

## **AC 2009-351: DEVELOPMENT AND EVALUATION OF A CHARACTERISTIC IMPEDANCE CALCULATOR**

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# Development and Evaluation of a Characteristic Impedance Calculator

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## Abstract

This paper presents our development and evaluation of a transmission line characteristic impedance calculator that offers visualization and immediate feedback. Several impedance calculators based on closed-form equations are reviewed in terms of their functionality, accuracy and validity. Two example problems from our calculator are then presented to show how our calculator operates. The accuracy of our calculator for characteristic impedance and per-unit-length capacitance and inductance compares well with that of a commercial 2-D field solver for a range of geometries such as microstrip, stripline, embedded microstrip and asymmetric stripline. Our calculator is written in Matlab and its executable program will be made available on the internet.

## 1 Introduction

Simulation tools with visualization and immediate feedback are effective in engaging student's interest and in helping them to learn. Visualization appeals to a broad range of learning styles and helps students develop solid conceptual understanding vital for engineering design and application<sup>1</sup>. Instant feedback permits students to vary model parameters and to see their affect on the system's response immediately<sup>2,3</sup>. In short, simulation tools which permit both visualization and immediate feedback provide an engaging space in which the user can experiment and explore freely.

This paper reports on the development and evaluation of a transmission line characteristic impedance calculator with visualization and instant feedback. The calculator will be made freely available on the internet. Several calculators are currently available on the internet, some being free-of-charge and some available for purchase. Many of these calculators have limited functionalities and have little or no performance evaluation and validation data.

Our calculator employs established closed-form equations from the open literature. We have made great effort to verify them against commercial tools as described in this paper. It includes geometries such as microstrip, stripline, embedded microstrip, and asymmetric stripline. It is compared in terms of functionality, accuracy and validity against other free online impedance calculators also based on closed-form equations. In addition, performance evaluation data of our calculator are also presented over a broad solution space against those obtained from a 2-D field solver, Hyperlynx from Mentor Graphics. The results of the evaluation show that our calculator produces results comparable to Hyperlynx for microstrip and stripline.

The calculator will be especially helpful to engineering students as they explore a variety of transmission lines line behavior and develop the intuitive knowledge needed for design,

application, and innovation. It will be a valuable tool for teaching signal integrity of high speed digital design. The calculator is intended as the first in a series of modeling tools in signal integrity and high-speed digital design.

To encourage the adoption and adaptation of this tool, it will be made available on a number of websites including those of the IEEE EMC Society and Rose-Hulman Institute of Technology as well as that of the EMC industrial consortium associated with the EMC laboratory at Missouri University of Science and Technology.

## 2 Survey of Free Online Characteristic Impedance Calculators

Some free characteristic impedance calculators are surveyed in this section in terms of available geometries, accuracy, formulas, and references.

Table 1 shows some free impedance calculators on the internet. As can be seen from the table, the calculators cover a variety of geometries such as microstrip, embedded microstrip, stripline, dual stripline, asymmetric stripline, coaxial, differential stripline, and differential microstrip. Most of them focus on PCBs <sup>4, 6-9</sup> and one deals with more general geometries <sup>5</sup>.

Some of them have simple graphical user interfaces (GUIs) while others have more elaborate GUIs. Among them the most impressive one in terms of GUIs is the one from <sup>5</sup>.

All of the online calculators are based on empirical formulas that would only work under certain constraints. Some of them check for valid boundaries of parameters while others do not give any indication of accuracy and validity of their results.

**Table 1.** Online Free Impedance Calculators

	Microstrip	Embedded Microstrip	Stripline	Dual Stripline	Asymmetrical Stripline	Two wires	Coaxial	Differential Stripline	Differential Microstrip
<a href="http://www.chemandy.com">www.chemandy.com</a>	X								
<a href="http://www.amanogawa.com">http://www.amanogawa.com</a>	X		X			X	X		
<a href="http://www.ideaconsulting.com">www.ideaconsulting.com</a>					X			X	
<a href="http://www.ultracad.com">www.ultracad.com</a> uccalc.exe	X	X	X		X				
PCB Impedance and Capacitance Calculator from <a href="http://www.technick.net/">http://www.technick.net/</a>	X	X	X	X	X			X	X
PCB Trace Impedance Calculator <a href="http://www.cvel.clemson.edu">http://www.cvel.clemson.edu</a> and <a href="http://emclab.mst.edu/">http://emclab.mst.edu/</a>	X	X	X	X					

