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Who needs the method of sections and the method of joints? Just pick a strategy and define your system!

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Who needs the method of sections and the method of joints? Use a unified problem-solving approach!

Abstract

Statics courses can sometimes give students the misconception that there are many different approaches to solving statics problems, such as the “method of sections” or the “method of joints.” Students may also believe that trusses, frames, and machines are all fundamentally different and that they therefore require different approaches. As a result, many students believe that there is a “right” approach to use in solving a particular problem and that they need to remember it and use it. In this paper, we present a simple, unified problem-solving approach for solving all statics problems, including particle problems and rigid body problems; the calculation of internal forces or external forces; and problems involving a frame, a machine, or a truss. This approach is also applicable to solving problems in other courses such as Strength of Materials and Dynamics. In this approach, the first step in solving any problem is to articulate a “Strategy.” This simple step requires the students to take a few moments to reflect on the problem and write down a strategy rather than trying to pattern match or “find the right equation.” If the strategy is Newton’s 2nd law, which it often is in Statics, then the next step is for students to “Choose a System.” Students are required to define the system by drawing a dotted line around it or by stating the system in words. Once a system is chosen, and only after it is chosen, then students draw a free-body diagram (FBD) for the system. The mnemonic BREAD (B-Body, R-Reaction forces, E-External forces, A-Axis, D-Dimensions) has been found to be very helpful in teaching students how to draw complete and accurate FBDs. In this paper, we will present this problem-solving approach with a specific focus on defining the system and drawing a complete FBD.

Introduction

Statics is typically the first engineering course students encounter, and it is often the first exposure students have to engineering problem solving. Statics is also one of the most foundational courses in the mechanical engineering curriculum; students will continuously draw upon the skills they learn in Statics throughout their engineering education. Students with a strong understanding of statics will likely have an easier time with related and more challenging concepts in subsequent courses. Specifically, Statics has been shown to be an effective predictor of how students will perform in Dynamics [1], and instructors in Capstone Design courses often state that lacking an understanding of statics concepts hinders achievement in design [2]. It is therefore worthwhile to examine common teaching practices in Statics and to develop instructional methods that will enable students to confidently apply the skills they learn in Statics to a wide variety of engineering problems.

In many courses – Statics as well as others – it is common to present a solution process as tailored to a particular "type" of problem; for example, in Statics, students are taught separately a "method of sections" and a "method of joints" to solve for unknown forces acting on the members of a truss. Although this problem-solving structure may seem more procedurally simple for students, it creates the impression that the two types of problems are inherently different and must be solved differently, when in reality, the same foundational principles underlie both, and the choice of which method to use is simply one of convenience. This may teach students to rely heavily on identifying the "type" of problem they're trying to solve, as well as an explicit set of steps associated with that problem type, rather than on fundamental engineering concepts. Therefore, we believe it is beneficial to present to students a more universally applicable problem-solving framework that is can be used for solving many different types of engineering problems.

The systematic problem-solving approach presented in this paper is intended to free students from a reliance on limited problem-solving approaches that they may perceive as being applicable to only a small number of circumstances. This approach emphasizes a few basic steps which can be applied to a wide variety of problems in statics or in other courses. Several textbooks use a systematic, structured problem-solving approach, including Sheppard and Tongue [3], Plesha, Gray, and Costanzo [4], and more recently, Beer and Johnson et. al. [5]. In the engineering education literature, a variety of systematic approaches to problem solving have been proposed, including the Wankat and Oreovicz Model [6], the Plesh, Gray, and Costanzo Model [7], the Litzinger, Van Meter, Wright, and Kulikowich Model [8], and the Mettes, Pilot, Roossink, and Kramers-Pals Model [9]. A good summary of these approaches can be found in [10]. These models have many of the same steps, such as "list knowns and unknowns," "identify assumptions/constraints," "determine principles involved," "draw a figure," etc. The approach discussed in this paper is not as detailed or comprehensive as these approaches, but because it is relatively concise compared to these other approaches, it is the authors' hope that it is more useful for students and can be easily implemented by them. Although the paper will focus primarily on solving problems in Statics, the general framework is applicable to other courses such as Dynamics, Strength of Materials, and Thermodynamics.

A systematic approach to solving problems

In this paper, we propose a unified problem-solving approach rather than approaches such as the "method of sections" or "method of joints." A flowchart of this approach is shown in Figure 1. This figure is generally not presented to students because it can be a bit overwhelming when first seen. The primary focus of this figure is on presenting the process when the strategy is applying Newton's 2^{nd} law (which is often the case in Statics), while other principles that may need to be used are included in the single block called "Apply a different strategy." The boxes in red indicate branch points, making it clear that this is not a linear process. The two key steps we will focus on in this paper are identifying a strategy and defining the system.

