SOME THOUGHTS FOR TEACHING ENGINEERING STATICS

ABSTRACT

Engineering statics is a core technical and gate-keeper course for any mechanical engineering program. During the last several years, it has been noticed that there are two common critical issues for this course: implementation of the parallelogram law and drawing free-body diagrams. The parallelogram law is used to conduct vector addition and the reversed parallelogram law is used to resolve a vector into two components. This paper will present and explain how to jump over the general applications of the parallelogram law and directly discuss Cartesian vectors addition. We will present and systematically explain all possible five types of free-body diagrams to facilitate students to draw free-body diagrams.

1. INTRODUCTION

Engineering statics is typically offered in sophomore year for a mechanical engineering program. It is a gate-keeper course for any mechanical engineering program. Engineering statics is a challenging course for engineering students because vector operations, free-body diagrams, the concepts of moments, reactive forces, behaviors of different supports, force analysis on trusses, structures, machines, etc. are difficult to be mastered [1~4]. Engineering statics is a very important course for any mechanical engineering program. Lots of literature proposed and implemented different approaches to facilitate students to be successful in engineering statics, such as mechanical breadboard [1], different teaching styling [5], hands-on design project [6], sketch tool in SolidWorks [7], applying numerical modeling techniques [8], animated GIF files [9], task-analysis-guided deliberate practice [10], the explored view [11] and the inquiry-based approach [12], etc.

During the last several years, authors have noticed that there are two common critical issues for this course: implementation of the parallelogram law and drawing free-body diagrams. In the first two weeks of the engineering statics, the parallelogram law is normally taught for vector additions and resolution of a vector along two specific directions. Some students had difficulty in understanding the parallelogram law for vector additions and were in big trouble in the application of the parallelogram law for conducting vector additions. Some student lost their fight and failed the engineering statics due to the difficulty related to the parallelogram law at the beginning two weeks of this course. However, the parallelogram law is only one tool used at the beginning of the statics course and is not used in the rest of the engineering statics course. A thorough understanding of drawing a free-body diagram is of primary importance for solving problems in engineering statics. Some students have difficulties in drawing a proper free-body diagram. They said that they didn't know how to start and how to draw a free-body diagram because there were so many different possible situations. Without a free body diagram, students cannot properly solve engineering statics.

We proposed to jump over the general application of parallelogram law and directly discuss Cartesian vectors and Cartesian vectors addition as a special example of the parallelogram law.

As for the free-body diagram, we proposed to systematically explain and demonstrate all possible five types of free-body diagrams to facilitate students to overcome the issue of drawing free-body diagrams.

2. CARTESIAN VECTOR APPROACH

2.1 BACKGROUND

The Engineering statics in our mechanical engineering program is offered in the first semester of their sophomore year and is a 4 credit course in a fifteen-week fall semester with four-hour lectures per week and without a lab section. The textbook for this course is "Engineering Mechanics: Statics" by Dr. R.C. Hibbeler [13]. In the first two weeks, we typically discuss the definitions of vector and scalar. Then we start to discuss how to run vector addition and resolution of a vector along two specified directions by using the parallelogram law [13]. After a sketch of the vector addition of two vectors or a resolution of one vector along two lines is drawn, students are guided to use two trigonometry laws (Sine Law and Cosine Law) for the calculations of a vector's magnitude or angle.

Some students had difficulty understanding the parallelogram law. Some students had difficulty applying Sine Law or Cosine law to calculate the magnitudes or angle of a resultant vector or the magnitudes of two components along two specified lines of a vector. They were frustrated and lost confidence in learning engineering statics due to the difficulty in understanding and applying the parallelogram law in the first two weeks of the engineering statics course. Some students gave up, lost the fight, and failed the engineering statics.

Authors have been trying to find approaches to help students in this issue. The authors proposed an approach which is to sketch vector addition per the parallelogram law in SolidWorks and the magnitude and angle of the resultant vector can be directly measured through the SolidWorks sketch tool [7]. This approach helps them a lot, but they must have SolidWorks software and must use the SolidWorks sketch tool.

