Demonstrating Techniques for Estimating the Constant of Variation in Commonly Occurring Variation Problems in College Algebra Textbooks

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Abstract

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In many College Algebra Textbooks, the section on variation contains direct variation problems of which Hooke's law for an elastic spring is an example, and inverse variation problems of which the illumination produced by a light source is an example. Each requires that the constant of variation be determined first based on a given set of values for the unknowns. Using this constant, the student is then asked to find one of the unknowns given the values of the others.

In the engineering component of LaPREP, a nationally acclaimed intervention program in engineering, math and science for high-ability middle and early high school students held on the LSU-Shreveport campus, hands-on activities were developed to estimate constants of variation for both direct and inverse variation problems. Using Hooke's law as an example of a direct variation problem, several combinations of springs and weights were used to estimate coefficients of variation and the distant a spring stretched when a new weight was applied. In the inverse variation example, several combinations of distances and bulb wattages were used to estimate coefficients of variation and the corresponding illumination at specific distances from the light source. This paper gives a brief history and some of the accomplishments of LaPREP as well as specifics of the experiments and the several problems encountered in conducting them.

LaPREP Program

LaPREP (Louisiana Preparatory Program) is a two-summer enrichment program which identifies, encourages, and instructs competent middle and early high school students, preparing them to complete a college degree program, in engineering, math, or science[1]. It is not a remedial program; rather, it seeks to engage and challenge students at a time when they are particularly vulnerable to nonacademic distractions. LaPREP, which takes place on the LSU-Shreveport campus seven weeks a summer over two consecutive summers, emphasizes abstract reasoning, problem solving and technical writing skills, mainly through mathematics enrichment courses and seminars. Class assignments, laboratory projects and scheduled exams are integral parts of LaPREP. The faculty is drawn from LSU-Shreveport and the local school system.

LaPREP targets bright students who may be financially disadvantaged and those who are first generation college bound. In the eleven years of LaPREP, approximately 80% of its more than 300 participants have been minority students and have come from virtually every middle school in the Shreveport-Bossier area.

Each summer 30 first-year participants join with approximately 25 returning second year participants for a summer of intellectually stimulating work and fun on the LSUS campus. Students successfully completing the first summer session with a 70% or better average are eligible for the second summer session in 2004.

The topics studied over two summer sessions include:

- Engineering
- Logic
- Algebraic Structures
- Probability and Statistics
- Problem Solving
- Technical Writing
- ACT Preparation
- Medical Career Preparation
- Drug, Alcohol, and Gang Awareness and Prevention

Other features include: field trips to local industries, visiting lecturers and minority speakers, college and career awareness, swimming, basketball, ping pong, pool and other recreation.

Since a significant number of LaPREP students come from low-income families, LaPREP charges no tuition or fees. LaPREP provides free transportation to and from the program site via Sportran bus passes, free lunches in the University Center, books and other materials needed for classes, and cost-free field trips.

LaPREP Accomplishments

LaPREP will begin its twelfth annual summer session on the campus of LSUS in June of 2003. Evaluations contributed by the participants of the program, their parents, and by local and state officials who have visited the program have shown the program to be highly successful. Participant interest in attending college and majoring in math or science has greatly increased. No former LaPREP participant has dropped out of high school, and all who have been eligible have enrolled in college. Eighty-four percent of exiting participants have indicated LaPREP has increased their desire to study math and science. Moreover, the first 89 LaPREP graduates who became eligible enrolled in college, and almost 90% of them responding to a survey indicated they were majoring in engineering, math, or science.

LaPREP has received honors both locally and nationally. Dr. Carlos Spaht, LaPREP founder and director, has received prestigious awards resulting from his work with LaPREP: The Jefferson Award for outstanding contribution to public service; the Jacqueline Kennedy Onassis Award, the highest public service award offered nationally; and the White House Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring. At the state and regional level, Dr. Spaht has been awarded the Carnegie Foundation's Louisiana Professor of the Year award and the LA/MS Mathematical Association of America's award for Distinguished Teaching of Mathematics and the Governor's Award of Excellence, all due in part to his work with LaPREP.

