AC 2008-2566: THE STRUCTURE OF HIGH SCHOOL ACADEMIC AND PRE-ENGINEERING CURRICULA: MATHEMATICS

Mitchell Nathan, University of Wisconsin - Madison

Natalie Tran, University of Wisconsin - Madison

Allen Phelps, University of Wisconsin - Madison

Amy Prevost, University of Wisconsin - Madison

The Structure of High School Academic and Pre-engineering Curricula: Mathematics

Abstract

Our curriculum content analysis examines how the pre-engineering curriculum *Project Lead The Way* as compared to the *academic curricula* focus high school students' understanding of mathematics that would prepare them for future studies and careers in engineering. We address the mathematics topics that are presented in these curricula and how the topics are sequenced and presented to students. The results of our content analyses reveal differences in the organization of the intended *pre-engineering* and *academic* curricula. The *PLTW* curriculum addresses far fewer mathematic content and process standards when compared to *academic* curricula, and also exhibit far fewer points of potential integration of mathematics knowledge than expected, given the clarion call made in recent national policy reports and the Perkins Act.

Curriculum Analysis

Curricula—the textbooks, activities and materials that make up a course— provide a critical link between standards and accountability measures, as well as serving as the primary connections between instruction and learning. Curricula shape and are shaped by the professionals teachers who use them. The curricula influence the content of the subjects being taught[9] as well as the way the teaching is enacted. This investigation explores the structure of high school curricula for mathematics and for pre-engineering in order to understand the learning experiences that are intended to prepare students for future studies and careers in engineering and other technical fields. It is part of a larger collaboration between the School of Education and the College of Engineering investigating the challenges and remedies for the development of a broader, more diverse and more able pool of engineers in the US by looking at engineering education systemically as a continuous, developmental experience from post-primary education through professional practice.

The initial questions posed in the curriculum analysis research presented here are predicated on the major needs identified in the NRC (2007) report, *Rising Above the Gathering Storm*[9]: The United States must compete in the global economy by optimizing its knowledge-based resources, particularly in science, technology, engineering, and mathematics (STEM), and by sustaining the most fertile environment for new and revitalized industries and the well-paying jobs they bring (p. 4). In response to this report, more than 1700 high schools in 49 states are implementing new, integrated courses such as *Principles of Engineering* and *Introduction to Engineering Design*, from the nationally distributed *Project Lead the Way* curriculum[12], which create new ways of engaging students in learning math, science and technical knowledge[10]. *Project Lead the Way* (*PLTW*) is a four-year sequence of pre-engineering courses currently offered in 7-10 percent of America's high schools. When combined with academic mathematics and science courses, *PLTW* strives to introduce students to the scope, rigor and discipline of engineering and engineering technology prior to entering college.

The structure of the high school pre-engineering curricula that students encounter is not well understood. Furthermore, the study of the idealized, or *intended* curriculum addresses only one component of the complex system within which engineering and technical education arises. The *enacted* curriculum--the actual student learning behaviors and student-teacher interactions--is a critical piece that is not addressed here. Yet it is essential to document how curricula are structured, apart from their enactment, for several reasons. First, curricula institutionalize certain views of learning and development by selecting what is and is not covered, and the sequence of their organization[8]. Secondly, educators appear to internalize the views of knowledge and development as they appear in textbooks, even when those are tacit, and even when they conflict with basic principals of educational reform that are adhered to by teachers[7]. These internalized views then shape the instructional and assessment practices of teachers, and so directly influences the learning opportunities and experiences of learners. Finally, curriculum analyses help to inform studies of the complexities of the classroom learning processes and instructional interactions that develop around these specific lessons and activities.

Pre-engineering Education at the Secondary Level

Engineering as a field sits on the interstitial boundary between the pure natural sciences (e.g., physics, biology, mathematics) and the sciences of the artificial[16]. As Shulman[15] notes, engineering is “a lovely juxtaposition between the formal requirements entailed in learning math and science and the creative challenges that accompany ‘messing with the world.’” If we are to advance engineering preparation and education, we need to better understand how engineering as a field and a body of knowledge is portrayed in K-12 curricula, how pre-engineering concepts and procedures relate to those presented in the academic mathematics classes—Shulman’s “lovely juxtaposition”—and identify the points of discontinuity and overlap among the two. We take this as a central aspect of the study of the integration of technical and academic education[9].

Pre-engineering at the high school level can be considered part of the reform of vocational education that is now more closely identified with Career and Technical Education (CTE). Historically, the purpose of vocational education has been to prepare students for entry-level jobs in occupations requiring technical mastery, but less than a baccalaureate degree. Over the last 15 years, however, this objective has shifted toward broader preparation that develops the academic, vocational, and technical skills of students. The United States is shifting from a manufacturing-based economy to one that overwhelmingly provides services and information. Consequently, the traditional focus of vocational education is giving way to a broader purpose—one that includes greater emphasis on academic preparation and provides a wider range of career choices[5]. This emphasis includes more education and training requirements in areas like critical thinking and collaborative skills.

Courses in career and technical education (CTE) are rich with physical, practical, situated and collaborative attributes of learning that are called for in current educational reform[2,17]. However, these same learning settings are often devoid of the theoretical and formal content that we expect to be in place to support later generalization, abstraction, and transfer that is more typical of the liberal arts education agenda. Thus, vocational education in the US faces a fundamental paradox[13]: Public education is entrusted with the intellectual development of our youth, but, in practice, the development for those who opt for a technically oriented education instead of a college-bound one is restricted through the choices made about content and pedagogy.