After the parallelogram law has been discussed, the Cartesian vector will be discussed in the engineering statics course. later in this engineering statics course, the Cartesian vector is the main tool for force vector operations such as force vector additions, position vector, dot product, and cross-product [13]. The parallelogram law is not be used anymore in the later study of engineering statics. In the last years, authors proposed another approach to help students to overcome the issue in the parallelogram law. The authors proposed that we could jump over the general parallelogram law and directly discuss the Cartesian vector for vector operations as a specific example of the parallelogram law. We tried this approach in the 2021 fall semester and found that students were able to effectively conduct force vector operations. The authors will implement this approach again and will collect some comparison data to see the effectiveness of this approach. This approach will be explained in detail in the next section.

2.2 USE THE CARTESIAN VECTOR TO REPLACE THE PARALLELOGRAM LAW

The vector addition must follow the parallelogram law. Two key applications of the parallelogram law in vector operations are (1) to explain the principle of vector addition and conduct the vector addition of two vectors and (2) to conduct the resolution of a vector in two specified directions or lines. When we directly jump over the general parallelogram law and directly discuss the Cartesian vector and vector addition as a special example of the parallelogram law, we must be able to explain and conduct these two mentioned tasks. we implemented this approach by doing the following three actions.

A special example of the parallelogram law

After the definition of a vector and its three fundamental elements: magnitude, line of action (angle) and sense have been discussed in detail, we start to discuss the Cartesian vector as a special example of the parallelogram law. The horizontal vector $\overrightarrow{\boldsymbol{F_x}}$ has a magnitude F_x along the x-axis. The vertical vector $\overrightarrow{\boldsymbol{F_y}}$ has a magnitude F_y along the x-axis as shown in Figure 1. Per the parallelogram law, the vector $\vec{\boldsymbol{F}}$ as shown in Figure 1 is the resultant vector of $\overrightarrow{\boldsymbol{F_x}} + \overrightarrow{\boldsymbol{F_y}}$, that is:

$$\vec{\boldsymbol{F}} = \overrightarrow{\boldsymbol{F_x}} + \overrightarrow{\boldsymbol{F_y}} \quad (1)$$

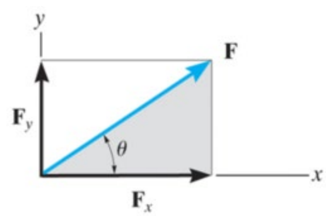

Figure 1 A special example of the parallelogram law for a vector addition [13]

This is a special example of the Parallelogram law. The two vectors are normal to each other and one is along the x-axis and another is along the y-axis. Since the shaded triangle in Figure 1 is a right-angle triangle, the magnitude and the angle of the resultant vector $\vec{\boldsymbol{F}}$ are:

$$F = \sqrt{F_x^2 + F_y^2}, \qquad \theta = tan^{-1}\left(\frac{F_y}{F_x}\right) \quad (2)$$

It was found that students easily accepted and understood the parallelogram law for this special case because the sketch of the parallelogram is a rectangular shape and the triangle for the calculation is the right angle. Another reason that students easily accepted this special example of parallelogram law is that students have already known about the right-angle triangle.

Expression of a vector as a Cartesian vector

After the special case of the parallelogram law as shown in Figure 1 had been explained and accepted by students, we introduced the Cartesian vector and explained how to express a vector as a Cartesian vector as shown in Figure 2.

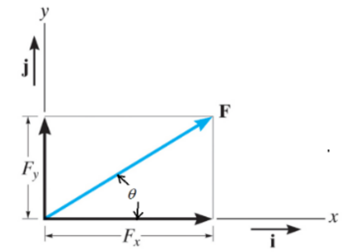

Figure 2 Expression of a vector as a Cartesian vector [13]

A vector $\vec{\boldsymbol{F}}$ has a magnitude F, an angle θ and the sense as shown in Figure 2. The angle θ is the angle between the vector $\vec{\boldsymbol{F}}$ and the positive x-axis in counter-clock direction. The horizontal vector $\overrightarrow{\boldsymbol{F_x}}$ with a magnitude F_x along the x-axis and can be expressed as:

$$\overrightarrow{\boldsymbol{F_x}} = F_x\vec{\boldsymbol{\imath}} = Fcos(\theta)\vec{\boldsymbol{\imath}} \tag{3}$$

Where $\vec{\boldsymbol{\imath}}$ is a unit vector along the x-axis.

