

A Multidisciplinary Mid-Level Electrical and Mechanical Engineering Course

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Abstract

Linear system theory and feedback control are commonly included in the curricula of both Mechanical and Electrical and Computer Engineering programs. Both majors typically offer these subjects in similar, yet separate courses. In contrast, we have created an integrated, multidisciplinary course that effectively covers these topics for students in both majors. The learning objectives, structure, and example content, including a culminating, contextual project, are presented here. This class successfully integrates two subject-matter, specific-content courses, Signals and Systems (ECE) and Dynamics (ME), into a single course that focuses on the development and application of general mathematical modeling and analysis tools to support the engineering design process. It is taught in a studio-setting and serves as a prerequisite for advanced courses in either major. The material is motivated by the classic problem of controlling an inverted pendulum on a translating cart. We have developed an easy-to-implement but robust, affordable system based on a commercial Arduino-like platform that allows students to experiment and quickly iterate on proposed control algorithms. Our implementation of the project requires students to perform cycles of symbolic and numerical mathematical analysis followed by experimentation and iteration. Student evaluation data provides evidence of the efficacy and advantages of concept integration which helps build a shared language applicable to future academic projects and professional practice.

Introduction

Course requirements for many Electrical and Computer Engineering (ECE) programs include a course with a title similar to “Signals and Systems.” Likewise, many Mechanical Engineering programs require a course with a name similar to “System Dynamics.” Both courses involve time and frequency domain mathematical analysis tools for Linear, Time-Invariant (LTI) systems but use content-area specific examples, *e.g.*, RLC circuits or mass-spring-damper systems. Much of the fundamental mathematics underlying these courses is the same yet there exist significant differences in the course material. For instance, a System Dynamics course may include rotational dynamics and motion. On the other hand, a Signals and Systems course will emphasize filtering and introduce concepts such as sampling and discrete-time signal processing, topics rarely covered in an ME System Dynamics course.

In order to expose students to perspectives and approaches from both ME and ECE, while simultaneously providing them with the opportunity to learn discipline-specific content and skills, we designed and offered three half-semester courses: Engineering Systems Analysis (ESA), Engineering Systems Analysis: Dynamics (ESA-D) and Engineering Systems Analysis:

Signals (ESA-S). All ME and ECE students take ESA during the first half of the semester. In the second half of the semester, ME students take ESA-D and ECE students take ESA-S. The collection of these courses constitutes the equivalent of 1.5 regular courses at Olin College. They are scheduled so that if desired, students can take both ESA-D and ESA-S, after completing ESA. All three courses were offered for the first time during the Spring 2020 semester.

In this paper, we focus on the 7-week ESA course, functioning as the conceptual precursor to the discipline-specific ESA-D and ESA-S that follow. ESA was taught by a multidisciplinary team comprising two ECE faculty members and one ME faculty member in a studio classroom setting. In the current iteration (ongoing at the time of authorship of this paper), the course is taught by one ECE faculty member and one ME faculty member online due to the COVID-19 pandemic. These courses serve as prerequisites and prepare students for more advanced courses in ME or ECE, such as Control Theory, Digital Signal Processing or Structural Dynamics.

The curricular goals of ESA are to further develop students' skills and expertise in the engineering analysis process, increase their self-directed and peer learning abilities, and to convey content that is common to ME and ECE programs. The focus on quantitative analysis is part of a broader effort to educate students in this area. The course material is built around a hands-on project to control an inverted pendulum on a cart, a classic problem in control theory [1] which is often included in Signals and Systems and System Dynamics courses [2], [3].

To this end, we developed a project using an affordable system based on an Arduino-like platform, the Balboa 32U4 Balancing Robot Kit (Pololu Robotics and Electronics, Las Vegas, NV). Since this classic ECE and ME problem is usually presented as either a demonstration or simulation, rather than a physical project that students realize themselves, we believe an opportunity exists for the ECE and ME education community to adapt this system for their own courses.