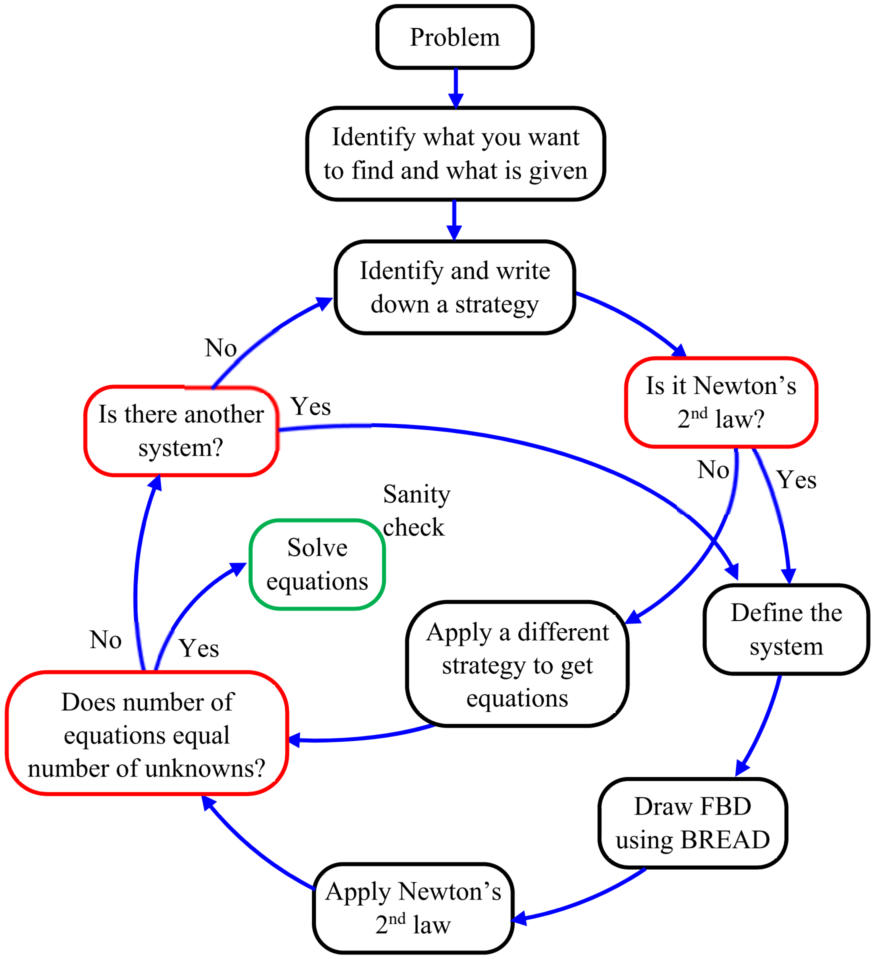


Figure 1 – Statics problem-solving approach flowchart

Identifying what you want to find and what is given

The first step in solving any problem is to clearly identify what is given and what needs to be determined. In most Statics books, unless the problem is written poorly or is intended to be a design problem, this information is typically found in the problem statement.

Identifying a strategy

This is one of the key steps in this approach. The purpose of this step is to force the student to reflect on the problem and think about principles rather than asking themselves the dreaded question, "What equation do I use?" In Statics, sometimes students want to draw a FBD or use Newton's 2^{nd} law for every problem, but this is not always the correct approach. Sometimes the correct strategy would be to use the definition of moment or the definition of the 2^{nd} moment of area. A listing of common strategies used in Statics is shown in Table 1. This table also contains some common strategies used in Dynamics and Strength of Materials.

Table 1 – Sample strategies for several engineering courses.

Course	Sample Strategies
Statics	• Newton's 2^{nd} law • Definition of moment or couple • Definition of centroid • Definition of resultant force (distributed loads, for example) • Definition of 2^{nd} moment of area or centroid • Geometric constraints
Strength of Materials	• Newton's 2^{nd} law • Definition of stress or strain • Stress-strain relationships • Deformation • Mohr's circle
Dynamics	• Kinematics • Newton's 2^{nd} law • Work-energy • Impulse-momentum

From Figure 1, it is clear that if Newton's 2^{nd} law is not the appropriate strategy, or it has already been applied to every system in the problem, then additional equations need to be determined using information in the problem statement or one of the additional strategies shown in Table 1. If the strategy to be used is Newton's 2^{nd} law, the next critical step is to clearly **define a system**.

Defining a system

We require students to define a system by drawing a dotted line around the desired system on the original figure associated with the problem or by clearly defining it in words. For 3D problems, it is harder to draw a clear line around the system and see what is being included, so the authors recommend the system be clearly defined in words. This is also a useful way to define a system when dealing with frames and machines. The system should be chosen based on what the student is interested in determining.

As an example of how to define a system, let's look at the truss shown in Figure 2. (This figure and many others in this paper are modified from those found in Ref. [5]. One of the co-authors of this paper is a co-author of this textbook). If the students are asked to determine the force in member *AC*, they could use the system shown in Figure 3, or if they are asked to determine the force in member *BE*, they could use the system shown in Figure 4. The free-body diagrams (FBD) associated with these systems are also shown in these figures. There is no need to call these the method of joints or

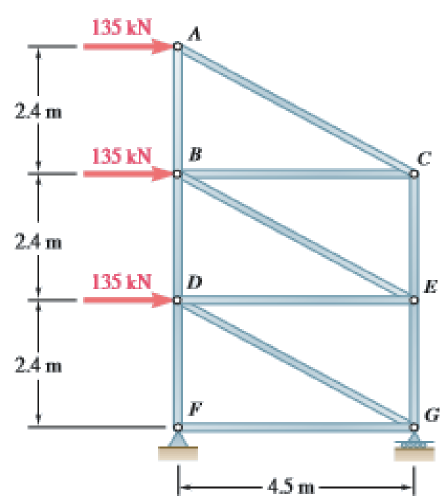


Figure 2 – Truss example used to show how to define systems

the method of sections. All students need to know how to do is to clearly define a system. Once a system has been chosen, the next step is to have students draw a FBD using the mnemonic BREAD.