The vertical vector $\overrightarrow{\boldsymbol{F_y}}$ with a magnitude F_y along the x-axis can be expressed as

$$\overrightarrow{\boldsymbol{F_y}} = F_y\vec{\boldsymbol{\jmath}} = Fsin(\theta)\vec{\boldsymbol{\jmath}} \tag{4}$$

Where $\vec{\boldsymbol{\jmath}}$ is a unit vector along the y-axis.

Per the special example of the parallelogram demonstrated in Figure 1, we have:

$$\vec{\boldsymbol{F}} = \overrightarrow{\boldsymbol{F_x}} + \overrightarrow{\boldsymbol{F_y}} = Fcos(\theta)\vec{\boldsymbol{\imath}} + Fsin(\theta)\vec{\boldsymbol{\jmath}} \tag{5}$$

This is a Cartesian Vector. This is the way to express a vector as a Cartesian vector. After we discuss the expression of a vector as a Cartesian vector, we use the Cartesian vector to discuss the addition of multiple vectors and other vector operations.

The resolution of a vector along two specified lines

Another important application of the parallelogram law is to dissolve a vector along two specified lines by using the reversed-parallelogram law. When the general case of the parallelogram law is not discussed, we can still use the Cartesian vector to solve this type of problem as shown in Figure 3. The vector $\vec{\boldsymbol{F}}$ with a magnitude F and its angle θ is resolved into two components along the line a and line b. The angles for line a and line b are γ and β. Let's assume that the magnitude of components along line a and line b are F_a and F_b. By using Cartesian vectors, we have:

$$Fcos(\theta)\vec{\boldsymbol{\imath}} + Fsin(\theta)\vec{\boldsymbol{\jmath}} = F_a cos(\gamma)\vec{\boldsymbol{\imath}} + F_a sin(\gamma)\vec{\boldsymbol{\jmath}} + F_b cos(\beta)\vec{\boldsymbol{\imath}} + F_b sin(\beta)\vec{\boldsymbol{\jmath}} \tag{6}$$

Per Eq. (6), we have:

$$Fcos(\theta) = F_a cos(\gamma) + F_b cos(\beta) \tag{7}$$

$$Fsin(\theta) = F_a sin(\gamma) + F_b sin(\beta) \tag{8}$$

From Eq. (7) and Eq. (8), we can solve F_a and F_b as follows:

$$F_a = \frac{\sin(\beta-\gamma)}{\sin(\beta-\theta)} F, \quad F_b = \frac{\sin(\gamma-\beta)}{\sin(\beta-\theta)} F \tag{9}$$

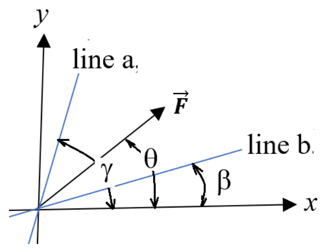

Figure 3 Resolution of a vector along two specified lines

3. FREE-BODY DIAGRAMS

A thorough understanding of drawing a free-body diagram is of primary importance for solving problems in engineering statics. All textbooks clearly describe how to draw all kinds of free-body diagrams, but some students have difficulties in drawing proper free-body diagrams for their problems. They said that they didn't know how to draw a free-body diagram because there were so many different possible situations. Without a free body diagram, students cannot properly solve engineering statics.

