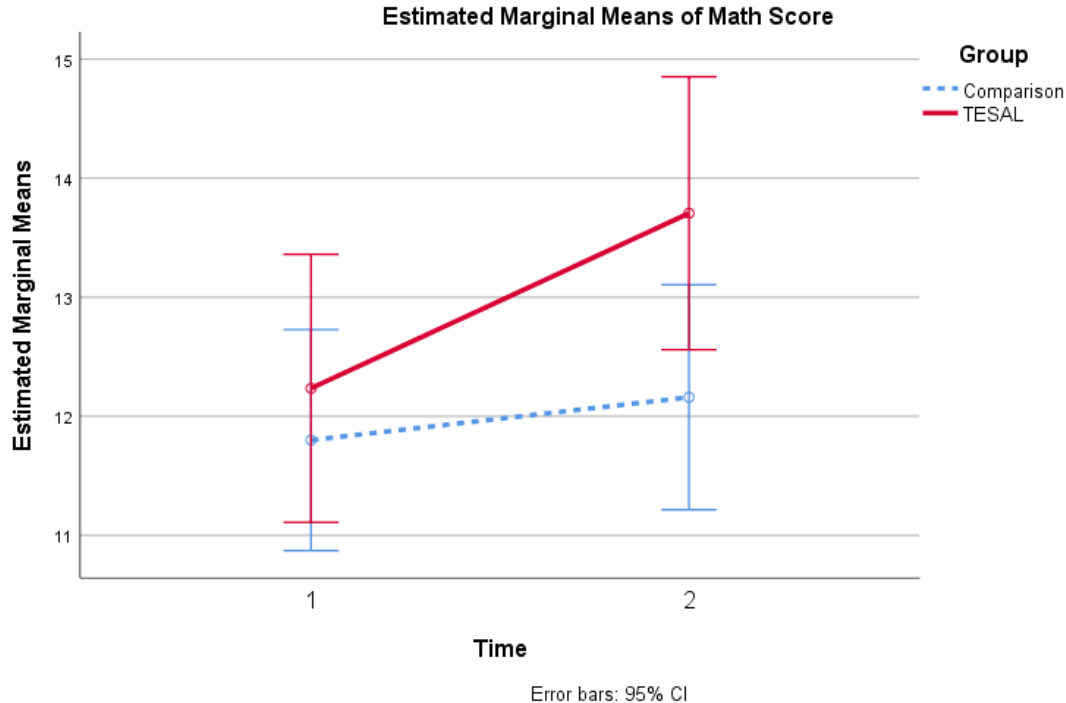
Significant univariate interaction effects included Engineering Design Teaching Efficacy and Beliefs [ $F(1, 45)=33.61, p<.01$ ], Student Technology Use [ $F(1, 45)=14.93, p<.01$ ], STEM Career Awareness [ $F(1, 45)=38.35, p<.01$ ], and 21<sup>st</sup> Century Learning Attitudes [ $F(1, 45)=30.10, p<.01$ ] (Figures 5 through 8, respectively). In three of four cases, participating teachers increased while comparison teacher reduced or maintained. In one case (21<sup>st</sup> Century Learning), participating teachers maintained a high value while comparison teachers increased over time.

Mean (standard deviation) DTAMS Mathematics subscale scores pretest and posttest for participating TESAL and comparison group teachers are provided in Table 3. A 2 (time) by 2 (group) multivariate mixed ANOVA was utilized to examine these scores. There was a significant main effect of time [ $F(1, 40)=7.34$  Pillai's Trace,  $p<.01$ ], no significant main effect of group, and no significant interactions at the multivariate level (see Figure 9).

**Table 3. Math score pre- and post-test descriptive statistics**

	Pretest	Posttest
TESAL (n=17)	12.24(1.95)	13.71(1.93)
Control (n=25)	11.80(2.50)	12.16(2.58)

*Note: Numbers in parentheses are standard deviations.  $n<25$  due to missing data.*



**Figure 9. Estimated marginal means on 21<sup>st</sup> Century Learning Attitudes relative to significantly different change across time for each group.**

End of summer focus group discussions with working teams of participating teachers were recorded and transcribed for coding and analysis in NVIVO, a qualitative data analysis software package. Discussion prompts focused in year 1 on perceived impact on teachers, anticipated impact on teaching practice, and anticipated challenges, and in year 2 focused on changes in teacher perspective and experienced impact on teaching practice and student learning.