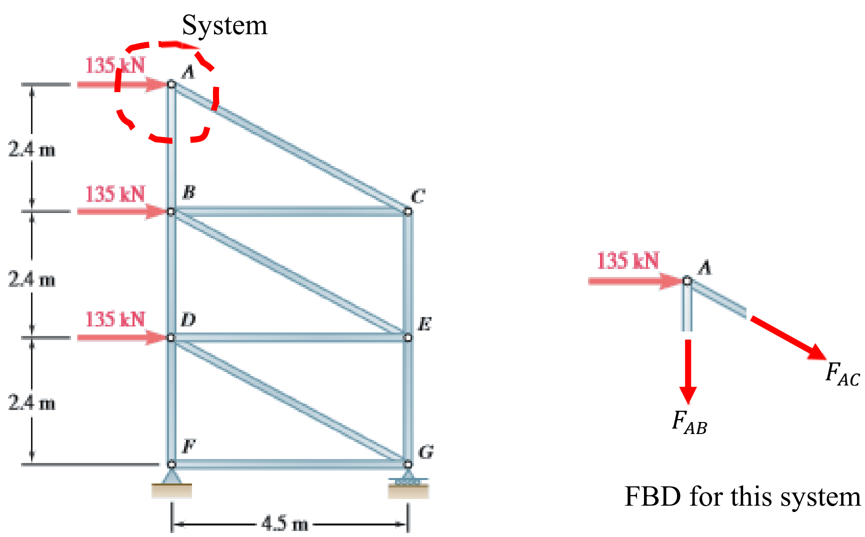


Figure 3 – System that could be used to solve for the force in member *AC*

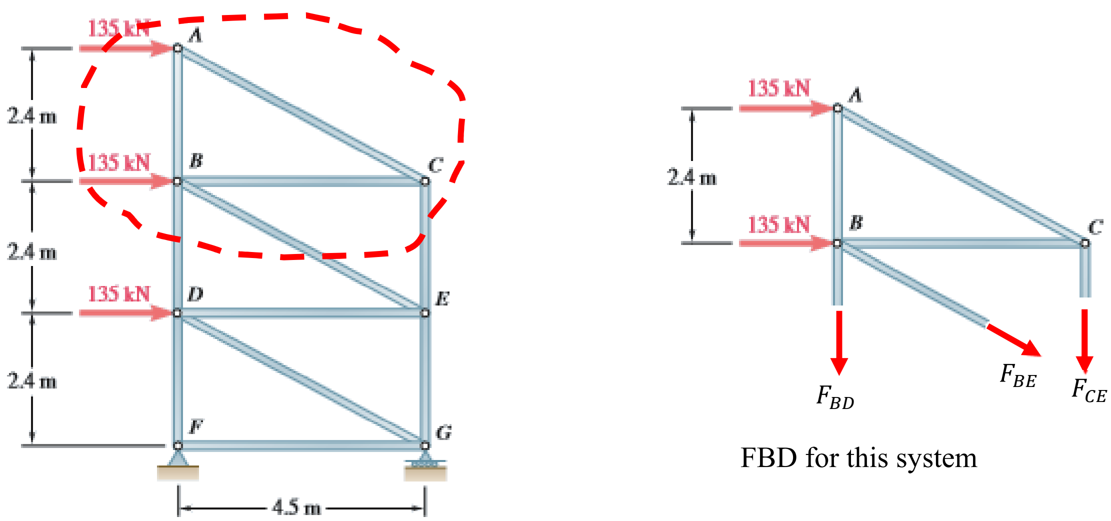


Figure 4 – System that could be used to solve for the force in member *BE*

Drawing the FBD using BREAD

A previous paper discussed two mnemonics that can be useful when drawing FBDs [9]. The mnemonic BREAD has been found to be very helpful in teaching students how to draw complete and accurate FBDs. How to use this mnemonic with a clearly defined system is described below.

B −Body: As discussed in the previous section, the system is defined by drawing a dotted line around it. The body is everything inside the dotted line.

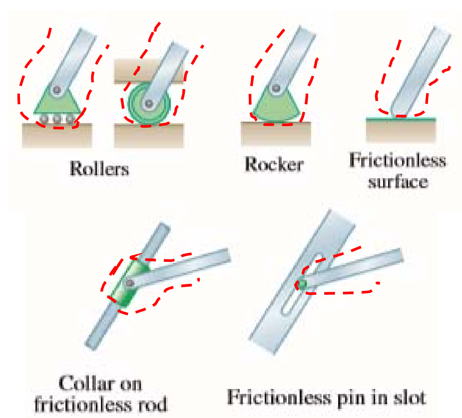
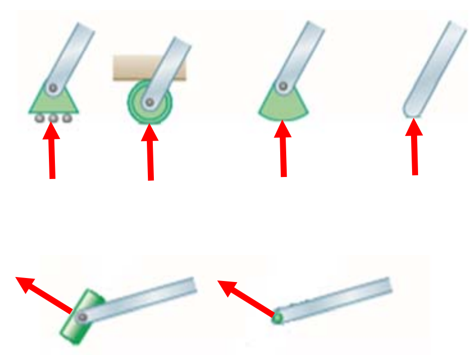
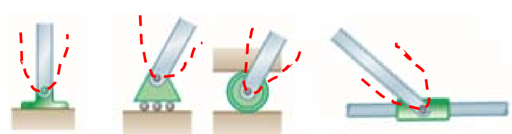
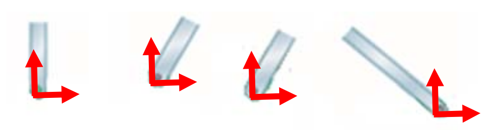
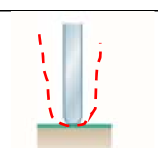
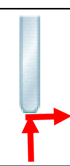
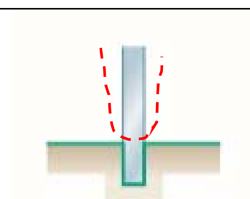
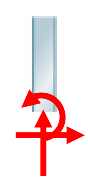
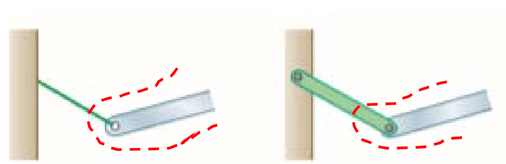
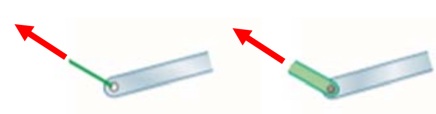
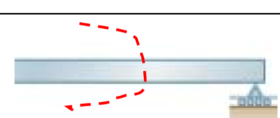
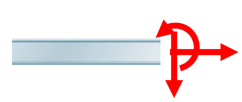
R – Reaction forces: The reaction forces are found by examining the system boundary, determining where the system interacts with the surroundings, and representing these interactions as forces on the FBD. Examples of how interactions between the system and the surroundings are represented on a FBD are shown in Table 2. Most Statics books have a table similar to Table 2. The main difference between those found in most textbooks and Table 2 is that we try to be very consistent in representing various interactions between the system and the surroundings. For example, if the system boundary cuts a pin, then the effect of the surroundings on the system is represented as two orthogonal forces. This is true even if the pin is connected to a frictionless slider as shown in the 2nd row of Table 2. If the interaction is a frictionless surface, roller, or slider, then the interaction is represented as a single normal force. In many Statics books, the slider or roller is not included in the body that is used for the FBD. In Statics, this will not make a difference, but in Dynamics, where a roller or slider can have mass, it is important to include it in the FBD, and therefore, by requiring it in Statics, students are developing good habits that can carry over into Dynamics. When a two-force member is cut, then the interaction is represented as a single force pointing between the two endpoints of the two-force member. Finally, if a rigid member is cut, as in the last row in Table 2, the interaction is represented as a force parallel to the cut surface, perpendicular to the cut surface, and a moment.