The objective of the free-body diagram is to facilitate students to run force analysis for solving reaction forces or internal forces. There are a lot of variations of free-body diagrams in engineering statics. After reviewing all of the possible types of free-body diagrams, we systematically group them into five possible types of free-body diagrams. We believe that if all possible types of free-body diagrams can be systematically explained, it might help students to have a better understanding of it. We proposed to tackle this free-body diagram issue in two steps. First, we clearly explained the definition of the free-body diagram in detail and then demonstrated all five possible types of free-body diagrams for engineering statics courses.

Definition of a free-body diagram

There are a lot of descriptions of a free-body diagram. A free-body diagram is a sketch of the outlined shape of a body or a system under the consideration, which represents it as being isolated or "free" from its surroundings, i.e., a "free-body" [13]. Several key points were explained in detail during lecturing.

- All external forces including body forces such as weight must be shown in the free-body diagram.
- On the cut-off section of a component, the internal forces must be applied. The types and number of internal forces on the cut-off section vary per actual questions. This was explained in detail in different examples.
- On the cut-off of the supports or "freeing" supports, the reaction forces must be applied. Types and number of reaction forces vary per different supports. This was explained in detail in different examples.

After the definition of a free-body diagram and three important notes were fully explained, we demonstrated five types of free-body diagrams along with the progress of the engineering statics course.

The free-body diagram of a particle

The first implementation of a free-body diagram in engineering statics is in Chapter 3 "Equilibrium of a particle" [13]. The components discussed in the chapter are mainly cables, springs, and bars, so the free-body diagram is relatively simple and easily drawn. We explained how to draw the free-body diagram of a particle. Figure 4 a) is one problem with a particle in equilibrium. Figure 4 b) and Figure 4 c) are two free-body diagrams. We had the following important notes for these free-body diagrams.

- Components such as cable, spring, and a bar are all subjected to axial load only. The cable can only have a tension force. Spring or a bar can have tension or compression.
- When a particle is used for the free-body diagram, every connection to the particle can be treated as a support and is removed or cut off. The reaction force on the particle can be assumed to be tension force, that is, the force is outwards from the particle along each component. The picture b) in Figure 4 is such an example.
- When the body around the particle is cut off as a free-body, the internal force on the cut-off sections of components (cable, spring, or bar) can be assumed to be tension along the components, that is, the forces are outwards from the cut-off section along components. Picture c) in Figure 4 is such an example.

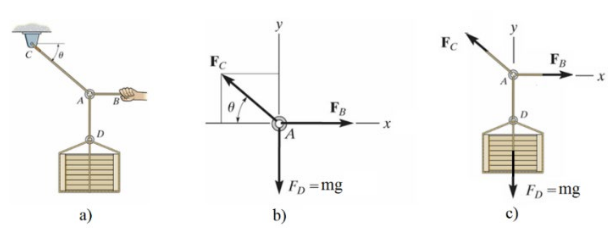

Figure 4 An example of a free-body diagram of a particle [13]

The free-body diagram of a structure with supports

The free-body diagrams are extensively used in Chapter 5 "Equilibrium of a Rigid Body". The textbook provides two tables (Table 5-1 and Table 5-2) which include all basic types of supporters and corresponding reaction forces [13]. Partial of these two tables are duplicated and displayed here as Figure 5 and Figure 6.

We spend a decent amount of time explaining typical supports and their corresponding reaction forces as shown in Figures 5 and 6.

For a free-body diagram of a structure with supports, we explained this in detail.

- When support is removed or cut off, we must apply reaction forces on the cut-off place of the support. That is, we use the reaction forces on the supporting location to replace the supports in a free-body diagram.

- The types and number of recreation forces are all listed in Table 5-1 and Table 5-2 of the textbook [13].