The federal legislation that defines and funds vocational education tries to address this shortcoming. Historically, vocational and academic tracks were explicitly separated. Today these programs are supposed to work together. In 1990, through amendments to the Carl D. Perkins Vocational Education Act of 1984, the federal government mandated that vocational and academic education must be integrated. The amendments made funds available "to provide vocational education in programs that integrate academic and vocational education . . . so that students achieve both academic and occupational competencies." Now, studies of the prevalence and impact of CTE require that one look not merely at students' track designations—there often are none in most of today's high schools—but at students' high school transcripts, so that the more sensitive course-taking patterns that reveal CTE affiliation are properly identified.

Analyses of students' transcripts show that *nearly all* students currently participate in some form of CTE, despite its elective status, and that 25% are considered "concentrators," taking three or more courses in a common career track alongside their regular high school program[3]. Furthermore, there are *no* significant differences by race or gender between CTE participants and the current general student population[3]. CTE also shows some important impacts: The majority of CTE concentrators go on to college, not directly to work; 80% complete the same number of math and science high school courses as their academic-only peers; and although CTE concentrators as a group enter high school less well prepared than academic-only students, that gap is narrowed and may even be eliminated by the time they reach graduation[5,11].

However, the integration of "academic" and "vocational" course material is difficult for many reasons: Turf battles across school departments; new demands placed on teachers to expand their knowledge and pedagogical practices; and organizational impediments of existing graduation requirements, curriculum guidelines, high-stakes testing, and expectations from parents and the community. There may be one other significant barrier to integration: CTE must alter deeply entrenched stereotypes about who can learn what, and who should have access to the mantel of higher education and employment in high-status, high-pay technical fields. Rose (2004) is blunt in his assessment: "My sense is that, with a few exceptions, most policy and curricular deliberations about vocational education have embedded in them assumptions of cognitive limitation[13]" (p. 185). Inevitably, the history and policies of vocational education reveal deeply held beliefs about the inherent and unchangeable nature of intelligence and the cognitive capabilities of those who come from low-SES backgrounds.

In framing our investigation, we focused on this question of integration of vocational and academic education. To address this, we considered ways that we could identify potential points of synergy between the curriculum materials used in pre-engineering courses and the content covered in their academically oriented counterparts. In order to capture these points of integration, we examined mathematics curricula through the lenses of national and state teaching and learning standards.

The Role of National Standards to Curriculum Analyses

As Schmidt, Wang, and McKnight have suggested, there are currently no mandatory national standards in the U.S[14]. Instead, there are recommendations developed by the national professional organizations such as the National Council of Teachers of Mathematics (NCTM) and the National Research Council (NRC). The lack of mandatory standards allows for varying standards at the state and district levels.

Schmidt and his colleagues have argued that coherence is necessary to have quality content standards. That is, "sequence of topics and performances consistent with the logical and, if appropriate, hierarchical nature of the disciplinary content from which the subject matter derives … must evolve from particulars… to deeper structures" (p. 528). Their research using the data from the Third International Mathematics and Science Study (TIMSS) showed a difference in patterns of math and science content between the six highest-achieving countries and the US. In these higher-achieving countries, new topics are gradually introduced with each grade level, and topics are part of the instructions for only a few grades. In contrast, US national standards indicate that the topics are introduced at each grade level and they tend to persist across grades. Analyses of state and district standards in the US showed similar patterns. In general, the NCTM *Standards* for mathematics are distributed across grade-level groupings. In contrast, the six highest-achieving countries' topics are sequenced to reflect the hierarchical and logical structures of the mathematics discipline[14].

Analyses of secondary mathematics textbooks used in TIMSS also showed variation in content, presentation, and task. The variation found in the textbooks suggests that textbook content may not be compatible with students' mathematical conceptions, and this may hamper learning[6]. Studies have even shown that textbooks can have organizational structures that are at odds with what is empirically known about students' mathematical development. For example, in algebra education "textbooks organized around the principle of symbol precedence," which introduce algebra initially through equations and other symbolic formalisms before presenting students with word problems, "are not optimally tailored to the many students who appear to follow a verbal precedence trajectory of algebra development" that capitalizes on using language and context as the basis for algebraic thinking[8]. A corpus analysis showed that most pre-algebra and algebra textbooks exhibited a symbol precedence view, where students must first demonstrate mastery of symbolic representation and procedures before moving on to verbally presented problems. Algebra-level textbooks intended for high school students showed this pattern even more strongly than pre-algebra textbooks marketed to middle school classes. The results of this study also indicate that textbooks published after 1990 (following the mathematics reform of the late 1980's and the release of the landmark1989 NCTM *Principles and Standards*) placed less emphasis on early mastery of symbolic representation, as compared to older textbooks that were published before mathematics education reform took hold[8], suggesting that some systemic impact of the reform affects curriculum organization.

Another curriculum analysis effort was conducted by Project 2061, funded by the American Association for the Advancement of Science (AAAS) to help all Americans become literate in science, mathematics, and technology[1]. Using the expertise of teachers, researchers, and scientists, Project 2061 developed a procedure for evaluating textbooks and assessments. The curriculum-analysis procedures include the following steps: (a) Identify specific learning goals to serve as the intellectual basis for the analysis; (b) Make a preliminary inspection of the curriculum materials to see whether they are likely to address the targeted learning goals; (c) Analyze the curriculum materials for alignment between content and the selected learning goals; (d) Analyze the curriculum materials for alignment between instruction and the selected learning

goals; (e) Summarize the relationship between the curriculum materials being evaluated and the selected learning goals. The validity of this curriculum-analysis procedure has been verified by a research study using assessment items and student work. The results of this study suggest that this procedure is an effective tool for analysis of mathematical content of assessment items and of a set of standards. The analysis of student work also suggests that student thinking does not always reflect the standard identified as best aligned with the learning goals of an item[4].
The National Research Council (NRC) also commissioned a curriculum study to evaluate the quality of evaluations of a total of 19 curricula, including 13 mathematics curricula supported by the National Foundation (NSF), and 6 commercially generated mathematics curricula[2]. After examining 147 studies, classified into four categories of evaluation methodologies (content analyses, comparative studies, case studies, and syntheses), the committee developed a framework to guide curriculum evaluations based on three major components: (a) the program materials and design principles; (b) the quality, extent, and means of curricular implementation; and (c) the quality, breadth, type, and distribution of outcomes of student learning over time. In our larger research study of engineering education, we plan to address all three components when examining the *PLTW* curriculum and those in mathematics and the natural sciences. However, the scope of this paper is limited to the first component of *PLTW* curriculum--studying the program materials and design principles present in the first three core courses.

