Work-in-Progress: Ambiguous Reaction Couples: A Universal Approach to Analyzing Bearing and Hinge Support Reactions in 3D Statically-Determinate Problems

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Abstract

Having students develop an understanding on how to draw proper FBDs is perhaps the main objective of any Statics course and reactions due to supports are often a challenging part of this process. The most unfamiliar supports that students encounter in Statics are hinges and bearings in three dimensions, which may or may not require reaction couples that need to be accounted for. Different textbooks have treated this topic in very different and sometimes confusing ways. In this paper, we introduce the ambiguous reaction couple approach to analyzing bearing and hinge supports in 3D statics problems. In this method, all the possible reaction couples are first considered during the drawing of the FBD and translation of the forces and moments to the equilibrium equations and subsequently a decision is made on whether to keep or discard each reaction couple with the help of an equations/unknown table. We provide three examples to show the application of this universal approach to different types of problems that involve bearing supports. We have found assessing the effectiveness of this approach difficult in a Statics class, but plan on implementing an assessment in Capstone Design.

Introduction and Motivation

The fundamental purpose of a Statics course is to initiate and encourage the development of a student's engineering judgment, at least with regards to mechanics, by employing the free-body diagram (FBD) as a tool. In a standard undergraduate civil or mechanical engineering curriculum, students build upon the concepts introduced in earlier courses utilizing tools such as the FBD (while ideally enhancing their engineering judgment simultaneously) until their Capstone course, in which the tools, concepts, knowledge and judgment are all employed in the service of solving an open-ended problem. Gainsburg[1] presents engineering judgment as the integration of theory and practicality, which implies that incorporating good engineering judgment requires a strong foundational understanding of the theory, significant practical experience and being able to connect the two in order to address an engineering problem. From an instructor's perspective, it can be challenging to present a new topic to students, whether in a Statics class or in Capstone, where there can be ambiguity and the student will need to rely on their knowledge and judgment in order to make a decision. We present one such topic in this work and aim to provide a general enough solution approach that will be understandable to undergraduate engineering students in any year of study.

From experience, one of the most difficult-to-teach aspects of FBDs is the proper consideration of reactions due to supports. Two important types of mechanical supports are hinges and bearings. Planar systems that include these supports are not problematic to a student when drawing the FBD since they are modeled as simple pin joints. In three dimensions, however, hinges and bearings can behave "strangely", in that depending on the problem, reaction couples may or may not be needed to be considered when drawing the free-body diagram. The reason for this is that while bearings can generally resist a small reaction couple, the primary purpose of utilizing bearings in most applications is to constrain motion in a certain direction, reduce friction and maintain proper alignment. If bearing supports or hinges are perfectly aligned and properly manufactured, the reaction couples would be zero in the FBD assuming a rigid body.

The unusual nature of three-dimensional bearing and hinge supports has been treated differently in eight common Statics textbooks reviewed by the authors. By far, the most comprehensive treatment of bearing supports was found in Costanzo, Plesha and Gray[2], who discuss self-aligning and perfectly-aligned bearings and when and why the reaction couples moments should or should not be considered. Sheppard, Anagnos and Billington[3] divide their bearing supports into two groups: individual bearings which will have reaction couples and perfectly-aligned bearings which have no reaction couples. Riley and Sturges[4] discuss several types of bearing designs and explicitly mention that the reaction couples are negligible if the bearings are perfectly aligned. Bedford and Fowler[5] and Hibbeler[6] mention that reaction couples may be zero and provide examples, but do not explicitly discuss the reason for this and are somewhat vague on the scenarios in which the reaction couples are zero. Meriam and Kraige[7] only briefly mention that the reaction couples on a thrust bearing may be zero to provide statistical determinacy while Beer and Johnston[8] only mention that radial and thrust bearings may have up to two reaction couples without providing a justification or an example. Lastly, Pytel and Kiusalaas[9] do mention radial and thrust bearings and hinges, but completely ignore the reaction couples and provide no explanation as to why they do so.

While each of the aforementioned texts presents the topic of the bearing and hinge support reactions in its own way, none of them provide a general guideline to tackle *any* statically-determinate three-dimensional rigid body that contains at least one bearing or hinge support. With this in mind, the *ambiguous reaction couple (ARC)* approach, which is a simple and intuitive way to deal with the reaction couples in three-dimensional bearing and hinge supports, is explored in this paper.