LaPREP has been recognized by the National Science Foundation, listing it in its directory of enrichment programs and the Mathematical Association of America, praising it for its contribution to mathematics in Louisiana. In addition, the Shreveport City Council passed a resolution applauding LaPREP "for making a positive impact on the lives of young people and for contributing to the future prosperity of the community and the nation."

Introduction to the Variation Problems

Many College Algebra texts contain a section on variation and accompanying exercises in which students must determine the constant of proportionality based on given information before they can complete the problem. One such exercise in which one estimates the constant of proportionality by experimentation has been reported in Proceedings of the 2002 ASEE Gulf-Southwest Annual Conference[2]. In this exercise, LaPREP students estimated the constant of proportionality for a column of given height, diameter, and load.

In the 2002 LaPREP Engineering component, experiments to estimate the constant of proportionality for two different variation problems – the inverse square law for illumination and Hooke's law for an elastic spring – were conducted. Appropriate versions of each law can be found in Algebra for College Students (Lial and Hornsby, 2000)[3].

Hooke's Law for an Elastic Spring

Hooke's Law is a typical variation problem found in College Algebra textbooks. Lial and Hornsby in their Algebra for College Students, 4th Edition[3] have: "Hooke's law for an elastic spring states that the distance a spring stretches is proportional to the force applied."

Initially, equipment used for the project included a metal spring stand having an adjustable horizontal arm and an adjustable scale, which could be moved up and down the vertical post. This unit bore no maker's mark so it was impossible to determine its origin. In addition, a set of eighteen disk-shaped weights, ranging from 10 to 500 grams, manufactured by the Welch Scientific Company, a weight tray on which to place the weights, and assorted springs were used in the project.

At the beginning of the experiment, students worked in groups of four or five, taking turns setting up the equipment. Each group selected a spring and weighs, helped determine the displacement of the spring when the weight was added to the weight tray and then estimated the constant of proportionality from the formula Y = k X where Y was the displacement in millimeters, X was the weight in grams, and k was the constant of proportionality.

Results

Three different springs were used in the project and for each spring, the constant of proportionality was determined for each of four different weights, using the formula Y = k X, where Y is the displacement in millimeters, X is the weight in grams, and k is the constant of proportionality.

Spring #1:

X (grams)	Y (displacement)	k (constant of proportionality)
1. 50	0.6	0.012
2. 150	1.2	0.008
3. 250	1.5	0.006
4. 500	2.2	0.0044

Spring #2:

X (grams)	Y (displacement)	k (constant of proportionality)
1. 20	10	0.50
2. 40	18	0.45
3. 60	24	0.40
4. 70	31	0.39

Spring #3:

X (grams)	Y (displacement)	k (constant of proportionality)
1. 20	9	0.45
2. 40	20	0.50
3. 60	30	0.50
4. 100	54	0.54

Discussion

The spring's displacement proved to be somewhat difficult to determine, as it was almost impossible to read the mark on the adjustable scale when aligned with the bottom of the weight tray. Some of the difficulty was due to the fact that the scale was chrome plated and the reflection from the plated surface washed out the marks on the scale. In an attempt to make it easier to line up the marks on the scale with the underside of the weight tray, it was decided to attach a thin straight piece to the underside of the circular tray, which would extend far enough to meet the marks on the scale. Several items were tried, but the one eventually used was a 3 ¾ " section of a hacksaw blade attached with rubber cement. Although this attachment helped some, all participants agreed that it was still difficult to get a good reading of the displacement, even though different viewing angles and sources of illumination were used.