E –External forces: The external forces that need to be included on the FBD are weight and applied forces and/or moments that act on the chosen system.

A –Axis: A coordinate system needs to be defined and drawn near the body that is to be used when applying Newton's 2nd law.

D – Dimensions: The only dimensions that are required to be put on the FBD are those that are necessary for the application of Newton's 2nd law, such as angles to resolve forces or distances required when taking moments.

Table 2 – Partial system boundaries and how to include the interaction between the system and the surroundings at the system boundary on a FBD.

Description	Partial system boundary	How to include the interaction on a FBD
Roller/ smooth surfaces		
Frictionless Pin		
Rough surface		
Fixed support		
Two-force member or cable		
Cutting a rigid member		

The remainder of the process

For each system, Newton's 2nd law can be applied, resulting in three equations for planar problems ($\sum F_x = 0, \sum F_y = 0, \sum M_p = 0$) or six equations for three-dimensional problems. At this point, the students number their equations and make a list of their unknowns. If the number of equations does not equal the number of unknowns, then more equations are needed. As shown in Figure 1, the first question students should ask themselves when needing more equations is, "Is there another independent system?" If there is, then the process of defining the system, drawing the FBD, and applying Newton's 2nd Law is repeated with the new system. If there are no additional systems, then students are asked to identify another strategy such as the ones listed in Table 1. Only when the number of equations equals the number of unknowns can the students solve the resulting equations. This process places a priority on formulating the governing equations and deemphasizes the algebraic solution. Software tools such as MATLAB, Mathematica, Maple, or Mathcad make the solution of algebraic equations relatively easy. Once the equations are solved, students are asked to perform a "sanity check," that is, to think about whether the answer makes logical/physical sense.

Example showing how to this process

In this section, we will present an example of how to use this approach, and in particular, we will focus on the importance of clearly defining a system and drawing a FBD for the system.

Example Problem 1: For the frame shown in Figure 5, determine the forces at pin *C* acting on member *ABCD*. Assume the weight and all the dimensions are known.

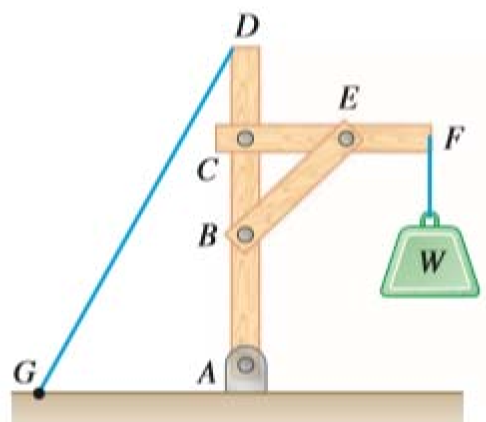


Figure 5 – Frame problem to illustrate the choice of systems

Solution: Since we are asked to determine forces, our **strategy** is Newton's 2nd law. Next, we need to **choose a system** to analyze. We try to stress that it doesn't really matter what system students choose. It is true that some systems make the algebra easier than others, but if students are allowed to use programs such as MATLAB, Mathematica, Maple, or Mathcad to solve the resulting governing equations, it is not particularly important to choose a system that makes the algebra easy. Students can choose bar *ABCD*, bar *CEF,* or the entire setup as a system.

The system boundaries and FBD for system *ABCD* are shown in Figure 6. The dotted red line indicates the system boundary, and where there might be confusion such as cutting the pin at *C*, a callout is used for clarification. Notice that in the FBD for *ABCD,* there are a total of six unknowns. Newton's 2nd law will only give us three equations, meaning that another system is required, for example system *CEF*, as shown in Figure 7. The reactions at *C* are drawn equal and opposite to those drawn in Figure 6, using Newton's 3rd law. There are no additional unknowns in this figure, and we can write three equations for this system, so we will end up with a total of six equations and six unknowns. Note that students could have chosen the system to be the entire setup, but this is not required.