Ambiguous Reaction Couple (ARC) Approach to 3D Bearing and Hinge Supports

In the context of statics, we define an "ambiguous reaction couple" or ARC as a reaction couple due to a 3D bearing or hinge support that has a unique property: It *may* or *may not* be needed to be considered in the force and moment equilibrium equations for a given situation. The ARC will be identified with an asterisk, M_O^*. The purpose of presenting this approach in this context is to develop a uniform series of steps that allows for the FBDs of three-dimensional rigid bodies or structures containing bearings and hinges to be properly diagrammed and analyzed from a mechanics perspective. Some of the steps that will be outlined below are not entirely new concepts, but serve as effective intermediate tools in helping us develop this uniform methodology.

Step 1: Identify the type of bearing/hinge support and draw the FBD There are different types of bearing and hinge supports and it is very important that the proper reactions are considered when drawing the FBD. A few common supports and their reactions are discussed below:

- *Ball bearing:* This bearing type is designed to transmit radial forces. Assuming the bearing to be frictionless, there will only be reaction forces in the non-axial directions in the FBD. Each ball bearing should ideally be in contact with the shaft at a single point, which means that there is no need to account for reaction couples since the bearing will not be able to resist shaft rotation along any direction.

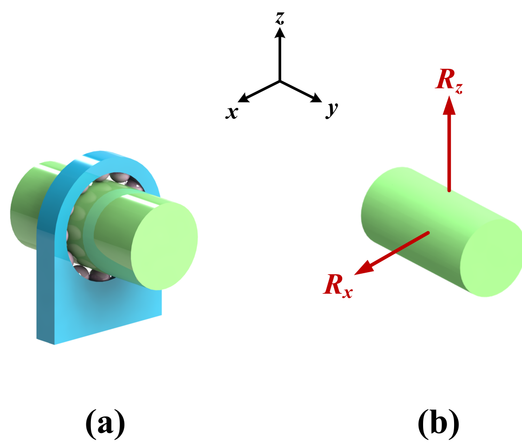

Figure 1 (a) Ball bearing; (b) Reactions on a shaft due to the ball bearing

- *Journal (or radial) bearing:* This bearing type is also designed to transmit radial forces. However, the friction experienced due to the presence of a journal bearing is significantly larger than a ball bearing. For this bearing type, the shaft is allowed to rotate about and translate along the axial direction. Hence, reaction forces and possible reaction couples (denoted as ARCs in Figure 2) should be included when drawing the FBD.

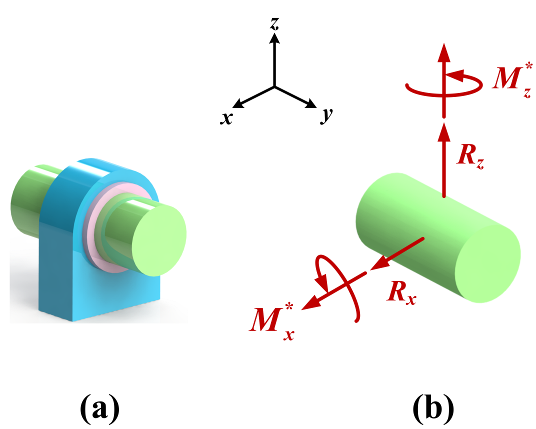

Figure 2 (a) Journal bearing; (b) Reactions on a shaft due to the journal bearing

- *Thrust bearing:* This bearing type is designed to transmit both radial and axial forces. Similar to the journal bearing, the shaft is allowed to rotate along the axial direction, but unlike the journal bearing, it is not allowed to translate axially, which implies that when drawing the FBD, reaction forces in all three directions and possible reaction couples about the non-axial axes (denoted as ARCs in Figure 3) should be considered.[1]

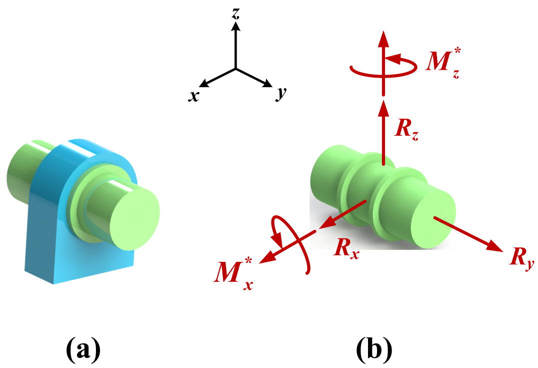

Figure 3 (a) Thrust bearing; (b) Reactions on a shaft due to the thrust bearing

- *Hinge:* A hinge limits the relative motion between two objects to an angular displacement with respect to a fixed axis. The reactions due to a hinge are similar to that of a thrust bearing, as shown in Figure 4.