During year 1 focus group interviews, the teachers were asked, "How has this professional development impacted you?" Their responses suggested positive impact that in some cases exceeded their expectations. Participating teachers spoke of how the professional development facilitated reflection and rethinking of their practice in the classroom. For example, one participant stated, *"This is good for me, it has made to try to think about the lessons that I do give my students and made me try and incorporate a little more hands on activities."*

Participating teachers also addressed how the professional development aided in *"expanding their horizons"* and *"overcoming inhibitions about integrating math into science"*. They described the potential of integrating Math and Science in their classroom as suggested by these responses.

*It has opened up a whole new world of integrating math and science. I have done my projects in my room before but integration of the two content areas would really benefit the kids, [and] that is what we are here for. So I think learning from the science teachers is awesome. Lots of opportunity.*

*I would say that it really enhanced my practice because I get a greater understanding of grading math and science with everything I do... like all the different activities but not just projects for science learning but also to combine those together so the kids understand it deeper.*

However, when asked, "How do you expect what you learn in this professional development to impact your work as a teacher this coming year?", some teachers expressed concerns about the lack of resources such as limited lab space in their school and availability of supplies as a hindrance to them using what they learned at the professional development.

Others addressed concerns around planning their lessons to ensure that the content standards and objectives were integrated well, as well as general time management concerns as noted by one participant's responses.

*I think just by incorporating more guided inquiry lessons it would probably save time on the back end. When you are not doing those sorts of lessons, you end up having to reteach some of [the] kids who did not pick it up the first time. So I mean I think it should hopefully save time and help us move a little faster.*

The teachers also emphasized the value that they gained by familiarizing themselves with standards across disciplines as a step toward greater integration across content areas. One participant noted:

*I think science teachers get science standards, math teachers look at math standards, and I think that was one of the reasons I was so skeptical about putting lots of math in my science lessons because I did not know what they [students] should already know. And I was always so scared to go above their head and I realized that that is what they should already be doing and knowing.*

The teachers also shared their excitement about trying the lessons that they created during the professional development, collaborating with teachers in their schools, and helping students make connections across disciplines.

*We do it as teachers; that is the way that we learn. Now we have to help construct those connections for our students. The importance of collaboration for students, like we are collaborating now. We learn so much from each other, how much more so will students learn from each other in the classroom.*

To accomplish participant checking, preliminary description of themes from year 2 were shared with the 17 teachers who participated in focus groups that year (see Table 4). All teachers indicated these themes described them “very well” or “extremely well”, and no teacher had substantive additions or contradictions.

The most relevant theme was tied to experiencing design lessons as learners. Teachers talked about productive struggle with critical mathematical and science content, understanding the process deeply enough to guide their students effectively, the importance of redesign, and how this professional development experience was dramatically different and more valuable than others they had experienced.

Additional qualitative content analysis of focus groups and individual interviews in year 3 focused on more deeply understanding participants’ perspectives related to productive struggle. Following a grounded theory approach [49] revealed five interrelated themes organized as interrelated ways of completing the stem, “In Project TESAL, productive struggle is...”

...Experiencing as Students

...Tolerating Ambiguity and Giving Students Autonomy

...Failing and Redesigning: It’s okay to struggle, It’s okay to fail

...Facing Content Gaps and Taking Students Deeper

...Working Together

Briefly, our participants found experiencing productive struggle as students, as learners themselves faced with engineering design, mathematical modeling, and other practices we presented, to have multiple benefits. These included allowing them to see the classroom “*in the shoes of students and in the shoes of teachers*”; allowing them to “*experience productive struggle because we are given very few directions*” and to “*step back and let them [students] do that. Let them figure it out, within their group*”. Participating teachers came to understand that failure and redesign are critical components of successful learning, and how to transmit that understanding to their students. In order to get there, participating teachers had to open themselves up to facing their own gaps in content knowledge, which in turn gave them the confidence to be “*better risk takers*” and “*not as hesitant to take [my] students deeper than actually is even what is expected of them*”. They found that “*Productive struggle would be a lot more of the struggle instead of productive if you didn’t have groups, and so, but the group work isn’t going to be effective if it’s not a group-worthy task as we call it... And a lot of those tasks do involve some sort of engineering design where they come together as a group and search for a solution and look at their solution, compare it to the criteria and go back and redesign. So it all kind of bleeds into each other, they all support each other.*”