As a tool for guiding our analysis, we draw on the *Principles and Standards* developed by the National Council of Teachers of Mathematics in 2001. The NCTM proposed a highly informed and comprehensive set of guidelines (revised from those proposed in 1989) for what mathematics students should know and what they should be able to do with their mathematical knowledge and skill. They lay out five *content standards* that constitute much of K-16 mathematics: Number and operations, algebra, geometry, measurement, data analysis and probability. They also identify five *process standards* that address how mathematics is practiced: Problem solving, reasoning and proof, communication, connections, and representation. This framework provides us with a common metric as we consider how to describe the integration of concepts and activities presented in the academic and CTE curricula under investigation.

The *Project Lead the Way* Pre-Engineering Curriculum

The high school program for *Project Lead the Way* is a multi-year sequence of foundation and specialization courses which, when combined with traditional mathematics and science courses in high school, introduces students to the scope, rigor and discipline of engineering prior to entering college. However, those not intending to pursue further formal education can also benefit greatly from the knowledge and logical thought processes that result from taking some or all of the courses provided in the curriculum. Foundation courses include: *Introduction to Engineering Design, Principles of Engineering, and Digital Electronics*. Specialization courses include: *Aerospace Engineering, Biotechnical Engineering, Civil Engineering and Architecture, and Computer Integrated Manufacturing,* with an engineering research capstone course entitled, *Engineering Design & Development*. Everyone teaching *PLTW* courses must attend an extensive professional development program, including training provided by *PLTW's* network of affiliate colleges and universities. In addition to hosting summer training institutes and ongoing professional development, national affiliates offer graduate college credits opportunities for teachers. Through the professional development training, teachers become proficient in project- and problem-based instruction[12].

Research Questions

In our comparative study of curriculum organization, we juxtapose three *academic curricula* for mathematics with the *PLTW* curriculum to investigate differences in how the intended curricula address national standards in mathematics, and to explore areas of synergy that allow for integration of vocational and academic learning experiences. Our curriculum content analysis examines how the pre-engineering curriculum *Project Lead The Way* as compared to the *academic curricula* formulate students' understanding of mathematics that would prepare high school students for future studies and careers in engineering. We perform curriculum analyses of *academic* mathematics and *PLTW* curricula to identify: (a) the presence of national and state mathematics content standards; and (b) identify specific areas of integration of content and process standards across the two scholastic contexts. In so doing, we hope to contribute to the national clarion call (e.g., NRC, 2007) for integration among vocational and academic courses[9].

Method

This study examined the structure of PLTW and mathematics curricula at the high school level. Due to the resource constraints, only the three *PLTW* foundation courses *Principles of Engineering, Introduction to Engineering Design,* and *Principles of Engineering* were included in the analysis. These are the most widely taken *PLTW* courses nationwide as well as in the school districts we are studying, and they have the largest pool of course-specific certified instructors. The fifth edition of *Principles of Engineering* and *Digital Electronics* curricula published in 2004 and *Introduction to Engineering Design* published in 2000 were used in the analysis. Each course is designed to teach high school students within a "typical" high school schedule. This means that a class which meets each day for 40 minutes, 175 days a year should be able to cover the content of this course.

The curriculum is composed of units, which contain lessons and activities. The following information was provided for each of the three foundation courses: Curriculum overview, national content standards that each course aims to address, major learning concepts, lesson outlines, course assessments, lesson units, and glossary appendix. The following topics are expected to be covered in *Principles of Engineering*: (a) definition and types of engineering; (b) communication and documentation; (c) design process; (d) engineering systems; (e) statics and strength of materials; (f) materials testing in engineering; (g) reliability in engineering; and (h) introduction to dynamics/ kinematics. *Introduction to Engineering Design* consists of the following topics: (a) history of design and; (b) introduction to design; (c) student portfolio development; (d) sketching and visualization; (e) geometric relationships; (f) modeling; (g) assembly modeling; (h) modeling analysis and verification; (i) model documentation; (j) presentation; (k) production; and (l) and marketing. *Digital Electronics* is intended to address the following topics: (a) fundamental principles of engineering; (b) number systems; (c) logic gates; (d) Boolean Algebra; (e) combinational circuit design; (f) binary addition; (g) flip-flop circuits; (h) shift registers and counters; (i) logic families and specifications; (j) microprocessors; and (k) student directed study topic such as design paradigm.

The *PLTW* sample included the three foundation courses, IED, POE, and DE. Each course contained between 8 and 10 curricular units (total n = 30). The academic sample included 12 high school math textbooks, each with 8 to 14 chapters (total n = 134). The length of each course unit and chapter in the textbook varied depending on the topic. Thus, we had to establish justifiably comparable units of analysis intended to cover bounded topics specific to their content areas. The unitization process of the curricula is described in the next section, below.