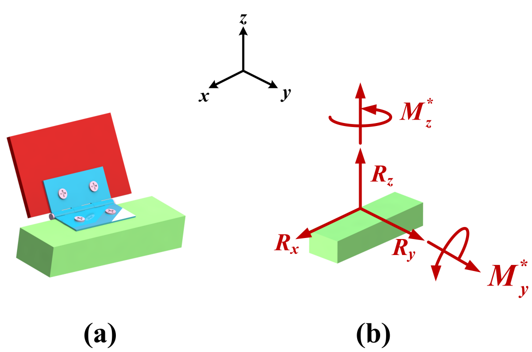

Figure 4 (a) Hinge support; (b) Reactions due to a hinge support

It is important to note that a student is not expected to memorize these reactions and should be able to derive them and draw the proper FBD when they encounter each case as long as they have a basic understanding on how the bearing or hinge is limiting motion. In terms of drawing the free-body diagram, which is the most important step in solving virtually any

[1]The term "thrust" bearing is a generic word used to describe a bearing that can transmit both radial and axial forces. In Statics texts that do discuss thrust bearings, they are graphically represented similar to Figure 3. However, different types of thrust bearings such as ball, cylindrical roller, spherical roller and tapered roller look quite different in real-world applications. The reader is encouraged to explore some of these bearings along with their applications from vendors such as Timken®.

mechanics problem, mnemonics such as BREAD[10] and techniques like the exploded-view approach[11] can assist the students in arriving at the correct FBD.

Step 2: Write the force and moment equilibrium equations Once the proper FBD is (hopefully) obtained, the equilibrium equations should be applied to the FBD:

$$\Sigma F_x = 0 \tag{1}$$

$$\Sigma F_y = 0 \tag{2}$$

$$\Sigma F_z = 0 \tag{3}$$

$$\Sigma M_{O,x} = 0 \tag{4}$$

$$\Sigma M_{O,y} = 0 \tag{5}$$

$$\Sigma M_{O,z} = 0 \tag{6}$$

Note that if we are dealing with a bearing or hinge, the ARC *should* be present in the FBD and equilibrium equations. A suggested approach to implement in order to solve such problems is to establish an *equation/unknown* table, similar to the one shown in Table 1. There are clear advantages to utilizing such a table. It allows the student to visually account for their unknowns and make sure that they have enough equations to solve the problem.[2] It also prepares the student to solve more complex problems in their upcoming engineering courses like system dynamics where multi-domain problems involving over 20 equations and unknowns are not unusual.

Table 1 Equations/unknowns table

Equations	Unknowns
(1)	F_1
(2)	F_2
(3)	F_3
(4)	M_1
(5)	M_2
(6)	M_3

Step 3: Identify whether the ARCs are true reaction couples or should be ignored After writing all of the equilibrium equations and establishing the equations/unknowns table, it is now time to make the important decision of whether to keep the ARCs as unknowns or neglect them from further consideration. Assuming that the problem is well-posed and statically determinate, there a handful of possible scenarios to consider:

[2] In statics, this is referred to as *static determinacy*. While the equations/unknowns table is *usually* helpful in identifying a statically determinate system, it by no means guarantees it. For example, if there are more than three unknown forces along the same direction, statics alone cannot be used to find the unknowns, even though the equation/unknown table might suggest an equal number of unknowns and equations in a given problem.

1. If there are as many equations as unknowns in the table: The ARCs will be necessary to maintain static equilibrium. Therefore, we can drop the asterisk and solve for all the unknowns.

2. If there are more unknowns than equations in the table: Eliminate all of the ARCs from consideration first. Two scenarios are possible:

   (a) If the number of equations and unknowns are now equal, the bearings or hinges were properly aligned and no reaction couples due to these supports need to be considered.

   (b) If there are now more equations than unknowns, then at least one of the ARCs should be brought back to maintain static equilibrium. This would require a careful reexamination of the problem schematic and the FBD equations. If any pair of bearings or hinges are properly aligned along the same axes, those ARCs should be eliminated. If the removal of any of the ARCs makes it impossible to achieve static equilibrium, that reaction moment should not be ignored.

Step 4: Solve for the unknowns Once the number of equations and unknowns match, we can solve for the unknowns using linear algebra techniques or through a computer software such as MathCAD, Maple, Mathematica or MATLAB.

Examples

Example 1: Need all of the ARCs A bent rod is subjected to external forces as shown in Figure 5 and held in static equilibrium by a rocker support at A and a journal bearing at C. The shorter leg of the rod is 30 cm while the longer leg is 40 cm. Points B and D are located at the center of each leg. Determine the reactions.