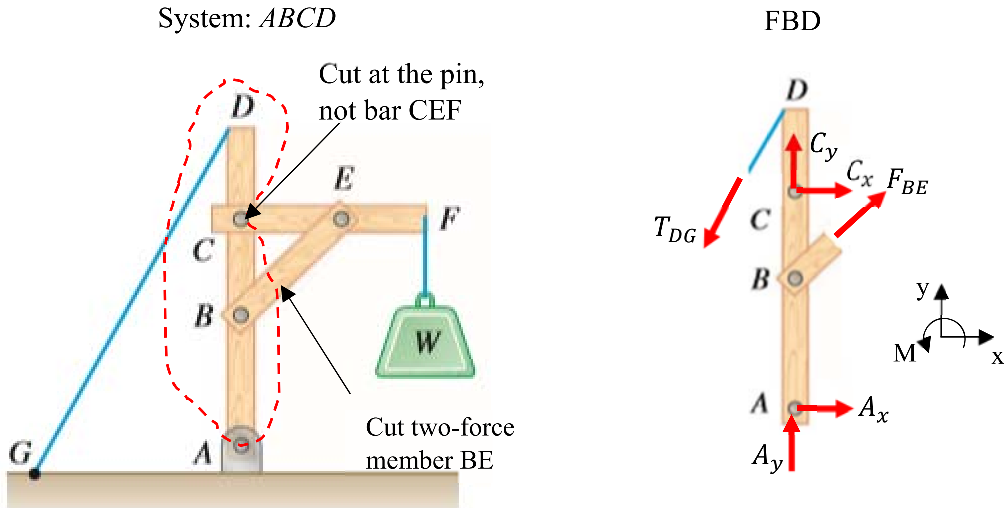


Figure 6 – System *ABCD* that cuts the pin at *C* and through the two-force member and the system's FBD. The dimensions are left off the FBD for clarity.

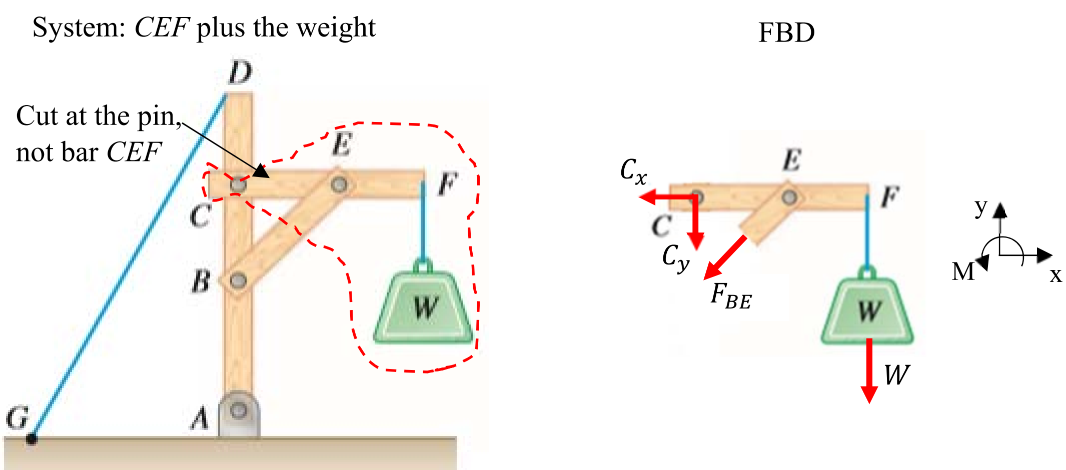


Figure 7 – System *CEF* plus the weight that cuts the pin at *C* and through the two-force member and the system's FBD. The dimensions are left off the FBD for clarity.

Notice in Figure 6 that when drawing the system boundary for *ABCD,* we are cutting the pins at *A* and *C* and through the two-force member *BE*. If the students did not recognized *BE* as a two-force member, they may have chosen the system boundary as shown in Figure 8. Notice that in the FBD for this system, there are a total of seven unknowns. Therefore, students will need at least two more systems, such as bar *CEF* and bar *BE* as shown in Figures 9 and 10. In the three FBDs shown in Figures 8-10, there are nine unknowns, and since we have three systems, we will be able to write a total of nine equations.

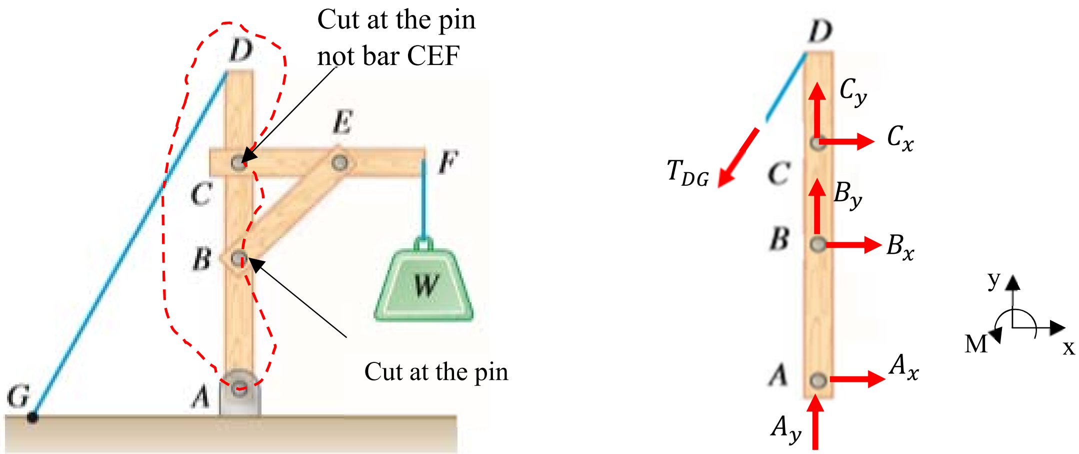


Figure 8 – System *ABCD* that cuts the pins at *B* and *C* and the system's FBD. The dimensions are left off the FBD for clarity.