\*Java based

**Table 2.** Availability of Equations and Comparison Data

	Microstrip	Embedded Microstrip	Stripline	Dual Stripline	Asymmetrical Stripline	Two wires	Coaxial	Differential Stripline	Differential Microstrip
<a href="http://www.chemandy.com">www.chemandy.com</a>	E, R								
<a href="http://www.amanogawa.com">http://www.amanogawa.com</a>	N		N			N	N		
<a href="http://www.ideaconsulting.com">www.ideaconsulting.com</a>					R			R	
<a href="http://www.ultracad.com">www.ultracad.com</a>	E,R	E,R	E,R	E,R					
PCB Impedance and Capacitance Calculator from <a href="http://www.technick.net/">http://www.technick.net/</a>	E,R	E,R	E,R	E,R	E,R			E,R	E,R
PCB Trace Impedance Calculator <a href="http://www.cvel.clemson.edu">http://www.cvel.clemson.edu</a> and <a href="http://emclab.mst.edu/">http://emclab.mst.edu/</a>	E,R Y	E,R Y	E,R Y	E,R Y					

E – Equations available. Y – Comparison data available. R – References available.

Table 2 summarizes availability of underlying equations, performance comparison data and references on the websites or with the calculators for those we could find on the internet. An extensive list of references is given in <sup>6</sup>. A number of well-written lecture notes on transmission lines are available from <sup>5</sup>. The calculator in <sup>8</sup> and <sup>9</sup> is based on references from <sup>7</sup>. Some comparison data with a 2-D numerical solver is available from <sup>9</sup>.

### 3 Introduction to Our Calculator

Our calculator can model microstrip, stripline, embedded microstrip, and asymmetric stripline as shown in Figure 1. This section describes how it models a microstrip and a stripline to show its flexibility, ease of use, and graphical user interface and data presentation. The examples are also compared with the results from commercial software Hyperlynx <sup>18</sup>.

Hyperlynx uses 2.5-D field solver to obtain transmission line parameters. It should produce more accurate results than those of analytical or empirical solutions that our calculator uses. For the two examples below, our calculator produces results almost identical to those from Hyperlynx Version 7.5.

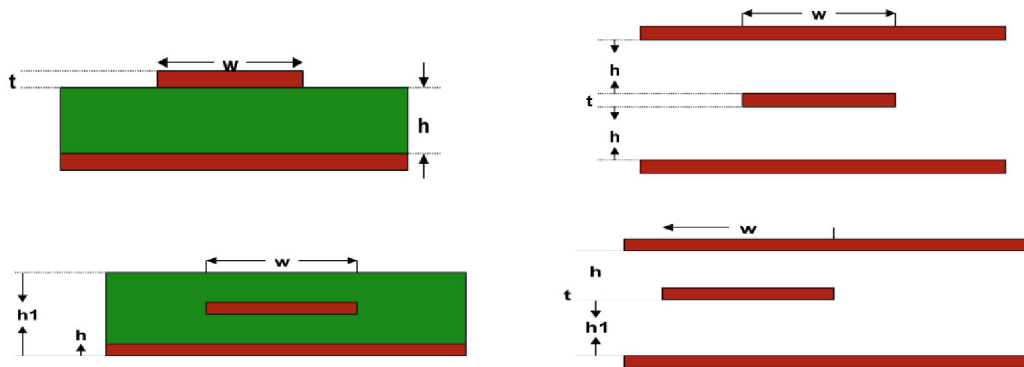


Figure 1. Available geometries: Microstrip, Stripline, Embedded microstrip, Asymmetric stripline.