The mathematics textbooks used in this study (see Table 1) were chosen because they were adopted by the cooperating school district and used by teachers in our larger engineering education research project. The district adopted a wide range of textbooks including Algebra, Geometry, Trigonometry, Precalculus, Calculus, and Statistics. Since the preliminary analysis of the *PLTW* curriculum indicated that mainly topics in Algebra, Geometry, and Trigonometry would be addressed in the foundation *PLTW* courses, only those textbooks were selected to represent the *academic* mathematics curriculum.

Algebra textbooks included topics on fractions, proportions, linear equations, functions, problem solving, probability, polynomials, and quadratic functions. Geometry textbooks covered reasoning and proofs, triangle properties, polygons, circle properties, area and volume, Pythagorean Theorem, volume, and trigonometry. Trigonometry textbooks addressed topics such as functions, transformations, roots, power, and logarithm functions, probability, and polynomial functions, and quadratic relations.

Table 1. Titles and Publication Information for Each Course and Textbook

Publisher	*PLTW*	*Algebra*	*Geometry*	*Trigonometry*
Clifton Park, New York	*Introduction to Engineering Design (2000)* *Principles of Engineering (2004)* *Digital Electronics (2004)*			
Glencoe		*Algebra 2*	*Geometry: Integrations, Applications, Connection*	
Core-Plus Mathematics Project (CPMP)		*Courses 1-4*	*Courses 1-4*	*Courses 1-4*
McDougal, Littell/ Houghton/Mifflin		*Algebra: Structure and Method*		

Key Curriculum Press	*Discovering Algebra* *Discovering Advanced Algebra*	*Discovering Geometry*	
Prentice Hall	*Focus on Algebra*		*Function, Statistics, and Trigonometry*

Procedure

Content Analyses

We performed content analyses using the framework suggested by the National Research Council 2004 Report, *On Evaluating Curricular Effectiveness: Judging the Quality of K-12 Mathematics Evaluations*[2]. These content analyses "focus almost exclusively on examining the content of curriculum materials; these analyses usually rely on expert review and judgments about such things as accuracy, depth of coverage, or on the logical sequencing of topics" (p.2). Content analyses include: (a) Disciplinary perspectives such as clarity, comprehensiveness, accuracy, depth of mathematical inquiry and mathematical reasoning, organization, and balance; (b) Learner-oriented perspectives such as engagement, timeliness and support for diversity, and assessment; and (c) teacher- and resource-oriented perspectives such as pedagogy, resources, and professional development. In this paper, we mainly discuss the disciplinary perspectives that include curricular organization, and comprehensiveness of the mathematical inquiry and reasoning as they are addressed in the *PLTW* and *academic* curricula in our sample.

Standards recommended by the National Council Teachers of Mathematics (NCTM) were used as frame of reference to compare and contrast *PLTW* and *academic* curricula. As reviewed above, the NCTM standards address the following content standards: (a) numbers and operations; (b) patterns, functions, and Algebra; (c) geometry and spatial sense; (d) measurement; along with the following process standards: (e) data analysis, statistics, and probability; (f) problem solving; (g) reasoning and proof; (h) communication; (i) connections; and (j) representation.

Establishing Common Units of Analysis Across the Curricula

Each unit in the *PLTW* course was treated equivalent to a chapter in the *academic* textbook. Curricular lessons are embedded in each unit for every *PLTW* course. When appropriate, each lesson was matched against the state math standards. These state standards were then reclassified to align with the *NCTM Standards*. While there was organizational uniformity in the lessons nested in the units of *PLTW* courses, establishing a common unit of analysis for the *academic* textbooks was a methodological challenge. For example, some textbooks aligned the lessons in the chapters to the *NCTM Standards* while others did not. For the former, the *NCTM Standards* were assigned to each unit within a chapter. For the latter, *checkpoints* at the end of each lesson were used as learning goals that could be aligned to the *NCTM Standards*. Each *checkpoint* within a lesson is treated as a unit of analysis. Table 2 below provides an example of the unit analysis.

Table 2. Examples of curriculum analysis

Topic	**Curricular Unit of Analysis**	**National Standards**	**State Standard**
Measurement *(PLTW Principles of Engineering*)	*Principles of Engineering Unit 6*: Students will be able to utilize a variety of precision measurement tools to measure appropriate dimensions, mass, and weight.	1.Understand measurable attributes of objects and the units, systems, and processes of measurement 2. Apply appropriate techniques, tools, and formulas to determine measurements	D.12.2 Select and use tools with appropriate degree of precision to determine measurements directly within specified degrees of accuracy and error (tolerance)
Data Analysis and Probability (*Core-Plus Mathematics Program*)	*(CPMP- 2A)* Checkpoint: a. Describe how to use Pearson's formula for a correlation coefficient. b. If the correlation coefficient is 1, what does that tell you about the points on the scatterplot? If the correlation coefficient is -1, what does that tell you? c. For what kind of data is it appropriate to compute Pearson's correlation coefficient?	1.Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them 2.Select and use appropriate statistical methods to analyze data 3.Develop and evaluate inferences and predictions that are based on data 4. Understand and apply basic concepts of probability	E.12.1 Work with data in the context of real-world situations by (a) formulating hypotheses that lead to collection and analysis of one- and two-variable data, (b) designing a data collection plan that considers random sampling, control, groups, the role of assumptions, etc., (c) conducting an investigation based on that plan, (d) using technology to generate displays, summary statistics, and presentations

Results

In this section we present the results gathered from the content analyses of *PLTW* and *academic* curricula. We address the following questions: What mathematics topics are presented in the curricula? How the topics are sequenced and presented to students?

Project Lead the Way Curriculum

Once the intended learning goals for each unit were identified, they were aligned with the national standards for mathematics, as established by the National Council of Teachers of Mathematics (NCTM). The number of NCTM *Standards* addressed in each lesson for each chapter was recorded.