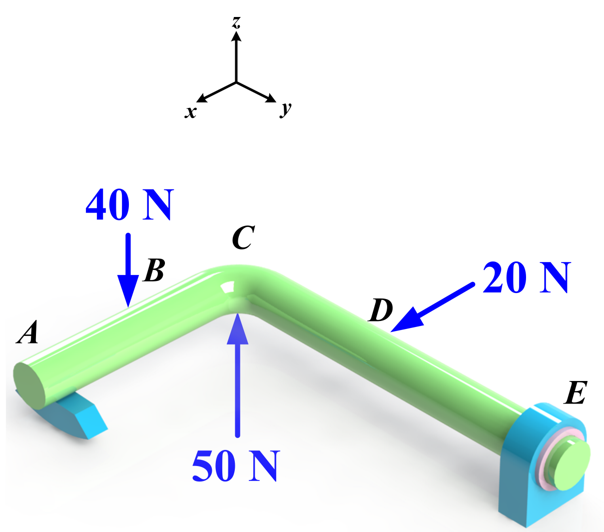

Figure 5 Example 1

Example 1 Solution The free-body diagram for this example is shown in Figure 6. In vector form, the applied/reaction forces and moments to the FBD are:

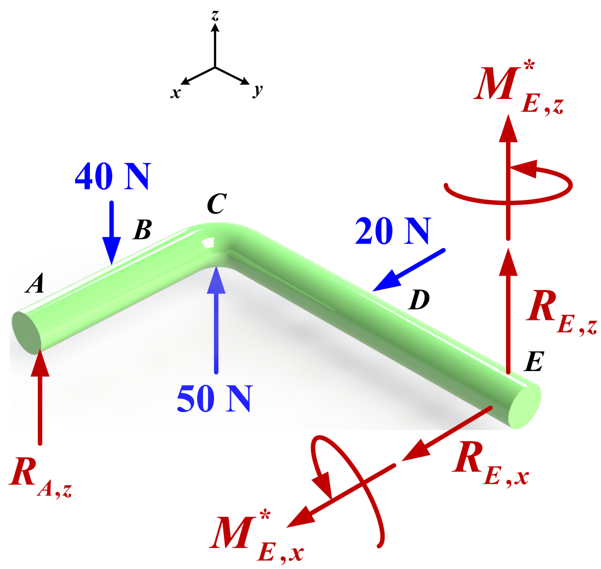

Figure 6 Example 1 FBD

$$\overrightarrow{F_A} = R_{A,z}\,\hat{k} \qquad \overrightarrow{F_B} = -40\hat{k}\ \mathrm{N} \qquad \overrightarrow{F_C} = 50\hat{k}\ \mathrm{N}$$

$$\overrightarrow{F_D} = 20\hat{\imath}\ \mathrm{N} \qquad \overrightarrow{F_E} = R_{E,x}\hat{\imath} + R_{E,z}\hat{k} \qquad \overrightarrow{M_E^*} = M^*_{E,x}\hat{\imath} + M^*_{E,z}\hat{k}$$

Equilibrium of forces results in:

$$\Sigma\vec{F} = \vec{0}$$

$$\hat{\imath}: \qquad R_{E,x} + 20\ \mathrm{N} = 0 \tag{7}$$

$$\hat{k}: \qquad R_{A,z} + R_{E,z} + 50\ \mathrm{N} - 40\ \mathrm{N} = 0 \tag{8}$$

Applying moment equilibrium about Point *E* results in:

$$\Sigma\overrightarrow{M_E} = \vec{0}$$

$$\overrightarrow{M_E^*} + \overrightarrow{r_{EB}} \times \overrightarrow{F_B} + \overrightarrow{r_{EC}} \times \overrightarrow{F_C} + \overrightarrow{r_{ED}} \times \overrightarrow{F_D} = \vec{0}$$

$$\hat{\imath}: \qquad M^*_{E,x} - (40\ \mathrm{cm})R_{A,z} - 200\ \mathrm{N\cdot cm} + 1600\ \mathrm{N\cdot cm} = 0 \tag{9}$$

$$\hat{\jmath}: \qquad 600\ \mathrm{N\cdot cm} - (30\ \mathrm{cm})R_{A,z} = 0 \tag{10}$$

$$\hat{k}: \qquad 400\ \mathrm{N\cdot cm} + M^*_{E,z} = 0 \tag{11}$$

As we derive each equilibrium equation, it is a good idea to properly account for the equations and unknowns instantaneously until a completed table similar to Table 2 is found.