**Table 4: Focus group themes.**

Barriers anticipated from last year for the most part were not actually problematic. In fact some teachers described finding this approach actually saved time because multiple CSOs are addressed in clusters rather than one at a time. In a couple cases, perceptions from their colleagues were challenging and in several cases their colleagues were very supportive. The main barrier turned out to be physical space to house the student projects themselves.
Anticipated student impact on motivation and learning did occur. Teachers universally agreed on this. Impact on motivation was most pronounced for inclusion students and those who had in the past been difficult to engage. "Advanced" students often struggled with the lack of directions but teachers all agreed this was productive struggle even if their students may not have seen it that way. In some cases teachers experienced pushback from parents who thought their children should be learning the way that they had learned.
Most teachers shared this approach with other teachers in their building, most often with their team members and curriculum coaches. In a few instances, colleagues or administrators were skeptical at first but then saw the benefits of the approach in how students responded. There was unevenness in support experienced from administrators ranging across consistently positive from the start, to starting skeptical but beginning to be won over as they saw impacts, to not being present or engaged in teachers classrooms to even see what was happening.
Teachers perspectives coming in to this year were generally more focused as compared to the first year. Most and maybe all teachers experienced productive struggle, especially in year 1 and 1st part of 1st week this summer. This was often very uncomfortable for them, to the point that many considered dropping out of the program, especially when it was not clear to them why we were asking them to do things. They felt their struggle sometimes went beyond what was productive. There was general consensus that now they understand the big picture of experiencing struggle as learners so that they can guide their students through the process more effectively. They almost uniformly see great value in this and talked at some length about how this PD is dramatically different and more valuable than most other PD they have experienced.
Many, but not all, teachers described the approach spreading in at least one of the following ways. A. To other lessons beyond those they were required to do. B. To other teachers in their teams. C. To other teachers across most or their entire school.
Suggestions for future: -More detailed feedback on lessons, both in planning stages and in observations. They want both validation and constructive criticism. -Some more communication on "why" and the big picture from our perspective, although they understand there are good reasons for not explaining everything.-More connections of specific math CSOs to the science and engineering. -More opportunity for them to see each other's lessons. One suggested videotaping observations and showing those to the group, or even them visiting each other's rooms although that may not logistically be possible. -Scaffolding their presentation of this approach to their colleagues, possibly with a PowerPoint that includes general description of approach, literature base for its effectiveness, and evidence from multiple teachers in this program of its effectiveness. They could then share this with their entire school at one of their early out PD afternoons. -Scaffolding development of parent involvement and buy-in in some way.

Participating teachers were required to develop two engineering design lessons and to implement at least one of those lessons in their middle school classroom during each fall from 2015 to 2017. Observations were conducted in 22 of the 25 teachers' classrooms during the fall with 20 of those observations occurring during implementation of a lesson developed specifically for Project TESAL. Five teachers self-reported regarding their implementation of engineering design lessons when observation was not possible.

As two examples, the "Gingerbread House" engineering design lesson plan and rubric is provided in Figure 10, and "Comparing Cell Phone Companies" lesson is provided in the

Appendix. Engineering design lesson plans developed by participating teachers covered the following topics:

- Design a Roller Coaster (3 science educators, 2 math educators, 2 special educators: 4 schools),
- Design process to make the perfect hard-boiled egg (1 science educator, 1 special educator: 2 schools)
- Design an ice cream cone business (1 math educator)
- Design a healthy meal plan from McDonalds (1 technology educator)
- Design a tall and safe gingerbread house (1 math educator; see Figure 10 below)
- Design a skating ramp (1 math educator)
- Design a shoe box that can be made from a single piece of cardboard (1 special educator)
- Design a scalable process to make a non-Newtonian fluid Oobleck (1 math educator, 1 science educator: 1 school)
- Use the design process to select a cell phone provider for a medium sized company (1 math educator; see Appendix)
- Design an air-bag (1 science educator)



Interestingly, we saw co-teaching and collaboration across content areas and across grade levels, on the Roller Coaster and on the Non-Newtonian Fluid Oobleck engineering design lessons. In addition, several teachers delivered more than the required number of engineering design lessons.