Institutional and curricular context

Students at Olin College major in ME, ECE or Engineering (the latter offering several possible concentration areas). ESA is taken primarily by sophomores and juniors who are ME and ECE majors, with a small number of Engineering majors, often those who are concentrating in Robotics. ESA students have not only completed prerequisites (differential equations, linear algebra, and mechanics) but have also taken a mechatronics course where they interface an Arduino platform with sensors such as shaft encoders. After completing ESA, along with ESA-D and/or ESA-S, they can take advanced courses in their respective majors.

ESA was created as part of an *analysis stream* or series of courses at Olin (initiated in 2016). The establishment of this stream was in part due to observations from core faculty that our students are proficient in the user-centered design process, *i.e.*, adept and willing to use the tools from their training in user-centered design. However, they were less comfortable drawing on analytical tools when confronted with real-world challenges. This stream was created to improve students' comfort, confidence, and competence in using analytical tools when faced with open-ended real problems, while helping them recognize that analysis involves more than just knowing content.

Specific goals of the analysis stream included helping students to 1) be comfortable with a quantitative analysis process, 2) know where in the process they are and what they should be doing at different points in the process, 3) know how to make use of analysis results and apply them in the engineering design process, and 4) be ready and willing to apply analysis to new problems. The analysis stream at Olin College starts with a sequence of three courses (in the current iteration) called Quantitative Engineering Analysis I, II, and III. Various aspects of early versions of this course are described in [4], [5], and [6]. This sequence integrates foundational mathematics, physics, and engineering in applied contexts through modules tied to specific practical applications. The ESA course described in this paper is a follow-on course covering material traditionally presented in a major-specific way in ME and ECE programs. It provides students with a toolkit of useful linear system approaches, the theory that underpins these tools, and practical experience conducting analysis and experimentation on a real system.

Key design principles and overall course goals

The following design principles were used to guide the development of the ESA courses.

1. *Promote self and peer learning.* The benefits of self-directed and peer learning are well recognized, including increased student motivation and understanding of material, as well as preparation for lifelong learning. This class uses these modes of learning as the primary approach for students to acquire skills and content knowledge. The course is run studio style using white-boards and props where students work collaboratively in pairs or groups of four on different tasks, *e.g.*, reading and working out problems. As the course progresses and students' self-directed learning skills develop, we reduce scaffolding around the assignments accordingly.

2. *Integrate the use of analytical, computational, and experimental tools in analysis, embracing real-world non-idealities, and demonstrating relevance through context.* We incorporated significant computational, experimental, and simulation-based exercises in the course, as compared to a mostly theoretical treatment common when introducing material on these topics. Our reasons for doing this are to help students use techniques and tools that are commonly used by practicing engineers and to help them understand the role theoretical models play in system design and analysis in the presence of the inherent limitations of real-

world non-idealities. We paid special attention to explicitly tying the introduction of theoretical material to real-world applications.

3. *Require working to completion, especially with the final project.* We placed a significant emphasis on ensuring that every student team was able to have a working minimum-viable-product for their final project. The main motivations for this requirement are to boost students' confidence in applying analytical tools and for them to fully appreciate the level of detail needed to complete a project. Additionally, since this is a classic and well-recognized problem in ME and ECE, the students would have the opportunity to showcase their completed projects in portfolios or on resumes.

4. *Technical content goals.* To achieve the above goals, we recognized that course content would have to be reduced in comparison to traditional treatments of the material. Noting that content knowledge relevant to an engineer's career is constantly and rapidly evolving, *e.g.*, see [7], we believe that a reduction in specific technical content is an appropriate tradeoff to facilitate higher-level learning goals while ensuring students leave with an enduring understanding of fundamental concepts. For this course, we emphasized the following content:

- A. Modeling of electrical and mechanical systems using ordinary differential equations.
- B. Understanding the unilateral Laplace transform, including its use in modeling, analyzing input-output systems, and solving governing equations.
- C. Understanding and analyzing canonical responses of first and second order systems and how they can be used to inform analysis of higher order systems.
- D. Connecting poles and zeros of transfer functions to system response and stability.
- E. Creating, manipulating, and analyzing block diagrams.
- F. Understanding basic Proportional and Proportional-Integral (PI) feedback control, including related concepts such as the Final Value Theorem and the Routh-Hurwitz criteria.
- G. Basic parameter estimation by curve fitting experimental data.