### 3.1 A Microstrip Example

Figure 2 shows how our calculator models a microstrip line that has the following parameters: width=6mils, height=3mils,  $\epsilon_r=3.9$ , trace thickness=1.4mils, loss tangent=0.02. The calculated line properties are  $Z_0=51.6\Omega$ ,  $C=1.1\text{pF/m}$ ,  $L=3.0\text{nH/m}$ ,  $T_{pd}=57.7\text{ps/m}$ . Total loss=1.7Np/m. Hyperlynx software from Mentor Graphics produces the following results for the same microstrip:  $Z_0=47.3\Omega$ ,  $C=1.2\text{pF/m}$ ,  $L=2.6\text{nH/m}$ ,  $T_{pd}=56.0\text{ps/m}$ ,  $R_0=0.033\text{m}$ , where 1 ounce copper is equal to 1.35mil thickness. It is clear that our results agree with those from Hyperlynx closely.

The losses are composed of conductor loss  $\alpha_C$  and dielectric loss  $\alpha_D$ . The total loss  $\alpha_T$  is therefore  $\alpha_T=\alpha_C+\alpha_D$ . The losses are shown in Neper/cm. Hyperlynx presents the AC resistance value  $R$  due to the skin effect. Therefore, it will not be useful to compare loss calculations of the two calculators.

Figure 3 shows our calculation of individual parameters as function of width to height ratio ( $W/H$ ) or frequency. Comparisons of these curves to Hyperlynx and measurements will be given later in Section 4.

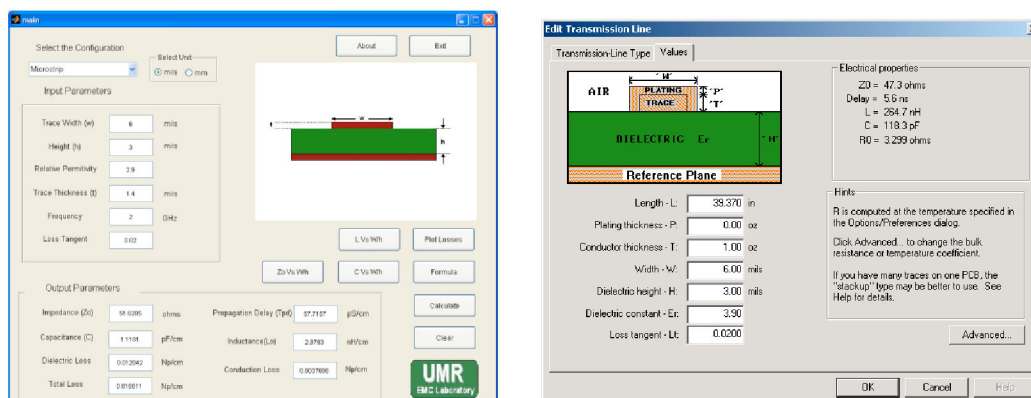


Figure 2. A Microstrip Example with our Calculator and Hyperlynx.