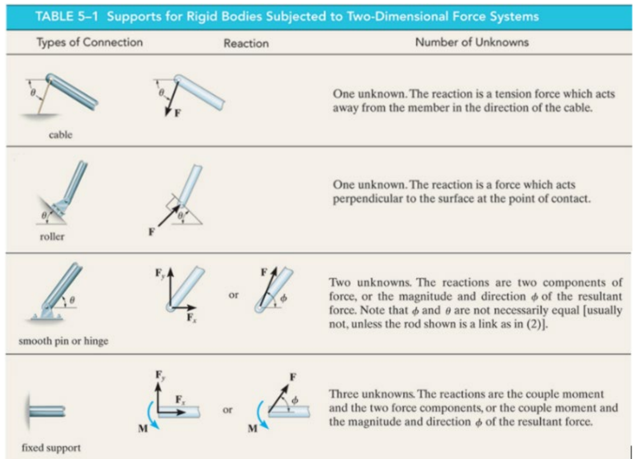

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
smooth pin or hinge	or	Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
fixed support	or	Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

Figure 5 Partial list of typical supports in 2D problems [13]

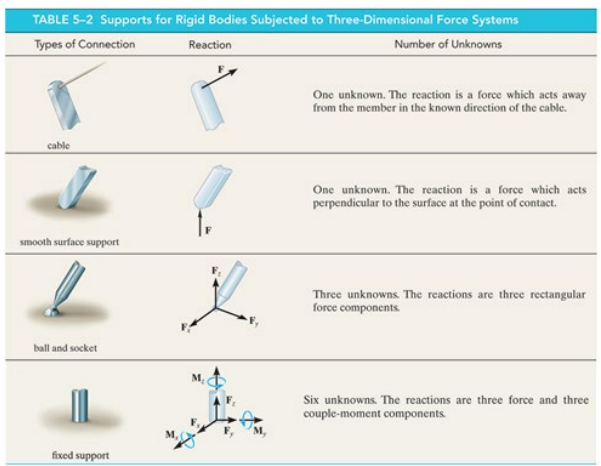

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
ball and socket		Three unknowns. The reactions are three rectangular force components.
fixed support		Six unknowns. The reactions are three force and three couple-moment components.

Figure 6 Partial list of typical supports in 3D problems [13]

An example of a free-body diagram in a 2D structure is shown in Figure 7. The a) in Figure 7 is a 2D structure problem. The b) in Figure 7 is the free-body diagram. In this free-body diagram, the pin support at point A and the roller support at point B are cut off. The reaction forces $\boldsymbol{A_x}$ and $\boldsymbol{A_y}$ are used to replace the pin support at point A. The reaction force $\boldsymbol{N_B}$ is used to replace the roller at point B.

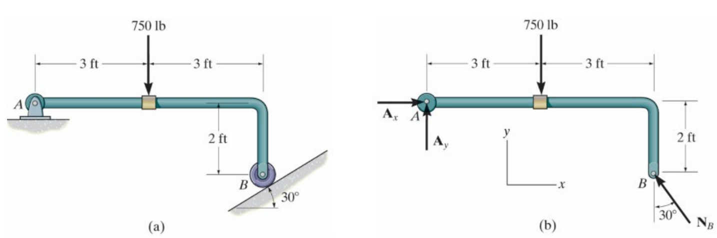

Figure 7 An example of a free-body diagram of a 2D structure [13]

An example of a free-body diagram in a 3D structure is shown in Figure 8. The a) in Figure 8 is a 3D structure problem. The b) in Figure 8 is the free-body diagram. In this free-body diagram, the ball and socket support at point A is cut off and replaced by three reaction forces $\boldsymbol{A_x}$, $\boldsymbol{A_y}$ and $\boldsymbol{A_z}$. The narrow bearing support at the point B is cut off and replaced by two reaction forces $\boldsymbol{B_x}$ and $\boldsymbol{B_z}$. The smooth surface support at the point C is cut off and replaced by one reaction force $\boldsymbol{F_C}$.