Course structure

ESA was structured around the development of engineering concepts in tandem with mathematical, computational, and experimental tools (Figure 1). The inverted pendulum robot platform used for the course project is shown at the center of the figure. The engineering concepts were developed through a set of in-class and outside-class exercises which combined readings and problem-set style questions. Mathematical tools and computational tools were also introduced through these exercises.

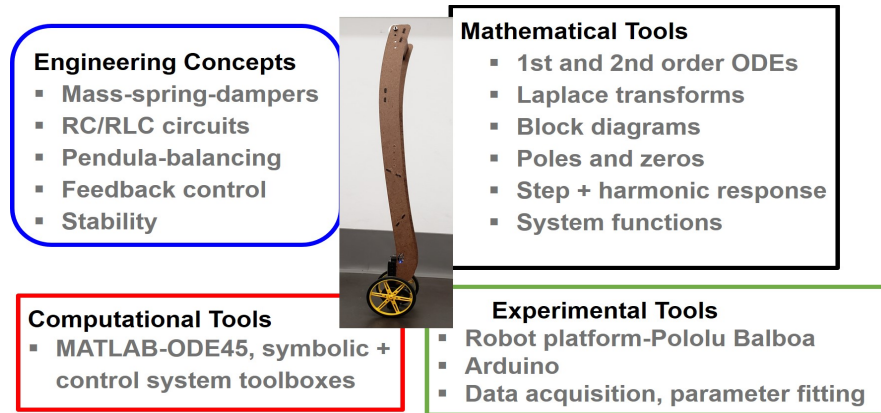


Figure 1. Illustration of key concepts and tools united by the inverted pendulum robot project.

The half-semester course schedule is illustrated in Figure 2. The first class introduces and frames the project with a presentation featuring devices that stand upright, such as the Segway scooter and bi-pedal robots. These contemporary applications demonstrate to students that the material they are learning, while at an introductory level, can be applied to real-world problems. We followed this with a review of 1st and 2nd order systems. Next was an introduction to the Laplace transform and its use in solving ODEs, leading to methods of modeling and analysis of LTI systems in the s-domain, including the use of block diagrams, transfer functions and poles and zeros.

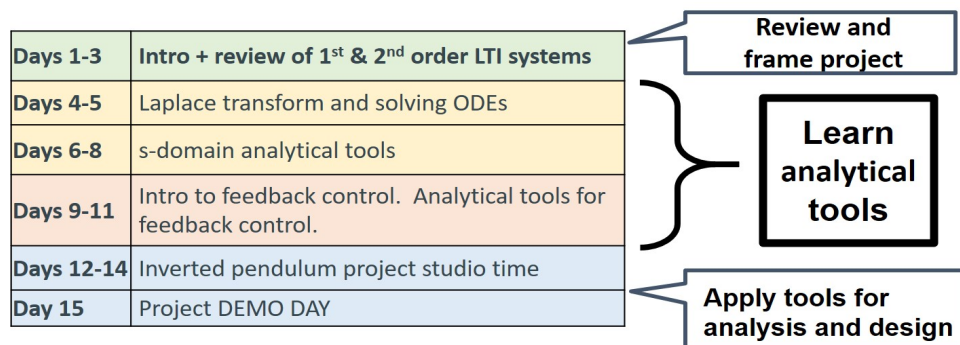


Figure 2. Course schedule.