Figure 1 provides a summary of the number of mathematics topics addressed in the three foundation *PLTW* courses. This reflects the exposure a typical "concentrator" would experience of a three-year period of CTE course taking. The absolute number of standards addressed in each of the PLTW courses was used to illustrate the variation of standard frequencies. Figure 1 also shows that the majority of the mathematics topics are covered in *Digital Electronics* and *Introduction to Engineering Design* with only very few topics addressed in *Principles of Engineering*, with greater emphasis on standards addressing reasoning and proofs and forms of quantitative representation.

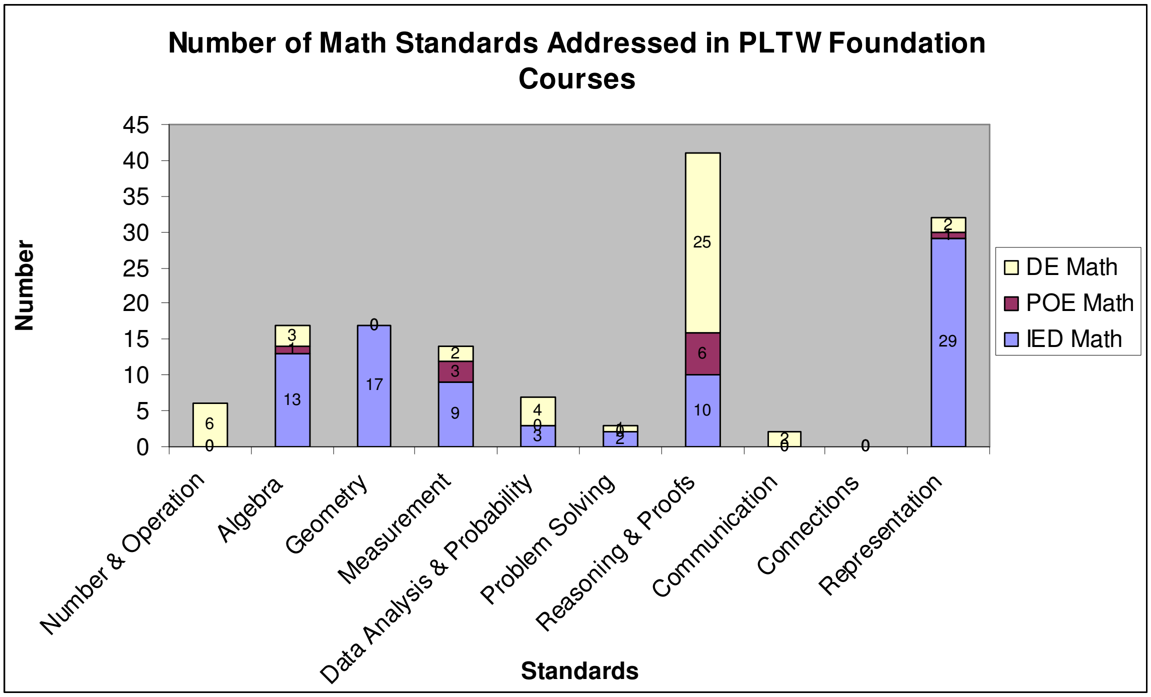

Figure 1. Number of mathematics standards addressed in *PLTW* foundation courses.

In contrast, Figure 2 illustrates the *academic* curriculum, as represented by the Key Curriculum Press *Discovery* series. It shows a greater number of topics being addressed throughout the three-year program. The data indicate that *PLTW* courses address far fewer mathematics topics, as defined by the *NCTM Standards,* compared to the *Discovery* curriculum.

To understand the relative emphasis of the NCTM *Standards* within each course over time, the proportion (percentage) of each standard within a given unit is calculated. This number is obtained by dividing the frequency of a given standard by the total number of possible standards addressed in that unit. Thus, the percentage represents the emphasis of each standard relative to each other within a given chapter.

Table 3 illustrates the intended math standards addressed in the three *PLTW* foundation courses *Introduction to Engineering Design, Principles of Engineering,* and *Digital Electronics.* It is recommended that high school students enroll in these yearlong courses in the above order beginning in the 9th grade, though in practice only a small proportion of concentrators enter this 3-course sequence as high school freshmen. The percentage cut-off points were arbitrarily chosen to represent the relative level of emphasis for each math standards within each textbook. As such, we are interested in the relative level of emphasis for each math standard rather than the absolute number of standards covered. By comparing here the relative level of emphasis, along with the earlier analysis showing the frequency of standards addressed (Figures 1 and 2), we are able to extract stronger patterns of content emphasis.