Table 2 Original equations/unknowns table for Example 1

Equations	Unknowns
(7)	$R_{E,x}$
(8)	$R_{A,z}$
(9)	$R_{E,z}$
(10)	$M^*_{E,x}$
(11)	$M^*_{E,z}$

We now have to evaluate whether the ARCs $M^*_{E,x}$ and $M^*_{E,z}$ are required for static equilibrium. From Table 2, since we currently have as many equations as unknowns, the ARCs will be needed for static equilibrium and thus, should be treated as needed reaction couple, as reflected in the final equations/unknowns Table 3.

Table 3 Final equations/unknowns table for Example 1

Equations	Unknowns
(7)	$R_{E,x}$
(8)	$R_{A,z}$
(9)	$R_{E,z}$
(10)	$M_{E,x}$
(11)	$M_{E,z}$

Simultaneously solving for 5 equations and unknowns results in:

$$\boxed{\begin{gathered} R_{E,x} = 20 \text{ N} \nearrow \\ R_{A,z} = 20 \text{ N} \uparrow \\ R_{E,z} = 30 \text{ N} \downarrow \\ M_{E,x} = 600 \text{ N} \cdot \text{cm} \circlearrowright \\ M_{E,z} = 400 \text{ N} \cdot \text{cm} \circlearrowright \end{gathered}}$$

This example demonstrates a scenario where static equilibrium would not be met without the support reaction couples due to the journal bearing. The three external forces acting on the rod contribute to moments about point *E* in all three directions which requires counteraction from support forces and couples. In this case, the reaction couples from the journal bearing counteract the net moment about the *x* and *z* axes, while the rocker reaction is responsible for opposing rotation about the *y* axis. Next, we will consider a case where the reaction couples should all be ignored.

Example 2: Do not need any of the ARCs A bent rod is subjected to a 40-N external force in the *xz*-plane as shown in Figure 7 and held in static equilibrium by a rocker support at *A*, a thrust bearing at C and a journal bearing at D. The shorter leg of the rod is 30 cm while the longer leg is 40 cm. Points B and C are located at the center of the shorter and longer legs, respectively. Determine the reactions.

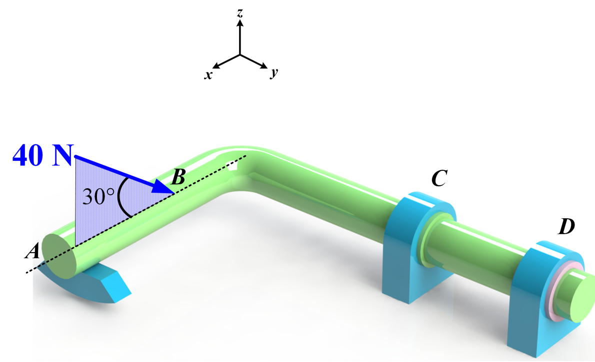

Figure 7 Example 2

Example 2 Solution The free-body diagram for this example is shown in Figure 8. In vector form, the applied/reaction forces and moments to the FBD are:

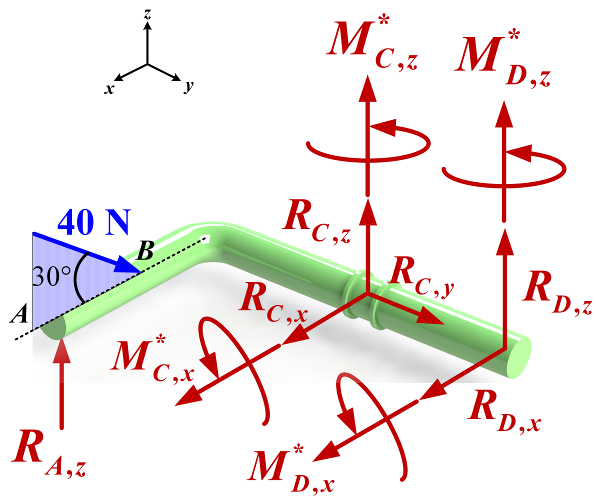

Figure 8 Example 2 FBD

$$\overrightarrow{F_A} = R_{A,z}\,\hat{k} \qquad \overrightarrow{F_B} = -10\sqrt{3}\hat{\imath} - 20\hat{k}\text{ N} \qquad \overrightarrow{F_C} = R_{C,x}\hat{\imath} + R_{C,y}\hat{\jmath} + R_{C,z}\hat{k}$$

$$\overrightarrow{F_D} = R_{D,x}\hat{\imath} + R_{D,z}\hat{k} \qquad \overrightarrow{M_C^*} = M^*_{C,x}\hat{\imath} + M^*_{C,z}\hat{k} \qquad \overrightarrow{M_D^*} = M^*_{D,x}\hat{\imath} + M^*_{D,z}\hat{k}$$