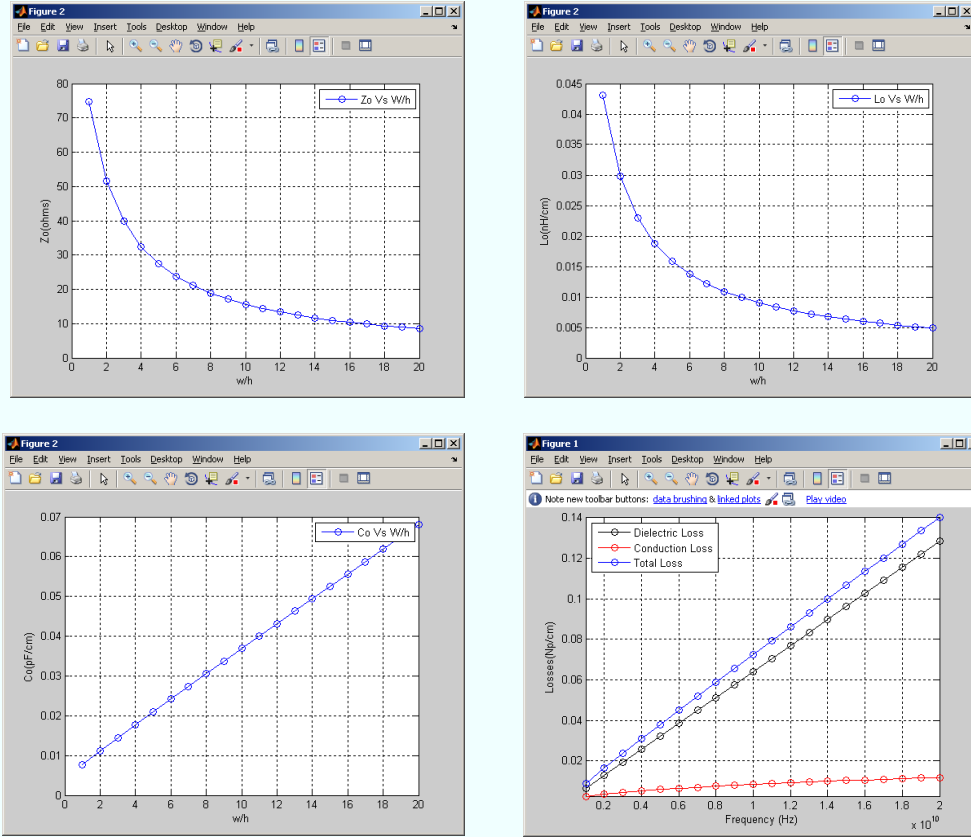


Figure 3.  $Z_0$ , L, C, and loss of the Microstrip as Function of W/H or Frequency.

Figure 4 shows the formulas used by our calculator to model a microstrip and the reference where the formulas are obtained. These formulas are available from our calculator as a push button on the operation menu.

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\epsilon_e}} \ln \left( \frac{8h}{w} + \frac{w}{4h} \right) & \text{for } w/h \leq 1 \\ \frac{120\pi}{\sqrt{\epsilon_e} \left[ w/h + 1.393 + 0.667 \ln(w/h + 1.444) \right]} & \text{for } w/h > 1 \end{cases} \quad \text{where } \epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12h/w}}$$

$$t_{pd} = 84.75 \times \sqrt{\epsilon_e} \text{ pS / inch}; \quad C_0 = \frac{t_{pd}}{Z_0} \text{ pF / inch}; \quad L_0 = C_0 \times Z_0^2 \text{ nH / inch}$$

where  $w$  = width(in metres);  $h$  = height(in metres);  $\epsilon_r$  = relative permittivity

$$w = w + \Delta w; \quad \Delta w = \Delta w + \left( \frac{1 + 1/\epsilon_r}{2} \right); \quad \Delta w = \frac{t}{\pi} \ln \left[ 4e / \sqrt{(w/t)^2 + \left( \frac{1/\pi}{(w/t) + 1.1} \right)^2} \right]; \quad \alpha_c = \frac{R_s}{Z_0 W} \text{ Np / m}$$

$$\alpha_d = k_0 \epsilon_r (\epsilon_e - 1) \tan \delta / 2 \times \sqrt{\epsilon_e} (\epsilon_r - 1) \text{ Np / m} \quad R_s = \sqrt{\pi f \mu_0 / \sigma}$$

Valid conditions  $0 < \epsilon_r < 16$ ;  $0.05 < (w/h) < 20$   $\epsilon_e$  = effective dielectric constant;  $R_s$  = surface resistance

Reference : D.M. Pozar, Microwave Engineering, Second ed, New York Wiley, 1998, chapter 3, pp 156-157, 162-163

Figure 4. Equations for our Microstrip Calculations.