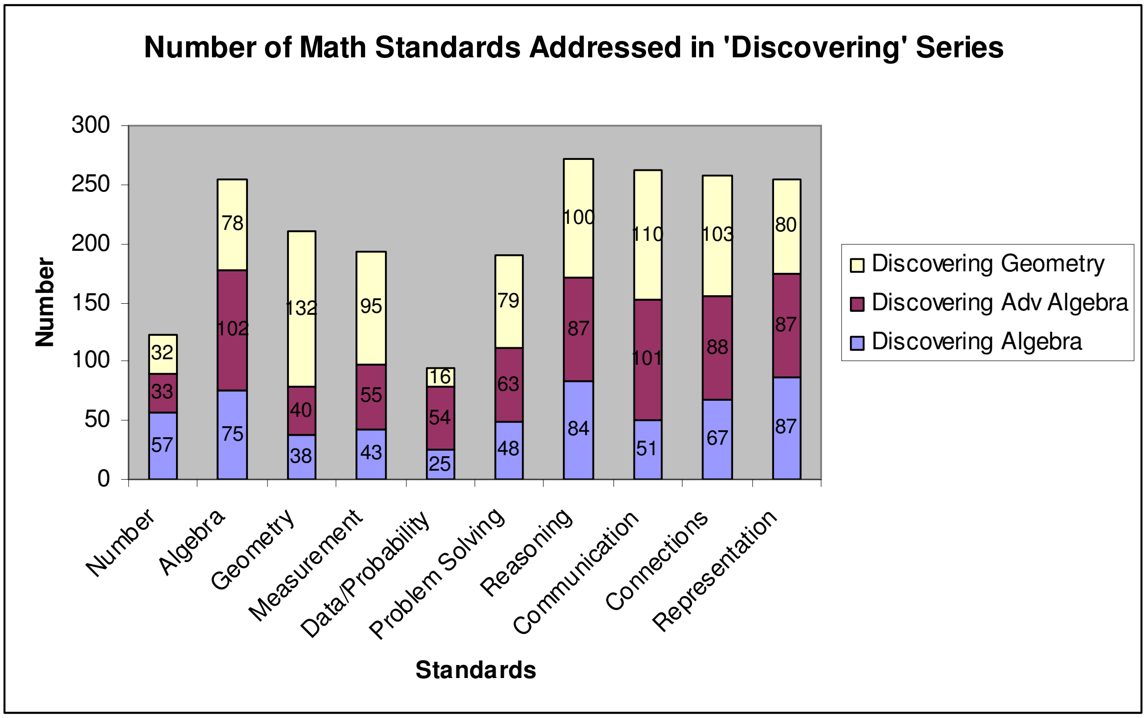

Figure 2. Number of mathematics standards addressed in the Key Curriculum Press *Discovery* series.

Tables 3-6 show the results using this procedure. The presentation also reveals how the relative emphases change over time as one moves from the initial curriculum unit to later units throughout the school year. The data suggest three distinctive patterns of topics addressed in each of these courses. For the year 1 course *Introduction to Engineering Design*, math topics relating to number, communication, and connections are not presented to students. Other math topics are neither introduced at the beginning or the end of the course. Instead, a large proportion of them are addressed mid-way into the course with strong emphasis on geometry, measurement, data and probability, and representation.

In contrast, as Table 3 illustrates, more math topics are addressed in the second-year course, *Principles of Engineering* (also see Figure 1), though they are presented in a less dense fashion than in *IED.* Math topics are widely presented throughout this course with varying levels of emphasis. In addition, the data showed greater emphasis on problem solving, algebra, geometry, communication, and representation at beginning of the course. However, as the course progresses (from Unit 1 to Unit 8), it is evident that other topics share equal emphasis. A noticeable pattern is observed in Unit 3 of this course with the greatest emphasis on number, algebra, and representation and the absence of all other topics.

Table 3. Mathematics standards intended for each unit in *PLTW* courses.

Introduction to Engineering Design	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12
Number												
Algebra					●	◘			◘			
Geometry				█	●	○			●			
Measurement					◘		█	▲	○			
Data/Probability						◘	█				█	
Problem Solving								▲				
Reasoning & Proof					◘	○		▲	◘			
Communication												
Connections												
Representation				▼	●	█			▲			

Principles of Engineering	U1	U2	U3	U4	U5	U6	U7	U8
Number	◘		●	○		◘		
Algebra		○	▼	◘	○	●		●
Geometry		▲		◘	◘		◘	◘
Measurement	○	○		●	◘			
Data/Probability		○		○	○	◘	●	●
Problem Solving	▼	○		◘	◘	◘	●	◘
Reasoning & Proof				○	◘		○	
Communication	●	▲		◘	○	●	◘	●
Connections	○			○	○		○	◘
Representation	○	◘	▼	◘	◘	◘	○	

Digital Electronics	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10
Number	●	◘		◘		●			◘	
Algebra	▲	●	●	●	●	▲	●	▲	●	▲
Geometry						●				◘
Measurement	●						○		▲	◘
Data/Probability				◘	◘					
Problem Solving	◘		●	◘	▲	▲	◘	▲		●
Reasoning & Proof				◘				◘		
Communication		▼	●	●	◘		○			◘
Connections	◘	◘		▲	●		◘	▲	▲	◘
Representation	◘		▼	◘	●		▲	◘		○

0.01-9% = ○
10% - 19% = ◘
20% -29% = ●
30%- 39% = ▲
40%-49% = ▼
50% or more = █

Data on *Digital Electronics* suggest that topics related to algebra are meant to be present throughout the course, reflecting the important role that algebraic models play in digital circuit design and analysis. In addition, in this third-year course, there is greater emphasis on process knowledge with topics such as problem solving, communication, connections, and representation, compared to content knowledge throughout the course.

In general, the topics introduced in the first year course, *Introduction to Engineering Design*, are mostly concentrated in the second half of the course. Topics in other two courses, *Principles of Engineering* and *Digital Electronics,* are widely distributed throughout the course. Thus, there is a lack of evidence to support the hierarchical sequencing of these topics.

Academic Textbooks

Table 4 portrays the topics covered in the three-year *Discovery* series from Key Curriculum Press. *Discovering Algebra, Discovering Advanced Algebra,* and *Discovering Geometry.* The data show that topics are widely distributed throughout the units in all three courses. *Discovering Algebra* consists of topics related to content knowledge such as number and algebra, and also reveals the intent to present many of the other content areas throughout the year. It also shows an emphasis on process standards, including reasoning, connections, and representation. A similar pattern is found in *Discovering Advanced Algebra* with less emphasis on topics related to number and more on communications. *Discovering Geometry* also indicates the same pattern as the two textbooks in the same series with an emphasis on geometry. Also, while number is emphasized early on in the course sequence, its focus diminishes as the series progresses. The opposite appears to be true for lessons addressing the measurement standard.