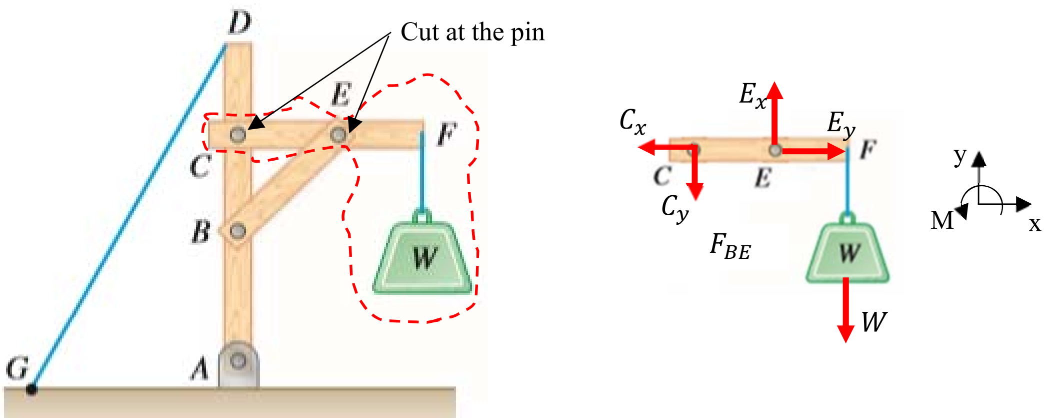


Figure 9 – System *CEF* plus the weight that cuts the pins at *C* and *E* and the system's FBD. The dimensions are left off the FBD for clarity.

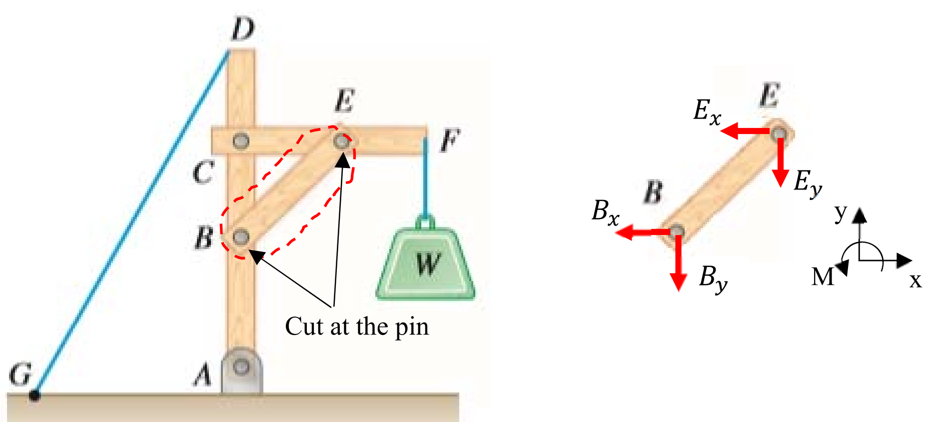


Figure 10 – System *BE* that cuts the pins at *B* and *E* and the system's FBD. The dimensions are left off the FBD for clarity.

Notice that if students try to cut one of the members, as in the system shown in Figure 11, the FBD will include the forces and moments at the cut. They will need to do this when they want to find internal forces or moments such as when they need to draw shear and bending moment diagrams or determine internal stresses in a member.

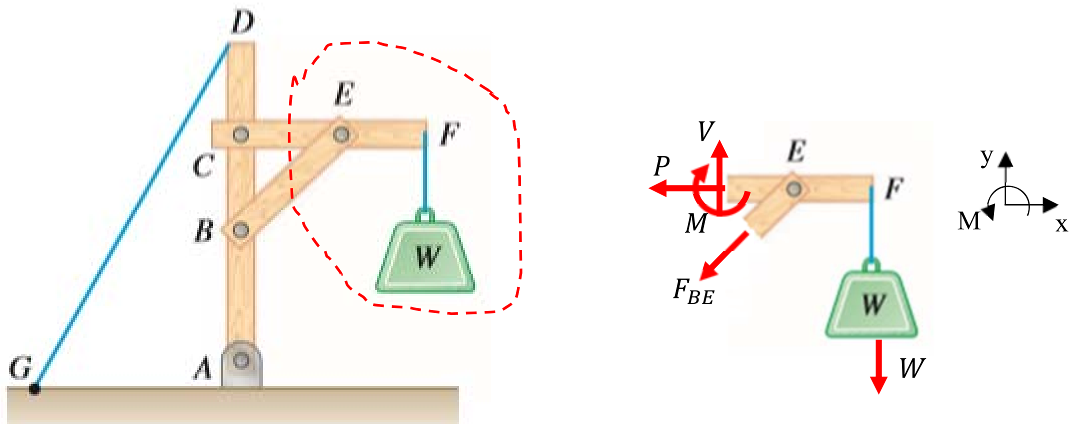


Figure 11 – System that cuts bar *CE* and the system's FBD. The dimensions are left off the FBD for clarity.

If the student had chosen a system that cut the cable connecting the weight to *F*, then on the FBD they would have included a tension, because when a cable is cut, as shown in Table 2, the force in the cable is represented by a tension. For this example problem, it is true that the tension is equal to the weight, but in Dynamics classes, this is usually not the case, so by using a consistent approach for representing the interactions as system boundaries, students are developing good habits that are transferable to other courses. A system that includes cutting the cable and the associated FBD is shown in Figure 12.

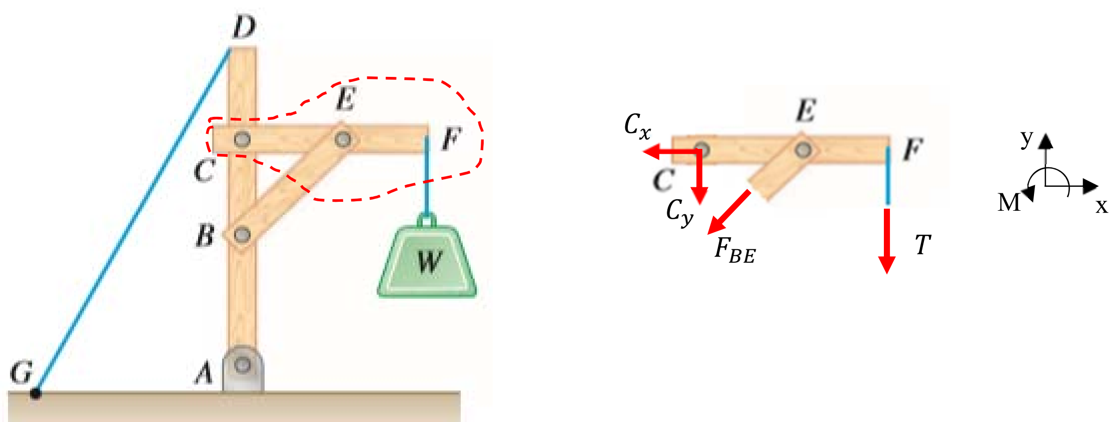


Figure 12 – System *CEF* that cuts the cable connected to the weight and pin *C* and the system's FBD. The dimensions are left off the FBD for clarity.