A number of themes emerged from classroom observations. Several teachers commented on how design based instruction increased the engagement of all students and that an increase in engagement of special education students was observed. One special education teacher in a low performing school first facilitated a design project to make a perfect hard-boiled egg. *“Students went home and asked their parents and grandparents to teach them to boil an egg and practiced with them. They then sat in the classroom carefully watching the water boil timing every step. I had never seen that level of engagement with this group of students. It has made me a believer”*. The same teacher then developed a second lesson to design a shoebox from a single piece of cardboard. The students made prototypes and a final scale model design. Two special education students who previously had limited success in the classroom designed an innovative triangular prism design – *“whenever they are struggling in class now I remind them of their success on the shoe box project. I am working on design projects to use in the Spring Semester with them now.”*

A number of the math teachers commented on challenges letting go and allowing students to work on more open-ended problems and how rewarding it was to see them succeed. In one math classroom, 6<sup>th</sup> grade math students performed measurements of the distance around a curved paper roller coaster built by 8<sup>th</sup> grade science students and the time to traverse the track. Students then calculated the average speed. Students performed repeated runs and calculated statistics for the average and spread of the data. Students were engaged for the entire lesson period and participated actively. Two months after facilitating the roller coaster lesson, this teacher decided



to develop her own lesson on building a gingerbread house (see Figure 10). She expressed that she was nervous to try something so hands on but had gained confidence from the previous hands on lesson. During observation the students were engaged and in particular she identified that one of the students in class who does not normally participate was engaged. We observed him taking the lead on measurement tasks and interacting well with other members of his group.

In addition to successful implementation by teachers in their classrooms, it was evident that teachers felt able to ask questions about content they were unsure of. The most common content areas that teachers felt uncomfortable with were the conversion of potential energy to kinetic energy and how to measure energy. We developed a series of design challenges to attempt to assist in this area, which we report on elsewhere.

### Gingerbread House Design

**Task:** Your task is to design and build a gingerbread house that has a solid structure and creative design using specific criteria.

**Day 1:** Design a gingerbread house on paper that meets the following criteria:

- Minimum area of 15 square inches
- Solid structure
- Functional roof
- One entrance
- Tallest house on Gumdrops Ave.

**Day 2:** Build the gingerbread house that your group designed using the materials available to you. Be creative!

Gingerbread House Fact Sheet

Height: \_\_\_\_\_

Area of the base: \_\_\_\_\_

Surface area: \_\_\_\_\_

**Day 3: Structure Assessment**  
According to meteorologists, on Thursday, December 17, 2015, Gumdrops Avenue will experience blizzard like conditions accompanied by a magnitude 4 earthquake.

**Day 4: Re-design**  
Does your gingerbread house need re-designed? Below, design a plan for improving your original design. How will you improve the structure of your existing gingerbread house?

### Gingerbread House Design Project Rubric

	Professional Engineer 10 points	Apprentice 6-9 points	Engineer In Training 2-5 points	Needs additional Training 1 point
<b>40 points possible</b>				
<b>Structure</b>	House has a sturdy structure, entrance and functional roof and a clear design	House has a sturdy structure and a clear design	House stands, but has a sloppy design	House does not stand and candy has no order
<b>Creativity</b>	House shows personality and creativity with a clear design	House shows personality and creativity	House shows some personality and creativity	House is lacking personality and creativity with poor design
<b>Collaborative Effort</b>	Outstanding collaboration with peers and respectful behavior	Moderate collaboration with peers and respectful behavior	Very little collaboration and respect	Lacks collaboration and respect
<b>Participation</b>	Maintained focus on the task, shows attention to detail, has a positive attitude and assists peers	Maintained focus on the task and has a positive attitude	Very little focus on the task and moderate effort	Not focused on the task, complaining and no effort. Eating candy

Figure 10: Gingerbread House Design lesson and rubric

## Discussion

Project TESAL successfully recruited a diverse group of mathematics, science, and special educators, and engaged them in professional development they found valuable. Project TESAL increased participating teachers' confidence to teach engineering design, their students' use of technology, and their knowledge of STEM careers significantly more than a non-participating group of teachers from similar schools. Participants valued being active participants in learning, opportunities for collaborating with peers and outside experts around the work of teaching, focusing on content across subjects and students' learning of that content, and the sustained nature of support and feedback through Project TESAL. These strengths align with best practices

for professional development, especially on math-science integration and engineering design [16] - [18], [28] – [32].