Table 5 describes the pattern of topic coverage found in the Glencoe textbook series, *Algebra 2* and *Geometry. Algebra 2* focuses on topics such as number, algebra, problem solving, communication, connection, and representation. *Geometry* consists of the same topics with the addition of Geometry and less emphasis on communication. In both courses, measurement and data/probability are less emphasized compared to other topics. Overall, the patterns found in these textbooks support previous analysis of the *NCTM Standards*, which had cluster organization that were distributed at the various grade levels[14]. Again, there is a lack of evidence for hierarchical sequencing of topics.

Table 6 describes the intended emphasis of each standard addressed in the *CPMP* courses (Glencoe/McGraw-Hill). The data show that with the exception of *CPMP Course 4*, which has greater coverage on number, algebra, data and probability, the emphasis is on the process knowledge and skills such as problem solving, reasoning and proof, communication, connections, and representation. Here, we observe a slight hierarchical sequencing of topics with increasing emphasis on content knowledge as the series progresses. The organization of topics found in this textbook series was consistent with previous research findings on verbal precedence view of mathematical development; that is students first exhibit mastery of verbally presented problems prior to success with symbolic representations and procedures[8]. The data show a greater emphasis on process knowledge that would involve problem-solving skills and less emphasis on symbols in the earlier courses. An increasing emphasis is placed on the introduction of content standards as the series progresses.

Table 4. Mathematics standards intended for the *Discovery* series from Key Curriculum Press.

Discovering Algebra

	C0	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
Number	◘	◘	◘	◘	◘	◘	○	◘	○	○	○	○
Algebra	○		◘	◘	◘	◘	◘	◘	◘	◘	◘	◘
Geometry	◘	○	○	○	○	○	○	○	○	◘	○	◘
Measurement	○	○	◘	◘	○	○	○	○	○	○	○	◘
Data/Probability		◘	○	○		○	○	○	○	○		
Problem Solving	○	◘	○	◘	◘	◘	○	○	○	○	○	○
Reasoning	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘
Communication	○	○	○	○	○	◘	◘	○	◘	◘	◘	○
Connections	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	○
Representation	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘

Discovering Advanced Algebra

	C0	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
Number		○		○	○	○	○	◘		○	○	◘	○	
Algebra	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘
Geometry	◘	○	○	○	◘		○	○	◘	◘	○	○	○	
Measurement	◘	○	◘	◘	○	○	○	○	◘	○	◘	○	○	○
Data/Probability		○	◘	○	○	○	○	○	○	○	○	○	◘	◘
Problem Solving	◘	◘	○	◘	○	◘	◘	○	◘	◘	○		○	○
Reasoning	◘	◘	◘	◘	◘	◘	◘	◘	◘	○	◘	◘	◘	◘
Communication	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘
Connections	○	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘
Representation	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘

Discovering Geometry	U0	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13
Number	○	○	◘	○	○	○		○	○	○	○	○		○
Algebra	○	◘	○	○	○	○	○	◘	◘	◘	○	◘	◘	○
Geometry	●	◘	◘	◘	◘	◘	◘	●	◘	◘	◘	◘	◘	●
Measurement	◘	○	◘	◘	◘	◘	◘	○	◘	◘	◘	◘	◘	○
Data/Probability		○	○	○	○	○	○		○		○			
Problem Solving	○	○	○	◘	○	◘	◘	○	◘	◘	◘	◘	○	○
Reasoning	○	◘	◘	○	◘	◘	◘	○	◘	◘	◘	○	○	●
Communication	○	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	●
Connections	●	◘	◘	◘	◘	○	○	◘	◘	◘	◘	◘	◘	○
Representation	◘	◘	◘	○	○	○	○	○	○	◘	◘	○	◘	◘

0.01-9% = ○
10% - 19% = ◘
20% -29% = ●
30%- 39% = ▲
40%-49% = ▼
50% or more = █

Table 5. Mathematics standards intended for *Algebra 2 and Geometry* from Glencoe.

Glencoe

Algebra 2 (Glencoe Mathematics)

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14
Number	◘	◘	◘	◘	◘	○	◘		○	◘	◘	◘	◘	◘
Algebra	◘	◘	◘	◘	◘	◘	◘	◘	●	◘	◘		○	◘
Geometry	○			○	○	○	○	◘		◘	○		◘	○
Measurement	○	○		○			○	○		◘	○		◘	○
Data/Probability		○		○		○	○			○		◘		
Problem Solving	○	◘	◘	◘	◘	◘	◘	◘	●	◘	◘	◘	◘	◘
Reasoning & Proof		○	○	○	○	○	○	○		○	◘	○		○
Communication	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘
Connections	●	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘
Representation	○	◘	◘	◘	◘	◘	◘	◘	◘	○	◘	◘	◘	◘
Geometry: Integrations, Applications, Connection														
Number	○	◘	○	○	○	◘	◘	◘	◘	◘	◘	◘	◘	---
Algebra	◘	◘	◘	○	◘	◘	◘	◘	◘	◘	◘	◘	◘	---
Geometry	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	---
Measurement	○	○	○	○	○	○	○	○	◘	○	○	○	○	---
Data/Probability	○	○	◘	◘	○	○	◘	◘	○	○	◘	◘	○	---
Problem Solving	◘	◘	◘	◘	○	◘	○	◘	◘	◘	◘	◘	◘	---
Reasoning & Proof	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	---
Communication	○	○	○	○	○	○	○	○	○	○	○	○	○	---
Connections	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	---
Representation	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘	---