Equilibrium of forces results in:

$$\Sigma\vec{F} = \vec{0}$$

$$\hat{\imath}: \quad R_{C,x} + R_{D,x} - 10\sqrt{3} = 0 \tag{12}$$

$$\hat{\jmath}: \quad R_{C,y} = 0 \tag{13}$$

$$\hat{k}: \quad R_{A,z} + R_{C,z} + R_{D,z} - 20\text{ N} = 0 \tag{14}$$

Applying moment equilibrium about Point *C* results in:

$$\Sigma\overrightarrow{M_C} = \vec{0}$$

$$\overrightarrow{M_C^*} + \overrightarrow{M_D^*} + \overrightarrow{r_{CA}} \times \overrightarrow{F_A} + \overrightarrow{r_{CB}} \times \overrightarrow{F_B} + \overrightarrow{r_{CD}} \times \overrightarrow{F_D} = \vec{0}$$

$$\hat{i}: \quad M_{C,x}^* + M_{D,x}^* + (20\text{ cm})R_{D,z} - (20\text{ cm})R_{A,z} + 400\text{ N}\cdot\text{cm} = 0 \tag{15}$$

$$\hat{j}: \quad 300\text{ N}\cdot\text{cm} - (30\text{ cm})R_{A,z} = 0 \tag{16}$$

$$\hat{k}: \quad M_{C,z}^* + M_{D,z}^* - 20R_{D,x} - 200\sqrt{3}\text{ N}\cdot\text{cm} = 0 \tag{17}$$

The initial equations/unknowns table should look like Table 4 below:

Table 4 Original equations/unknowns table for Example 2

Equations	Unknowns
(12)	$R_{C,x}$
(13)	$R_{D,x}$
(14)	$R_{C,y}$
(15)	$R_{A,z}$
(16)	$R_{C,z}$
(17)	$R_{D,z}$
	$M_{C,x}^*$
	$M_{D,x}^*$
	$M_{C,z}^*$
	$M_{D,z}^*$

From Table 4, it is clear that static equilibrium will be met if we eliminate the ARCs:

Table 5 Final equations/unknowns table for Example 2

Equations	Unknowns
(12)	$R_{C,x}$
(13)	$R_{D,x}$
(14)	$R_{C,y}$
(15)	$R_{A,z}$
(16)	$R_{C,z}$
(17)	$R_{D,z}$

Simultaneously solving for 6 equations and unknowns results in:

$$R_{C,x} = 20\sqrt{3}\ \mathrm{N} \swarrow$$
$$R_{D,x} = 10\sqrt{3}\ \mathrm{N} \nearrow$$
$$R_{C,y} = 0\ \mathrm{N}$$
$$R_{A,z} = 10\ \mathrm{N} \uparrow$$
$$R_{C,z} = 20\ \mathrm{N} \uparrow$$
$$R_{D,z} = 10\ \mathrm{N} \downarrow$$

In this example, the problem becomes statically indeterminate if the ARCs are kept as unknowns, which implies that if the two bearings fall out of proper alignment for any reason, additional equations from mechanics of materials will likely be needed to solve for all of the unknowns. Finally, we will consider a case where some of the ARCs will be required and some will not in order to maintain static equilibrium.

Example 3: Need some of the ARCs Two rods connected via two bevel gears are subjected to a 40-N external force along the *y* axis as shown in Figure 9 and held in static equilibrium by three journal bearings at *A*, *B* and *C*. The shorter leg of the rod is 30 cm while the longer leg is 40 cm. Point *B* is located at the center of the longer leg. Determine the reactions assuming that the reaction couple in the *z*-direction of the journal bearing at *A* is negligible and that the journal bearing reaction force at *C* is $10\ \mathrm{N}$ in the +*z*-direction and unknown in the *x*-direction.

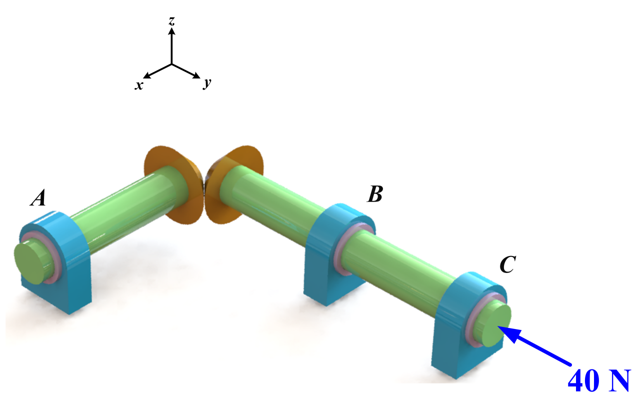

Figure 9 Example 3

Example 3 Solution The free-body diagram for this example is shown in Figure 10. In vector form, the applied/reaction forces and moments to the FBD are:

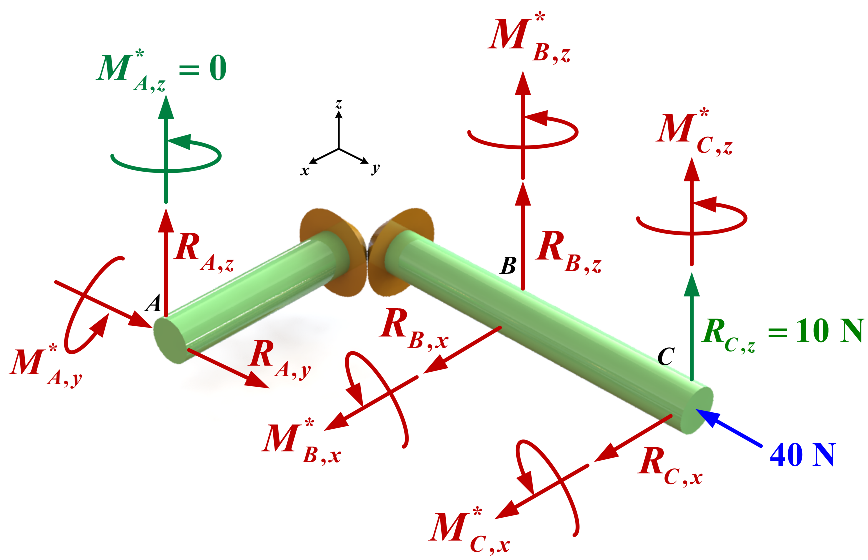

Figure 10 Example 3 FBD

$$\overrightarrow{F_A} = R_{A,y}\hat{\jmath} + R_{A,z}\hat{k} \qquad \overrightarrow{F_B} = R_{B,x}\hat{\imath} + R_{B,z}\hat{k} \qquad \overrightarrow{M_A^*} = M^*_{A,y}\hat{\jmath}$$

$$\overrightarrow{F_C} = R_{C,x}\hat{\imath} - 40\hat{\jmath} + 10\hat{k}\text{ N} \qquad \overrightarrow{F_D} = R_{D,x}\hat{\imath} + R_{D,z}\hat{k}$$

$$\overrightarrow{M_B^*} = M^*_{B,x}\hat{\imath} + M^*_{B,z}\hat{k} \qquad \overrightarrow{M_D^*} = M^*_{C,x}\hat{\imath} + M^*_{C,z}\hat{k}$$

Equilibrium of forces results in:

$$\Sigma\vec{F} = \vec{0}$$

$\hat{\imath}$: $$R_{B,x} + R_{C,x} = 0 \tag{18}$$

$\hat{\jmath}$: $$R_{A,y} - 40\text{ N} = 0 \tag{19}$$

$\hat{k}$: $$R_{A,z} + R_{B,z} + 10\text{ N} = 0 \tag{20}$$

Applying moment equilibrium about Point *C* results in:

$$\Sigma\overrightarrow{M_C} = \vec{0}$$

$$\overrightarrow{M_A^*} + \overrightarrow{M_B^*} + \overrightarrow{M_C^*} + \overrightarrow{r_{CA}} \times \overrightarrow{F_A} + \overrightarrow{r_{CB}} \times \overrightarrow{F_B} = \vec{0}$$

$\hat{\imath}$: $$M^*_{B,x} + M^*_{C,x} - (20\text{ cm})R_{B,z} - (40\text{ cm})R_{A,z} = 0 \tag{21}$$

$\hat{\jmath}$: $$-(30\text{ cm})R_{A,z} + M^*_{A,y} = 0 \tag{22}$$

$\hat{k}$: $$M^*_{B,z} + M^*_{C,z} + 20R_{B,x} + 30R_{A,y}\text{ N}\cdot\text{cm} = 0 \tag{23}$$

Table 6 Original equations/unknowns table for Example 3

Equations	Unknowns
(18)	$R_{B,x}$
(19)	$R_{C,x}$
(20)	$R_{A,y}$
(21)	$R_{A,z}$
(22)	$R_{B,z}$
(23)	$M^*_{B,x}$
	$M^*_{C,x}$
	$M^*_{A,y}$
	$M^*_{B,z}$
	$M^*_{C,z}$

The initial equations/unknowns table should look like Table 6. Eliminating all the ARCs in Table 6 will result in 6 equations and 5 unknowns, which means that one of the ARCs would be needed in order to maintain static equilibrium. From the problem diagram, journal bearings *B* and *C* are properly aligned, which leads to the final version of the equations/unknowns table in Table 7.