Valid and reliable assessment of teacher content knowledge coupled with available content expertise of project personnel is a strength that gives rise to a challenge in determining how to address and scaffold content needs of prospective groups. How much do middle school mathematics teachers need to know about science, and how much do science teachers need to know about mathematics, in order for them to meaningfully plan integrated instruction? Many teachers are uncomfortable opening their content knowledge gaps to remediation. In the context of somewhat low content knowledge scores and specific content deficiencies, especially outside of teachers' primary content area, how do we address content needs in safe and authentic ways? We found sustained engagement with our teachers critical, and teachers were more open to remediating gaps in the context of design projects focused on similar gaps their students are likely to have; gaps that just happen to overlap with content knowledge teachers need to develop more deeply themselves.

A compelling issue across focus group themes was productive struggle—in particular, that of teachers—seen in their comments about themselves, student effects, and parent responses. Project TESAL teachers experienced productive struggle authentically, and their misconceptions were similar to those of their students. Design based learning provided an experimental framework that was familiar to them and enabled further, richer experimentation that was targeted at understanding misconceptions and could be adapted for use in their classrooms.

Productive struggle is a key feature in learning that is conceptual, robust, and transferable [24], [39], [40]. The idea of struggle leading to learning is not new and research supports the benefits of learning through some form of struggle [24], [39], [40] – [46]. In mathematics, productive struggle has been noted as a fundamental “feature of teaching that consistently facilitates students’ conceptual understanding” and is highlighted as an essential practice for strengthening the teaching and learning of mathematics” [24], [47]. Yet is it difficult to understand and implement. Many teachers have not experienced this as a learner themselves. Part of that difficulty is the dominant cultural view of mathematics and science as only for “some people” or as a static body of knowledge that must be learned rather than created. Such beliefs often lead to instructional practices that tend to remove struggle from students rather than leverage it for learning [48]. Productive struggle hinges on instructional tasks that investigate content and create knowledge in meaningful ways. Therefore, instructional approaches that engage students in productive struggle also challenge existing notions of what it means to do mathematics and science; this can be uncomfortable for parents, other teachers, administrators, and some students. Making these experiences and related difficulties explicit may support teachers developing understanding of how to effectively engage their students in productive struggle.

A key strength of Project TESAL is that the collaborative project team involves WV Regional Education Service Area personnel who have authentic long-standing relationships with key schools and teachers in the area working closely with university faculty who have deep engineering, science, and mathematics content knowledge as well as education pedagogy, curriculum resource, literacy, and educational evaluation/research expertise. The fact that the project team brings together individuals with expertise in a wide variety of areas, all relevant to the success of the project, provides opportunity to model the benefits of cross-disciplinary collaboration. Each individual has the opportunity to draw on her expertise to contribute to the

work toward project goals and also gain new understandings from working with others. This sort of team is quite unusual in the mostly rural Appalachian area where we work.

Part of the Project TESAL vision is recognition that the engineering design process applies authentically to design and redesign of professional development, design and redesign of teachers' instruction, and to engineering design projects for students. Through all these applications of engineering design, productive struggle, evaluation, and redesign in the context of societally relevant scientific challenges are critical components that facilitate continuous quality improvement. The project team strives to both explicitly model and scaffold this mindset for and with participating teachers.

## **Conclusion**

The significance in this work lies in understanding how to effectively support teachers to buy-in to the value of productive struggle and develop a repertoire of instructional practices that effectively support their students engaging in and benefiting from productive struggle. In the data we have presented here, it is evident that our participants' experiences with struggle as learners have supported their developing understandings of the potential benefits of engaging their students in this practice. We have, over the last three years, developed a set of practices that helped move teachers who engaged in Project TESAL away from trepidation with engineering design, hidden gaps in their content knowledge, and negative views toward professional development and toward comfort with ambiguity, approaches to confront and improve gaps in content knowledge, and engaging a professional support network. Discussion of our practices, including sustained engagement, teachers experiencing engineering design as learners, and scaffolding through challenging content, has great potential to improve professional development and thereby positively impact student learning in STEM.