Note that for spring #1, there was considerable variability in the values for k. Since this was the first spring to be used in the project, the spring was carefully examined to determine if there was another explanation as to why the values for k varied so much in addition to the previously mentioned difficulty in reading the spring's displacement. Upon inspection, it was recalled that due to the spring's stiffness, absence of loops at each end from which to hang a weight tray or to attach to the arm of the adjustable arm, it was impossible for the spring to hang vertically. The values for k were more consistent for the other two springs, which suggests that spring #1 was not one suitable for this experiment.

Once the values for k had been determined through experimentation, students found the arithmetic mean of the k values for each spring, and then used this to predict for each spring the displacement when a different weight was used.

The Inverse Square Law

The Inverse Square Law as found in Lial and Hornsby's Algebra for College Students, 4th edition[3], states: "The illumination produced by a light source varies inversely as the square of the distance from the source." Equipment used in this project included a General Electric light meter type 214, a Cenco stand with spring loaded clamp to hold the light meter, a Cenco stand with light bulb socket, a fifteen watt Westinghouse Soft White incandescent bulb, a 40 watt incandescent light bulb with no maker's mark, a 60 watt equivalent Lights of America Mini-Twister compact fluorescent bulb, and a yardstick.

This project also was one in which students worked in groups of four or five, taking turns setting up the equipment, choosing the wattage of the bulb, and verifying the illumination in foot-candles. The procedure was simple – the stand with the light bulb was centered over the zero mark on the yard stick and two illumination readings were taken for each light bulb, one with the Cenco light meter stand positioned so that the illumination reading was 800 footcandles, then 200 footcandles. Once the corresponding distances were recorded, the values of k were determined by substituting in for d and I in the formula $k = d^2 \times I$ where I is the illumination in footcandles, d is the distance of the light meter from the light source in feet, and k is the constant of proportionality.

Results

Bulb #1 (15 watt incandescent):

I (in footcandles)	d (feet)	k
800	2.625/12	38.28
200	5/12	34.72

Bulb #2 (40 watt incandescent):

I (in footcandles)	d (feet)	k
800	5.25/12	153.13
200	10.5/12	153.13

Bulb #3

I (in footcandles)	d (feet)	k
800	6.125/12	208.42
200	13/12	234.72

Discussion

In selecting the I-values, two things were taken into consideration: (1) I-values which should allow for quick mental estimation of the required distances (doubling) to reduce the illumination by 75% were desirable; (2) The values on the light meter's scale ranged from 200 to 1000 footcandles, with values given at the 200, 400, 600, 800, and 1000 footcandle points and with a mark representing every 20 footcandles in between. A reduction in illumination of 75% required either the use of 1000 and 250 footcandles or 800 and 200 footcandles. It was much easier to use the 200-footcandle mark than to estimate the point on the scale where the illumination was 250 footcandles. Other options such as 900 and 225 were not considered because of the difficulty in reading the measurements.

Since it was expected that the distance would be doubled when the illumination was reduced by 75%, it is clear that there was a difference in the expected and observed values of the distances for bulbs #1 and #3. There are several factors which may have contributed to these discrepancies: (1) Ambient lighting. Since both daylight from the unshaded windows and that from overhead fluorescent light banks were present at the same time the measurements were being taken, this lighting, different from the light source, may have affected the illumination measurements; (2)

Difficulty in reading the light meter scale. The light meters needle's width was about 10 footcandles on the scale, so a measurement error of 10 footcandles, which would have been easy to make, could considerably change the corresponding values for k. For example, if when working with bulb #3, the footcandle readings were actually 810 and 190, the corresponding k values would have been much closer (211.03 at 810 footcandles and 222.99 at190 footcandles); (3) Difficulty in reading the marks on the yardstick. Both Cenco stands were centered over the yardstick, with the front of the light meter extended so that it was approximately parallel with the front of the stand, making exact measurement of distances a bit of a problem. It is conceivable that an error of 1/16", certainly within the realm of possibilities, could account for some of the differences in distance measurements, and in the resulting values for the constant of proportionality, k.

References

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