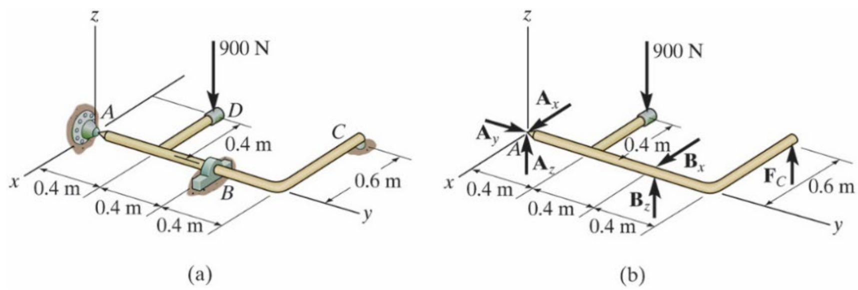

Figure 8 An example of a free-body diagram of a 3D structure [13]

The free-body diagram of a structure with a cutoff section of cable or a two-force member

A free-body diagram of a structure with a cut-off section of cable or a two force member is frequently used. Figure 9 and Figure 10 are two examples of such free-body diagrams. For this type of free-body diagram, we explain these in detail.

- On the cut-off section of supports, the reaction forces are applied such as the reaction forces on point A in b) picture of Figure 9 and the reaction forces on point A in b) of Figure 10.
- On the cut-off section of a cable or a two-force member, an axial load is applied on the cut-off section. For cable or a rope, the force on the cut-off section is always an axial load along the cable or rope such as b) in Figure 9. If a two-member can be identified, we could directly cut off the two-force member such as the DC member in the b) picture in Figure 10. The internal force on the cut-off section of a two-force member is always along the two pivot points of the two-force member.

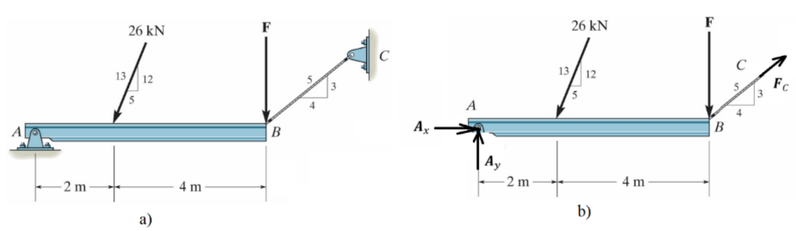

Figure 9 A free-body diagram of a structure with a cut-off of a cable [13]

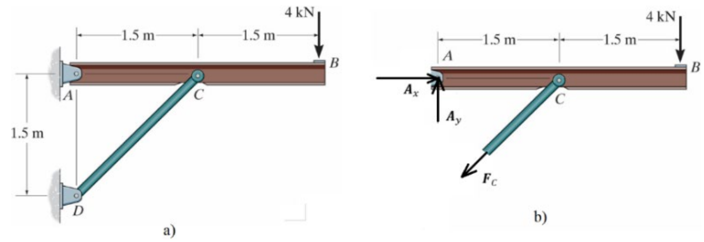

Figure 10 A free-body diagram of a structure with a cut-off of a two-force member [13]

The free-body diagram of a joint for truss analysis

For truss analysis, one approach is to use the joint method which is to draw a free-body diagram around a joint. This is the same as the free-body diagram of a particle. Since every member in a truss is a two-force member. The internal force of each member is along truss members. We can create two types of free-body diagrams for a joint. One is to remove all truss members connected to the joint and apply reaction forces on the joint. The reaction forces are assumed to be tension forces, that is, the forces are outwards from the joint along the members such the b) picture in Figure 11. Another type of free-body diagram is to cut off the members around the joint. Since all truss members are two-force members, the internal forces on the cut-off section are outwards from the cut-off section and along the truss members such as the c) picture in Figure 11.