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## Appendix

8th Grade  
**Grade Level**

2 -4 days  
**Date(s) of Activity**

Comparing Cell Phone Companies  
**Title of Activity**

### **Standards Addressed:**

#### **Math:**

8.F. 2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

**Science Literacy:** Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table)

#### **Engineering:**

S. 6-8.ETS.2	Evaluate competing design solutions using a systematic process to determine how well they meet the criteria and constraints of the problem
S. 6-8.ETS.4	develop a model to generate data for iterative testing and modification of a proposed object, tool, or process such that an optimal design can be achieved.

### **Learning Goals:**

1. **Engineering** - Students will determine criteria and constraints for their analysis.
2. **Math** - Students will construct functions from real world scenarios.
3. **Math** - Students will represent a function from a graph.
4. **Math**- Students will compare two or more functions.

### **Overview:**

**Key Question:** How do you construct and compare functions from real world scenarios?

**Key Task:** Develop a quantitative report analyzing cell phone companies based on their plans. Given: You and your team are part of a research company working for a business magazine. The company wants you to choose 3 cell phone plan providers and compare at least 3 different plans from each company. You will need to decide which plans are the “best” based on criteria that you develop. Be sure to list the criteria (at least 2 expectations) determining the “best” plan. You will also need to list the constraints (at least 2) used when comparing the plans. (ex. Contract length? ) Finally, you must use at least one equation and one graph.

**Prior Math Understandings and Skills:**

Students will be able to identify functions; Students will be able to represent functions using equations, tables, and graphs; Students will be able to interpret a function based on its graph.

**Assessment:**

1. Assessment task for <b>Learning Goal 1</b> (performance-based task? Presentation? Worksheet? Response to oral questioning?)	Performance based task – create a quantitative report comparing cell phone plans and companies
2. Assessment task for <b>Learning Goal 2</b> (performance-based task? Presentation? Worksheet? Response to oral questioning?)	3 functions should be written based on information from the cell phone plan (embedded into performance – based task)
3. Assessment task for <b>Learning Goal 3</b> (performance-based task? Presentation? Worksheet? Response to oral questioning?)	1 graph presented (embedded into performance – based task)
4. Assessment task for <b>Learning Goal 4</b> (performance-based task? Presentation? Worksheet? Response to oral questioning?)	Comparison of cell phone plans (embedded into performance – based task, quantitative report)

**Evidence of Student Understanding:**

<b>Learning Goal 1: Students will determine criteria and constraints for their analysis.</b>			
Misconceptions	Beginning Understanding	Proficient Understanding	Sophisticated Understanding
Confusing the difference between criteria and constraints	Students can list some information that may or may not be relevant to the project's outcome	Can list 2 criteria and 2 constraints related to the project	Students will list more than 2 constraints and criteria that will determine the best plan and all items listed will be relevant to the plan being the “best”
<b>Learning Goal 2: Students will construct functions from real world scenarios.</b>			
Misconceptions	Beginning Understanding	Proficient Understanding	Sophisticated Understanding
Understanding of what makes a relation a function	Students can make a table or chart to get values for their equation.	Students can identify $y = mx + b$ as the starting point or know what the slope and y intercept is, but cannot complete the equation for the function	Students can construct a function symbolically or algebraically.



<b>Learning Goal 3: Students will represent a function from a graph.</b>			
Misconceptions	Beginning Understanding	Proficient Understanding	Sophisticated Understanding
Confusion between the x and y intercept,	A coordinate grid is used but the function is not represented accurately	A function is represented accurately but there may be a flaw in a point graphed or labeling.	The student accurately labels the graph and precisely draws the function.
<b>Learning Goal 4: Students will compare two or more functions.</b>			
Misconceptions	Beginning Understanding	Proficient Understanding	Sophisticated Understanding
Students confuse the y intercepts and slopes when comparing	Students can compare two functions when presented in graphs and both functions are shown.	Students can compare two functions if the student changes them to the same representation (2 graphs, 2 tables, etc)	Students can compare multiple functions from multiple representations.