Following this, we introduced the idea of feedback control through several thought experiments, followed by proportional and proportional + integral control, analysis of stability, steady state errors, and linearization. In this portion of the course, we included simple problems, *e.g.*, automobile cruise control. We then followed with the analysis of a control system to balance an inverted pendulum on a cart. Students were assessed through a combination of homework, a take-home exam, and their performance on the final project.

Final project

The final project involves the balancing of an inverted pendulum on a robot cart by controlling its position and velocity. Intended as a challenging, fun, integrative experience, the “Rocky” project requires a combination of modeling, analysis, and experimentation. The project version described here is based on a prior iteration from the Quantitative Engineering Analysis course at Olin College. We set a minimum viable product of keeping the robot balanced within a one-foot square, as shown in Figure 3.



Figure 3. Inverted pendulum robot balancing in a square.

Starting with the Balboa 32U4 Balancing Robot Kit, we added an external attachment arm to the robot to increase its effective length and placed a mass at the top of the arm. The 32U4 is an Arduino-like system which is designed to be a balancing robot. While the manufacturers provided a heuristic method for balancing the robot, we required students to take a more analytical approach.

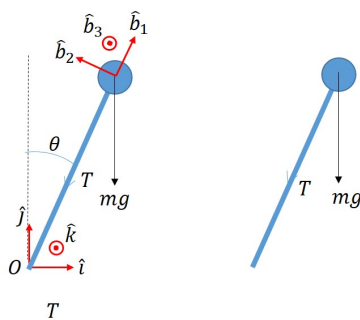


Figure 4a. Inverted pendulum.

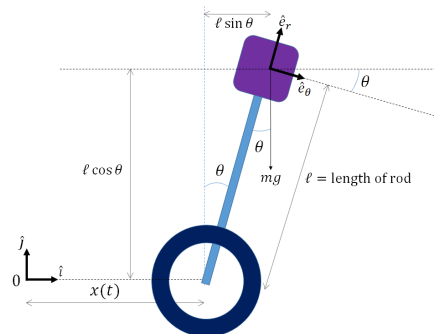


Figure 4b. Inverted pendulum on a cart.

After completing a hand analysis on a simple pendulum, students performed a hand analysis on an inverted pendulum as represented in Figure 4a. Students increased complexity by modeling

the dynamics of an inverted pendulum on a cart (Figure 4b). They derived equations of motion relating the displacement, velocity, and acceleration of the cart to the angle of the pendulum arm and found the transfer function $H_{v\theta}(s)$ relating the angle $\theta(s)$ to the velocity of the cart $V(s)$, i.e., $H_{v\theta}(s) = \frac{s}{g - ls^2}$.

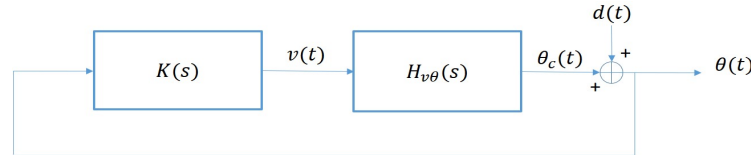


Figure 5. Block diagram for simple control of inverted pendulum on a cart.

The students then created a simple block diagram of the controlled system, given in Figure 5, where $\theta(t)$ is the angle of the pendulum with respect to vertical, $d(t)$ is a disturbance signal which perturbs the angle, and $K(s)$ is the controller. This is an active disturbance rejection control problem. Note that in the block diagram above, it is assumed that the desired angle of the pendulum is zero. Using the fact that an integral term in the feedback path will result in a zero-output signal (provided the system is stable), we argue that if the system above is stable and the controller has an integral term in the feedback path, $\theta(t)$ must approach zero for any bounded input.

Students continued developing their block diagram to include a first-order motor model illustrated by the block marked $\frac{ab}{s+a}$ in Figure 6. To ensure that the velocity and position of the cart do not change in steady-state, proportional and integral terms (J_p and J_i/s in the feedback loop) were placed around the motor velocity and position. Students conducted experiments to not only determine the effective length of their robots' pendulum arm l , but also the motor parameters a and b . The latter was done by fitting a curve to measured velocity data, an example of which is shown in Figure 7.