Table 7 Final equations/unknowns table for Example 3

Equations	Unknowns
(18)	$R_{B,x}$
(19)	$R_{C,x}$
(20)	$R_{A,y}$
(21)	$R_{A,z}$
(22)	$R_{B,z}$
(23)	$M_{A,y}$

Simultaneously solving for 6 equations and unknowns results in:

$$\boxed{\begin{gathered} R_{B,x} = 60\text{ N} \nearrow \\ R_{C,x} = 60\text{ N} \swarrow \\ R_{A,y} = 40\text{ N} \searrow \\ R_{A,z} = 10\text{ N} \uparrow \\ R_{B,z} = 20\text{ N} \downarrow \\ M_{A,y} = 300\text{ N} \cdot \text{cm} \circlearrowleft \end{gathered}}$$

The presence of an odd number of bearings in this problem leads to the unusual scenario of requiring only some of the ARCs to be eliminated and one ARC to be considered as a true reaction couple. This example also demonstrates one of the limitations of statically determinate systems with bearings: Since each bearing type requires at least two reaction forces to be accounted for, we cannot have any more than *three* bearings in a statically determinate problem without some of the reaction forces or couples being known apriori. In this case, since the journal bearing at A needed to counteract the moment due its own reaction force, its ARC in the y-direction needed to be considered, which meant that some of the reactions needed to be provided in the problem statement.

Conclusions and Future Work

In this work, the ambiguous reaction couple (ARC) approach to analyzing bearing and hinge supports in 3D statics problems has been presented and its utility is demonstrated with three examples. While most bearings are designed to maintain alignment in a mechanical system, most can also transmit a small reaction couple. This means that in problems where the bearings are not properly aligned, reaction couples may need to be accounted for. The dual nature of reaction couples for bearings led to the development of the ARC approach in which all the possible reaction couples are first considered during the application of the equilibrium equations and subsequently a decision is made on whether to keep or discard each reaction couple with the help of an equations/unknown table. As the three examples in this paper demonstrate, this approach can be universally applied to any statics problem involving bearings or hinges with a reasonably straightforward result. The ARC approach can also be helpful in explaining the proper application of bearings and hinges. Examples 1 and especially 3 are poor applications of bearings and hinges in a real-world scenario. When discussing this topic, instructors should encourage students to think about the situations that necessitate the application of bearings and hinges and why the case where the elimination of all of the ARCs leads to a statically determinate system is preferred to others. It is the authors' sincere hope that this technique will help make this topic less confusing for students compared to how we learned it when we were in their position.

The primary work that is still left to do is to assess the effectiveness of the ARC approach in the classroom. There are, however, a couple of impediments to this process *in a Statics course*. First, 3D supports, particularly those that involve bearings and hinges, can be a rather esoteric component of most Statics courses and are often either not taught at all or briefly covered without much depth. Collectively, the authors have taught Statics at three different institutions and at only one did the coverage entail bearing and hinge supports. Bearing and hinge supports are not a part of the curriculum of the Statics course at our current institution, Rose-Hulman Institute of Technology, and as such, we cannot assess the effectiveness of the ARC approach in our classes without significantly modifying the curriculum. Furthermore, 3D rigid-body problems in general are not as common as 2D rigid-body problems in Statics courses because they are more difficult to draw, the FBDs are generally not as complex, the translation of the FBD to equilibrium equations can be much more mathematical as opposed to intuitive and going from three to up to six equations and unknowns can be more time-consuming. All of this means that unless a 3D rigid-body problem is relatively simple, it is difficult to have a summative assessment of student performance in an exam-type setting, for example. This does not mean that making such an evaluation is impossible; it just means that the particular problem that is designed to assess this topic is very targeted. If any reader of this work is interested in assessing this technique in their Statics class, we would be more than happy to provide any assistance that we can.

Even though assessment of the ARC approach is rather difficult in Statics, we do still plan to study the effectiveness of this technique in a series of courses where its application may be very important - the Capstone Design sequence at our institution. One of the authors regularly teaches Capstone Design and has often seen students struggle with analyzing 3D components of their designs, especially when bearing are involved. We plan on utilizing the crux of this paper for seniors in Capstone Design as a self-study guide supplemented by a short video in order to evaluate the effectiveness of this approach in improving student understanding of the analysis of their designs and reporting our findings in a follow-up paper.

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