0.01-9% = ○
10% - 19% = ◘
20% -29% = ●
30%- 39% = ▲
40%-49% = ▼
50% or more = █

Table 6. Mathematics standards intended for *CPMP* Courses

CPMP1	**U1**	**U2**	**U3**	**U4**	**U5**	**U6**	**U7**	**Capstone**
Number	○	○	◘	○	○	○	◘	○
Algebra	○	○	◘	○	○	◘		◘
Geometry			○	◘	◘			○
Measurement		◘	○	○	○	○		○
Data/Probability	●	◘	○			◘	◘	○
Problem Solving	○	◘	◘	◘	○	◘	◘	◘
Reasoning & Proof	●	◘	◘	◘	◘	◘	◘	◘
Communication	◘	◘	◘	◘	◘	◘	◘	◘
Connections	○	◘	○	○	○	◘	◘	◘
Representation	◘	◘	◘	◘	◘	◘	◘	○

CPMP2	U1	U2	U3	U4	U5	U6	U7	Capstone
Number	◘	○	○	○	○	○	○	○
Algebra	○	◘	○	◘	○	◘	○	○
Geometry		◘	○	○	◘	◘	○	○
Measurement		○	○	○	◘	◘	○	○
Data/Probability	◘		◘	○	○	○	●	○
Problem Solving	◘	◘	○	◘	◘	◘	◘	○
Reasoning & Proof	◘	◘	◘	◘	○	○	○	○
Communication	◘	◘	◘	◘	◘	◘	●	◘
Connections	○	○	○	○	◘	◘	◘	◘
Representation	○	◘	◘	◘	◘	◘	○	○

CPMP3	U1	U2	U3	U4	U5	U6	U7	Capstone
Number	○		○			○	◘	◘
Algebra	◘		◘	○		◘	◘	◘
Geometry	○			●		◘	○	◘
Measurement	◘		○	○	○		○	◘
Data/Probability	○	●	○		●	◘	○	◘
Problem Solving	◘	◘	◘	○	◘	◘	◘	◘
Reasoning & Proof	◘	●	◘	▲	●	◘	◘	◘
Communication	◘	●	◘	▲	●	◘	◘	◘
Connections	○	○	○	○	◘	○	◘	◘
Representation	◘	○	◘	○		◘	◘	◘

CPMP4	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10
Number	○	○	◘	◘	◘	◘	○		○	
Algebra	◘	○	◘	◘	◘	◘	●	◘	◘	◘
Geometry		◘			○		○	◘		
Measurement										○
Data/Probability	◘	○	○	◘	◘	◘	◘	○	◘	◘
Problem Solving	○	○	○	○	○	○	○	○	◘	○
Reasoning & Proof	◘	◘	◘	◘	◘	◘	◘	◘	○	◘
Communication	◘	●	◘	◘	◘	◘	◘	◘	◘	◘
Connections	◘	◘	◘	◘	◘	◘	◘	◘	◘	◘
Representation	○	◘	◘	◘	◘	○	○	◘	○	◘

0.01-9% = ○; 10%-19% = ◘; 20% -29% = ●; 30%- 39% = ▲; 40%-49% = ▼; Over 50% = █

Discussion and Conclusions

The results of our content analyses suggest differences in the organization, emphasis and sequencing of mathematics content and process standards among the *pre-engineering* and *academic* curricula. Certainly not all topics in *PLTW* curriculum share equal coverage throughout the course; and our preliminary analyses of the distribution of science and technology standards suggest that the integration of these other areas within pre-engineering curricula also serve as a significant influence. Content analyses of the *academic* curricula indicate that topics are widely distributed throughout the course series. While these topics are distributed throughout the courses, they do not share equal emphasis relative to each other. There is, for example, a noticeable lack of emphasis is found among measurement and data/probability. It is speculated that these two topics are covered in other course series such as trigonometry, pre-calculus and calculus. Ironically, however, these may be particularly well suited as points of integration for these early pre-engineering experiences.

Although the *PLTW* curriculum addresses a fewer number of mathematic topics when compared to the *academic* curricula in our sample, the more relevant question focuses on the type of learning experience each student receives when these two curricula are integrated. That is, what is the mathematical learning experience of a student enrolled in both academic and *PLTW* courses during their high school years, and how do teachers and the students themselves construct an integrated understanding of mathematical and engineering based knowledge. The content analyses presented here show that there are very few points of integration across the technical education and academic courses, and these are largely in the first and third years, when students take *Introduction to Engineering Design*, and *Digital Electronics*, respectively. In terms of mathematics content standards, these courses overlap in number and algebra. In terms of process standards the integration is strongest when addressing representation and connections to real world applications. This analysis suggests that much can be improved in fostering integration between academic and vocational education classes. This additional integration could come about through the instructional activities that teachers are charged with, though long-term changes in curriculum should also be considered for sustained and systematic integration to occur. Whatever design choices curriculum developers make, findings in the learning sciences literature on the difficulties of fostering spontaneous transfer of complex mathematics and science concepts requires that these connects be made explicitly for students, rather than leaving it to them to make these all-important connections on their own[17].

While the content analyses presented in this paper provide information about the organization of the mathematics topics in *PLTW* and *academic* curricula such as what mathematics topics are presented in the curricula, how the topics are sequenced and presented to students, and what skills students will develop and when, it does not allow us to draw conclusions about the effectiveness of the curricula in preparing students future careers in engineering and technical fields. As the National Research Council has suggested, curriculum is effectively determined by using "an integrated judgment based on interpretation of a number of scientifically valid evaluations that combine social values, empirical evidence, and theoretical rationales"[2]. That is, multiple methods of evaluation are ultimately needed to strengthen the determination of effectiveness.

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