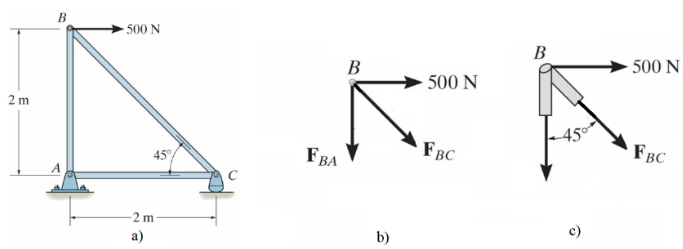

Figure 11 A free-body diagram of a joint [13]

The free-body diagram of a structure with sections

The free-body diagram of a structure with sections is extensively used for solving internal forces in truss, structure, and machine. Typically, we draw first the free-body diagram of a structure with support to solve the reaction forces at the support. Then, we make a section cut on the free-body diagram of the structure with supports to cut it into two parts. Then, we can choose one part for internal force analysis. For this free-body diagram, we must apply internal forces on cut-off sections. The b) picture in Figure 12 shows one example of the free-body diagram of a truss with sections. Since truss members are all two-force members with axial load only, the internal forces on the cut-off sections will be axial load along truss members. When the cut-off section is on a general structure, there will be 3 internal forces for a 2D problem as shown in b) picture of Figure 13 and will have 6 internal forces for a 3D problem

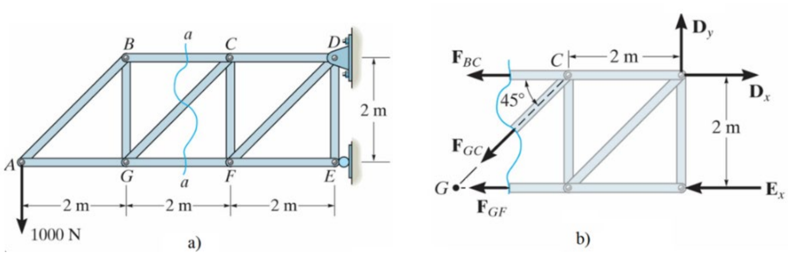

Figure 12 A free-body diagram of a truss with a section cut [13]

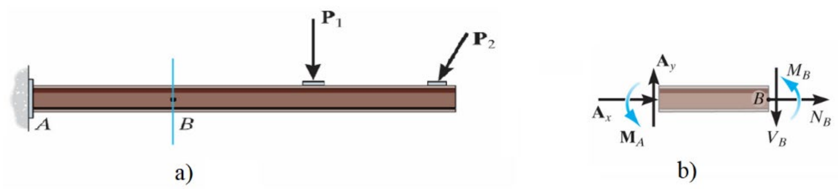

Figure 13 A free-body diagram of a structure with a section cut [13]

4. CONCLUSIONS AND DISCUSSIONS

The parallelogram law is the law for vector additions. The vector addition of two orthogonal vectors, which are vectors along the x-axis and y-axis, is a special example of vector addition by using the parallelogram law. The proposed approach used this specific case to demonstrate the parallelogram as shown in Figure 1. Students could easily understand the principle of parallelogram law. They could even verify the result by using the trigonometry formula of the right-angle triangle. Therefore, students could easily accept the parallelogram law for vector addition. This special case of vector addition could be used to introduce the expression of a vector as a Cartesian vector. Then, Cartesian vectors can be used to run the addition of multiple vectors and the resolution of a vector along two specified lines. Therefore, even the general applications of the parallelogram law are not discussed, this doesn't affect students' understanding and implementation of vector additions. Our observation and interaction with students seemed that this approach just removed some obstacles for some students and did not cause difficulty for students to understand Cartesian vectors and to conduct force vector operation. We will continue this approach and collect some data such as class surveys and exam grades to assess this approach.

A free-body diagram is an essential tool and skill for force analysis in engineering statics. All textbooks have clearly explained how to draw free-body diagrams in different chapters. The proposed approach for teaching how to draw a free-body diagram systematically group all possible types of free-body diagrams into five types of free-body diagrams. Then during engineering statics courses, when we need to use free-body diagrams, we explain the corresponding type of free-body diagram, which is one of the five types of free-body diagrams. We also used more examples to help students to have a better understanding of this type of free-body diagram. Our observation and interaction with students indicated that this approach did help them to draw free-body diagrams and to have a better understanding of how to draw free-body diagrams because all five types of free-body diagrams were systematically described and explained. We just implemented this approach last year and plan to continue this approach in our engineering statics course and will collect more feedback and comparison data in the future.

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