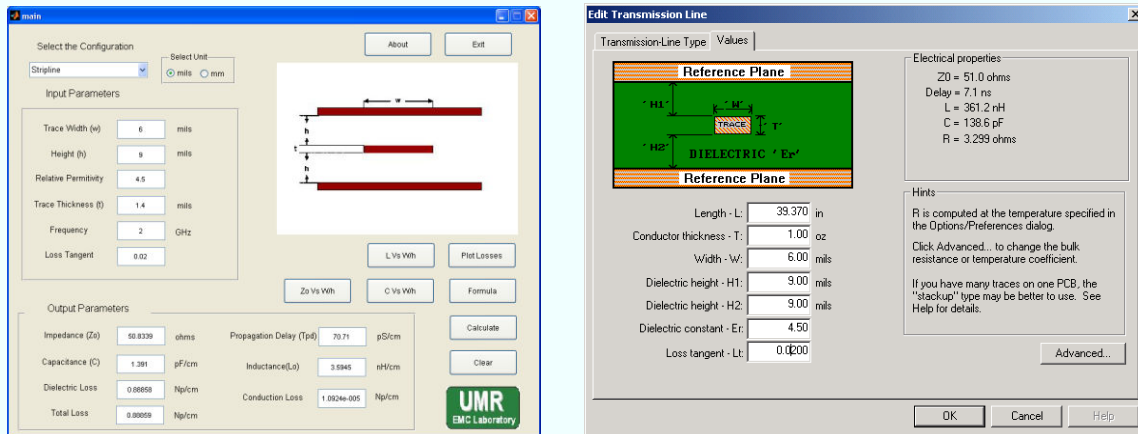


Figure 5. A Stripline Example with our Calculator and Hyperlynx.

### 3.2 A Stripline Example

Figures 5 to 6 show how our calculator models a stripline that has the following parameters: width=6mils, height=9mils,  $\epsilon_r=4.5$ , trace thickness=1.35mils, loss tangent=0.02. The calculated line properties are  $Z_0=50.6\Omega$ ,  $C=1.4\text{pF/m}$ ,  $L=3.6\text{nH/m}$ ,  $T_{pd}=70.8\text{ps/m}$ . Total loss=89/m. Hyperlynx software from Mentor Graphics produces the following results for the same microstrip:  $Z_0=51.0\Omega$ ,  $C=1.4\text{pF/m}$ ,  $L=3.6\text{nH/m}$ ,  $T_{pd}=70.8\text{ps/m}$ .  $R_0=0.033\text{m}$ . our results agree with those from Hyperlynx even more closely for this example.

Our calculator has incorporated boundary error checking and warning feature and will provide a warning message if a value is out of bound or in error as shown in Figure 7. The equations used in our calculator to estimate  $Z$ ,  $L$ ,  $C$ , and loss are all from open literature<sup>10, 17</sup>.

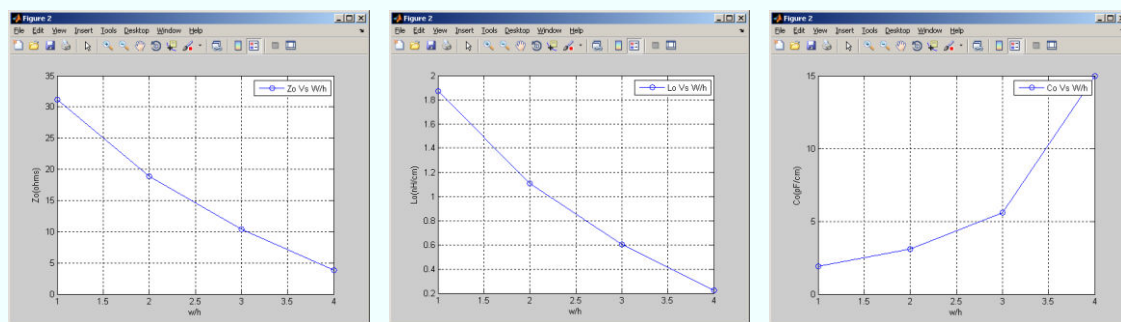


Figure 6.  $Z_0$ ,  $L$ ,  $C$  of the Stripline as Function of  $W/H$  or Frequency.