Example Problem 2: Two traffic lights are supported as illustrated. Calculate the tensions in cable segments AB, BC, and CD and the corresponding cable angles α, β, and γ. The total length of the cable is 36 ft.

Strategy: Since we are asked to determine forces, one strategy will be Newton's 2nd law. A quick look at the setup tells us that we will only be able to generate a maximum of four equations from Newton's 2nd law, but we have 6 unknowns. Since we are provided with dimensions and must determine angles, geometric relations will also likely be a strategy.

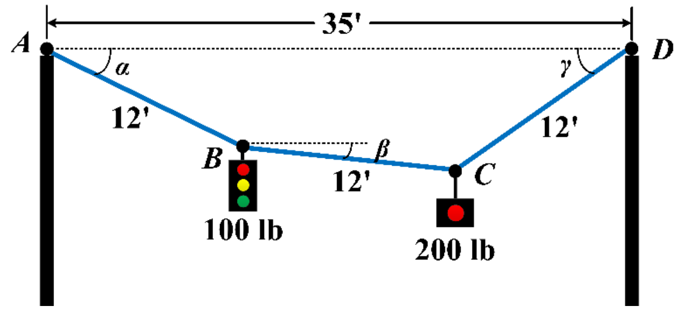


Figure 13 – Traffic lights problem

System: Two systems that can be chosen are the traffic light at B and the traffic light at C as shown with the FBDs in Figures 14-15.

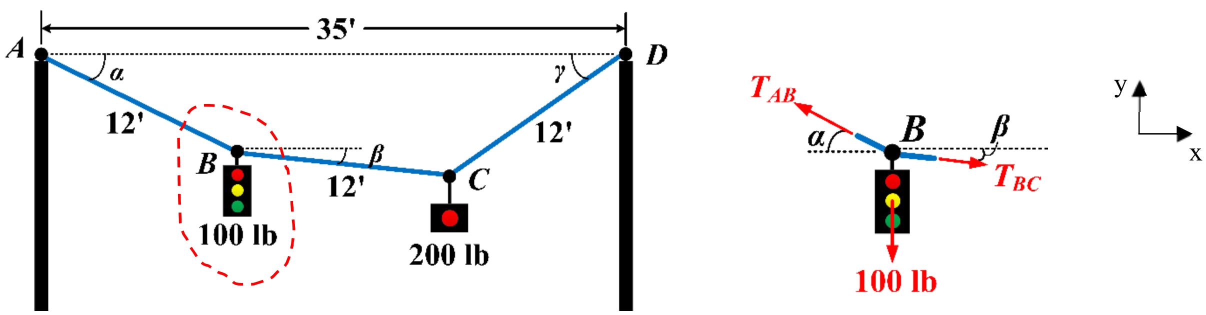


Figure 14 – System traffic light B and its associated FBD.

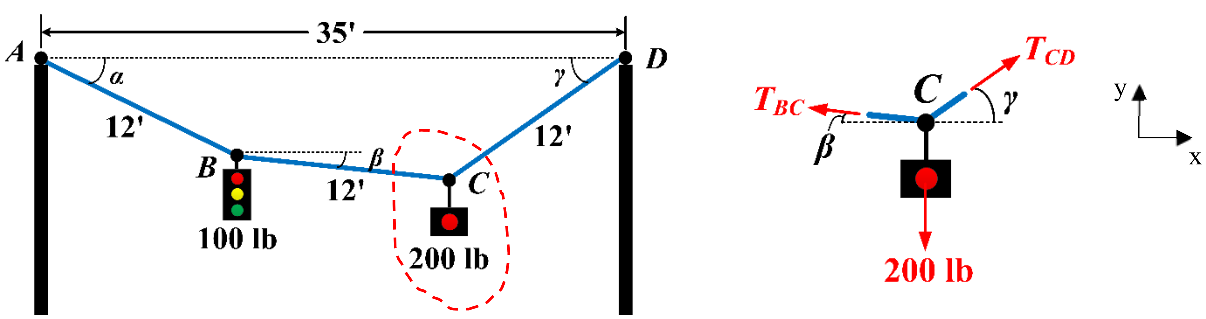


Figure 15 – System traffic light C and its associated FBD.

Analysis: Applying Newton's 2nd law to the FBD in Figure 14 will yield two independent equations and four unknowns (T_{AB}, T_{BC}, α, β). Doing the same to the FBD in Figure 15 will yield another two independent equations and an additional two unknowns (T_{CD}, γ). We are clearly still two equations short. A student might be tempted to take traffic light *B*, traffic light *C* and the cable connecting them as their system, draw their FBD and apply Newton's 2nd law, but neither of the two equations that will be generated are independent. We stress that if there are only two objects, in this case the two traffic lights, there will only be two independent systems. Thus, we have exhausted Newton's 2nd law as a strategy to solve this problem. Since we are given the length of the cables and have to solve for angles, our second strategy is simply using geometric constraints. From the original diagram, we can observe the following two geometric relations:

From the distance between the walls: $12\cos\alpha + 12\cos\beta + 12\cos\gamma = 35$

From the vertical position of the light at C: $12\sin\alpha + 12\sin\beta = 12\sin\gamma$

These two equations are independent from the previous ones, and no new unknowns are introduced. This brings the total number of equations and unknowns to six. The problem can now be solved for the unknowns.