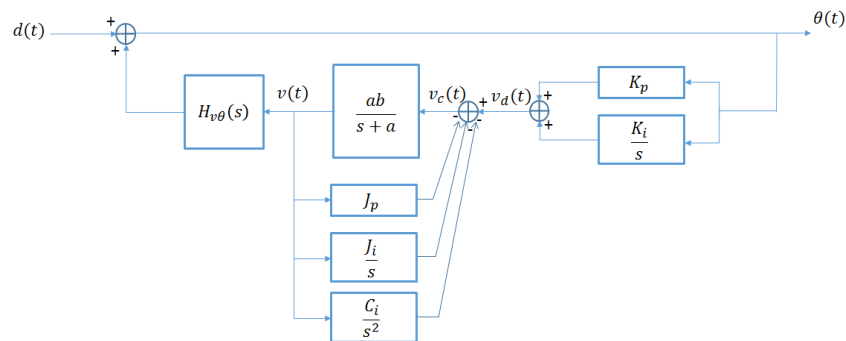


Figure 6. Block diagram of controlled inverted pendulum with motor model. Here $d(t)$ is a disturbance applied to the angle $\theta(t)$.

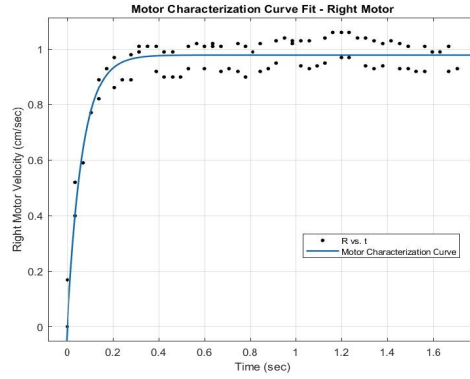


Figure 7. Motor model data fit; from Lacie Fradet, James Ho and Kerry McConnaughay. Used with permission.

It is important to note that the system shown in Figure 6 is fifth order and has exactly five free parameters: K_p , K_i , J_p , J_i , and C_i . As such, in theory, students will be able to set the poles of the system at locations of their choosing. While this is more complex than the lower order models often used to represent this system, we felt that going to a higher order has the benefit of enabling students to explicitly specify pole locations and promotes the use of symbolic mathematics tools to carry out analysis. An example of such code, given in Figure 8,

```
syms s a b l g Kp Ki Jp Ji Ci % define symbolic variables
Hvtheta = -s/l/(s^2-g/l) ; % TF from velocity to angle of pendulum
K = Kp + Ki/s ; % TF of the angle controller
J = Jp + Ji/s + Ci/s^2 ; % TF of the controller around the motor
M = ab/(s+a) % TF of motor
Md = M/(1 + M*J) % TF of motor + feedback controller around it
% J is applied on the feedback path
pretty(collect(Md)) % display Md(s)
Htot = 1/(1-Hvtheta*Md*K) % this is the total system function from
disturbance d(t) to % \theta(t)
pretty(simplify(Htot)) %display the total system function
```

Figure 8. Code for symbolically determining the fifth order transfer function.

can be used to show that the resulting transfer function of the system between the angle and disturbance, *i.e.*, $\theta(s)/D(s)$, is

$$\frac{ls^5 + (al + J_p abl)s^4 + (J_i abl - g)s^3 + (C_i abl - ag - J_p abg)s^2 - J_i abgs - C_i abg}{ls^5 + (al + J_p abl)s^4 + (K_p ab - g + J_i abl)s^3 + (K_i ab - ag + C_i abl - J_p abg)s^2 - J_i abgs - C_i abg}$$

Using symbolic and numerical tools, students can then determine values for the five control parameters in order to place system poles at different locations. The pole locations were selected to achieve certain desired responses (e.g., time constant, rate of oscillation, etc.).