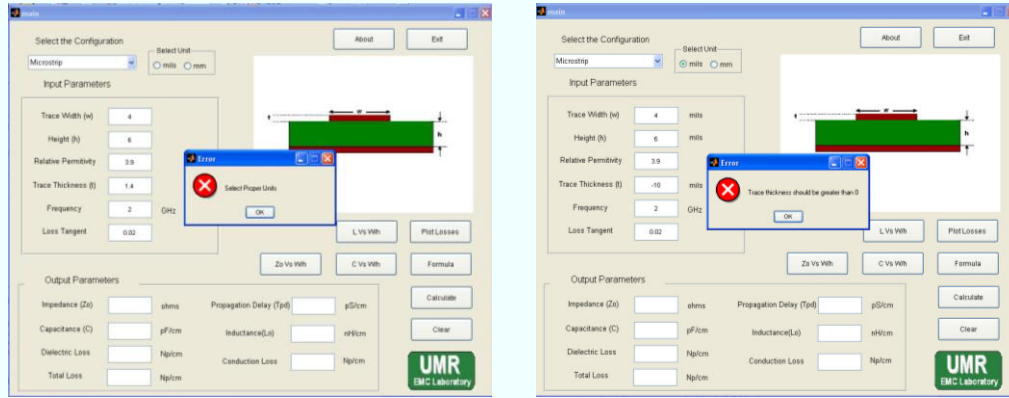


Figure 7. Our Calculator Provides Bound and Error Warning Messages.

#### 4 Performance Evaluation and Comparison with 2-D Field Solver

Results of our calculator for microstrip and stripline are compared with a 2-D field solver Hyperlynx from Mentor Graphics<sup>18</sup> to understand the accuracy and validity of our calculator in providing  $Z_0$ ,  $L$  and  $C$ .

Our results are grouped into two categories:  $W/H$  greater than one or less than one, where  $W$  is the width of a trace and  $H$  is the dielectric material thickness. The frequency range is up to 2 GHz. The thickness of traces is 1 ounce copper, i.e., thickness of 1.4 mils and relative dielectric constant  $\epsilon_r$  is 4.7. When  $W/H=2$ , a microstrip should have characteristic impedance of  $50\Omega$ .

The total attenuation loss is expressed as  $\alpha_T = \alpha_C + \alpha_D$ , where  $\alpha_C$  is conductor loss and  $\alpha_D$  dielectric loss. The losses are shown in Np/cm. The Neper (Np) uses the natural logarithm to express a ratio of two values. The loss calculations of our calculator are not compared to that of Hyperlynx because Hyperlynx does not provide losses as functions of frequency.

##### 4.1 Comparisons with Hyperlynx

Figure 8 and 9 presents comparison results for microstrips. For  $W/H > 1$ , our calculator has produced results close to those from Hyperlynx. For  $W/H < 1$ , our calculator and Hyperlynx differ a little bit.

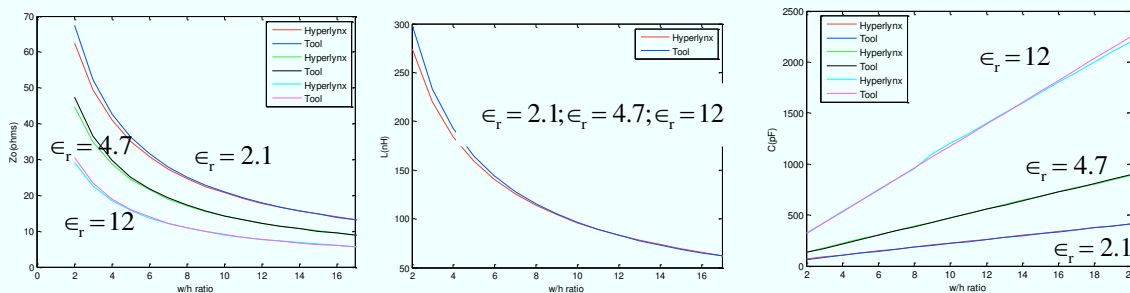


Figure 8. Microstrip of  $W/H > 1$ :  $Z_0$ ,  $L$ , and  $C$  for  $t = 1.4$  mils.

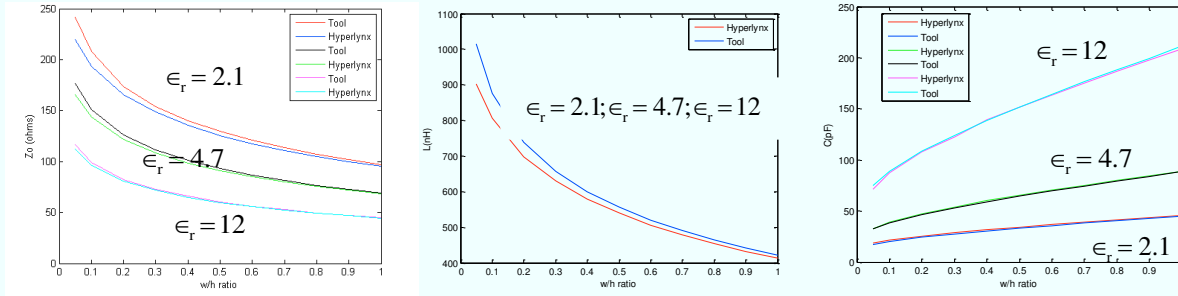


Figure 9. Microstrip of  $W/H < 1$ :  $Z_0$ ,  $L$ , and  $C$  for  $t = 1.4$  mils.