Classroom Observations and Pitfalls

The authors of this paper have each adopted the system-driven approach to problem solving presented in this paper and have used it in Statics as well as in other courses such as Dynamics and Thermodynamics. This approach has also been applied by one of the authors at the United States Air Force Academy (USAFA) in a large multi-section offering of a combined statics and mechanics of materials class. Anecdotal evidence indicates that this approach led to improved student performance in the course. In general, students who use this systematic approach seem to have an easier time applying fundamental principles to problems which do not fit the exact style of the examples they may have encountered during class. For instructors who may be interested in implementing this approach in their classrooms, this section identifies some common issues students face as beginners to this problem-solving approach and some suggestions for how to guide them in learning this process.

First, it is common for students to be less comfortable with problem-solving approaches which allow for more freedom of choice. These solution strategies tend to be less procedural and require more care in justifying choices made and steps taken. For students who do not have a strong understanding of the fundamental concepts underlying Statics, it may be possible to follow a procedure (such as the method of sections) and to arrive at a correct solution, but a more open-ended problem-solving approach may prove challenging. A related issue is that students struggle specifically with choosing a system. It has often become ingrained in them through previous coursework that there is a single, direct, "right" path to a solution, and this habit can be difficult to break. There are several effective ways to counter this discomfort in the classroom. First, it is important to emphasize repeatedly that there may be a variety of possible paths to a correct solution which may involve different choices of systems and different strategies. And second, to support this idea, it may be beneficial to solve a single example using two different combinations of systems or strategies and to encourage students to pursue problem-solving approaches which are more intuitive to them. Finally, as students begin to gain some comfort with the problem-solving process, it is helpful to draw on their own intuition by asking them to contribute suggestions for choices of systems, choices of strategies, and other steps. This suggestion is especially helpful for showcasing that even if a particular choice of system doesn't lead directly to the solution, it can often be combined with other systems which together will provide the necessary information.

Students also tend to struggle with knowing which strategy to implement and when. For example, they have trouble identifying when choosing another system and writing Newton's 2nd Law will no longer be helpful, requiring them to pursue other strategies (such as geometric constraints) instead. Most often, it is difficult for students to discern whether or not their choices of multiple systems are linearly independent. For example, consider the scenario shown in Figure 16. A student may first choose Pulley A as a system (labeled "System 1"), then choose the 150 kg weight as a system (labeled "System 2") and write Newton's 2nd Law equations for both. Lacking more equations to solve for all unknown variables, the student may next choose both the weight and pulley together as a third system (labeled "System 3"), failing to recognize that this choice of system is nothing more than a combination of the other two. An effective way to address this confusion is to spend some time while solving example problems in class showing what will happen if the third system is chosen and explaining why it will not provide new information.

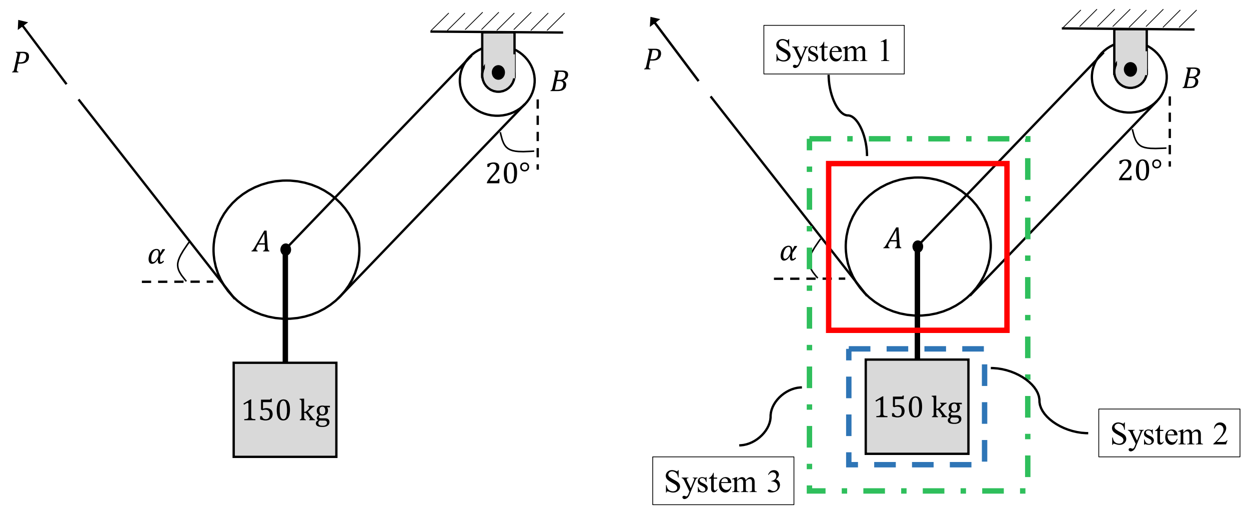


Figure 16. Choosing linearly independent systems

Conclusions

In this paper, we have presented a unified problem-solving strategy for Statics, but we have focused on its usefulness when applying Newton's 2nd law. The key step when starting any problem is to begin with reflection and ask, "What is the strategy?" If the strategy is applying Newton's 2nd law, then the next key step is to clearly "Define the system." This is done by drawing a dotted line around what is being set aside for analysis. Once a system is defined, we stress the importance of consistency when drawing the FBD by examining where the system interacts with the surroundings at the system boundary. We do not have any quantitative assessment results indicating that this method improves students' problem-solving abilities, but the anecdotal evidence from the authors' many years of teaching indicates that students benefit from this systematic approach.

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