Students then applied the resulting control parameters in the Arduino programming language on the robot platform. Starter code was provided for students to modify. This allowed them to focus on designing the control system, rather than on implementation details specific to the particular robot platform (e.g., reading sensor values or measuring the time elapsed time between iterations of the control loop). Based on different experimental measurements, students further tweaked the parameters of the control system. Students who desired to control the motion of the robot, modified the block diagram in Figure 6 to add additional inputs to offset the velocity and position.

Observations and course evaluation

At the end of the course, all student teams (primarily pairs) successfully balanced their pendulum robot, achieving the minimal viable product. Some are shown balancing in Figure 9. Several teams went beyond the minimum requirements including a radio-controlled version of the robot and one that could “dance” to music. These achievements were particularly encouraging because the closure of our campus due to the Covid-19 pandemic was announced a few days before the final demonstration day.



Figure 9. Inverted pendulum robots balancing together.

We collected responses to 10 custom questions as part of the school’s standard course evaluations; see Figure 10. Questions 1 and 2 assessed students’ perceptions of their preparation entering the course. Questions 3 - 6 address student perception of their acquisition and mastery of certain quantitative analytical tools. Questions 7 - 9 pertain to students’ confidence and willingness to use these tools in the future. The final question is intended to gauge interest in the course material. The students were asked to answer the questions on a five-point Likert scale (strongly disagree-SD, disagree-D, neutral-N, agree-A, and strongly

agree-SA). A total of 31 students responded to the survey out of the 41 students enrolled in the course.

		SD	D	N	A	SA	Mean	Std dev
1	I felt sufficiently prepared in differential equations for this class	0	2	3	14	12	4.2	.85
2	I felt sufficiently prepared in linear algebra for this class	0	1	5	11	14	4.2	.83
3	I can use Laplace Transforms to solve ODEs	0	0	0	13	18	4.6	.49
4	I understand how the poles of a system relate to stability and time response	0	0	0	13	18	4.6	.49
5	I feel comfortable working with systems in block diagram form	0	1	1	19	10	4.2	.66
6	I feel comfortable using MATLAB	0	0	5	8	18	4.4	.75
7	I feel confident I could apply material from this class in future courses	0	0	5	13	13	4.3	.72
8	After taking this course I feel more willing to apply quantitative analysis in a design process	0	1	6	15	9	4.0	.78
9	I feel comfortable working with both mechanical and electrical systems	0	3	9	14	5	3.7	.86
10	I would take the Controls course	1	5	9	9	7	3.5	1.1

Figure 10. Course evaluation survey results: 31 responses from 41 enrolled students.

The survey results provide some indication that students believed they learned the key concepts. For instance, 26 of the 31 respondents indicated that they feel confident they would be able to apply material from this course in future courses. Additionally, students assessed themselves as being able to use some of the important mathematical and analytical techniques introduced in this course. On the question of feeling comfortable working with both mechanical and electrical systems, the responses were more distributed, with 19 of the students either agreeing or strongly agreeing that they would feel comfortable working with systems from both domain areas. However, three students reported disagreeing with this statement which indicates that further effort may have to be put into improving students expertise in both electrical and mechanical systems.

Conclusion

We ran a successful experiment of delivering an integrated mid-level course for ME and ECE majors at Olin College of Engineering. Our approach was generally successful in that all student teams were able to put the theory developed in earlier parts of the course into practice to balance their inverted pendulum robots. Since this process required modeling, experimentation, simulation, numerical analysis and physical implementation, students were exposed to the major aspects of the engineering process through a compelling project experience. Furthermore, the ESA course provided a sufficient foundation for students progressing to follow-on, major specific courses. Currently, the second iteration of this course is being run with two faculty members, an ME and an ECE faculty member in an online

format, which makes use of writing tablets provided to each student for improved interaction. Simulink-based simulation of the inverted pendulum project has been developed for remote students. Those on campus will have a choice of the simulation-based final project or carrying out the inverted pendulum project on the robot platform.

Acknowledgements

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