Figure 10 and 11 presents comparison results for striplines. For both  $W/H > 1$  and  $W/H < 1$ , our calculator has produced results close to those from Hyperlynx. The smaller the ratio of  $W/H$ , the closer the results of the two calculators are.

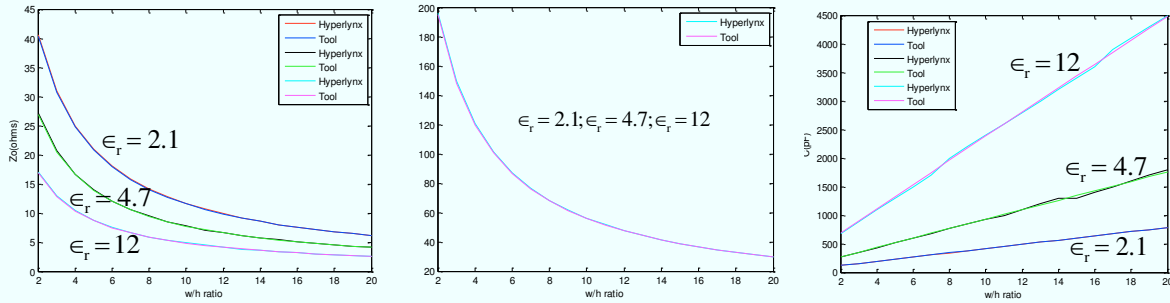


Figure 10. Stripline of  $W/H > 1$ :  $Z_0$ ,  $L$ , and  $C$  for  $t = 1.4$  mils.

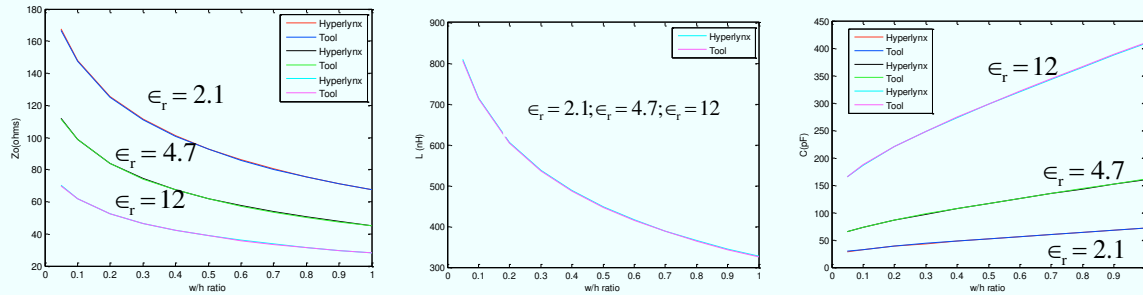


Figure 11. Stripline of  $W/H < 1$ :  $Z_0$ ,  $L$ , and  $C$  for  $t = 1.4$  mils.

## 5 Discussions

This paper presents the features and performance comparison of our impedance calculator for transmission lines such as microstrip, stripline, embedded microstrip and asymmetric stripline. It shows our calculator is easy to use and that its graphical data presentation is intuitive and useful. The accuracy and validity of its impedance and per-unit-length inductance and capacitance results are compared with these of a commercial 2-D field solver Hyperlynx from Mentor Graphics to show that our calculator agrees with Hyperlynx for microstrip and stripline. For the other geometries, our calculator and Hyperlynx produce similar results. Our calculator includes a set of comparison data points to help the user evaluate the

validity and accuracy of its results. It will be released to the public domain and will be a valuable addition to the free online calculators for the PCB design community.

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