AC 2009-430: A SIMPLE, AFFORDABLE STEADY-STATE FIN HEAT TRANSFER MINI-LAB/DEMO

Michael Maixner, United States Air Force Academy
Michael Rex Maixner graduated with distinction from the U. S. Naval Academy, and served as a commissioned officer in the USN for 25 years; his first 12 years were spent as a shipboard officer, while his remaining service was spent strictly in engineering assignments. He received his Ocean Engineer and SMME degrees from MIT, and his Ph.D. in mechanical engineering from the Naval Postgraduate School. He served as an Instructor at the Naval Postgraduate School and as a Professor of Engineering at Maine Maritime Academy; he is currently a member of the Department of Engineering Mechanics at the U.S. Air Force Academy.

James Baughn, UC Davis
James W. Baughn is a graduate of the University of California, Berkeley (B.S.) and of Stanford University (M.S. and PhD) in Mechanical Engineering. He spent eight years in the Aerospace Industry and served as a faculty member at the University of California, Davis from 1973 until his retirement in 2006. He is a Fellow of the American Society of Mechanical Engineering, a recipient of the UCDavis Academic Senate Distinguished Teaching Award, and the author of numerous publications. He completed an assignment to the USAF Academy in Colorado Springs as the Distinguished Visiting Professor of Aeronautics for the 2004-2005 and 2005-2006 academic years.
A Simple, Affordable Steady-State Fin Heat Transfer Mini-Lab/Demo

ABSTRACT
The engineering education literature is replete with recommendations on how to enhance student understanding of steady-state fin heat transfer. These range from the use of numerical programs which allow the user to change various parameters and observe the changes in other parameters or changes in graphic output, to pure laboratory experiments. The current authors feel, however, that the way for students to gain meaningful insight into the problem is through a lab experience which involves not only data reduction, but also analysis and use of a modified “Socratic method” to challenge students’ preconceived notions. Such a method and the associated device are described in this paper. This method has been employed at the United States Air Force Academy (USAF). The device itself is inexpensive and simple to manufacture and operate, and the analysis may be employed as either a “mini-lab” or as a classroom demonstration, since it allows rapid comparison of measurements with analytical predictions. Fabrication details, a recommended data sheet layout, and sample results are provided; additionally, initial student misperceptions are discussed, and how questions posed during the lab (the Socratic Method) aided in resolving them. Student feedback and performance on a graded “mini-lab” were most gratifying.

BACKGROUND AND IMPETUS
The perennial problem faced by most instructors is how to ensure that their students truly grasp the material in the time allotted. Recently, classes at the United States Air Force Academy (USAFA) were reduced to forty 53-minute sessions during each semester. Although the total time during each term was more or less the same as it had been before the change (forty-two 50-minute sessions), the scheduling of topics within the thermal fluids systems courses were nonetheless impacted. Among the measures taken to help ensure topics were covered AND actually understood by students in the reduced number of lessons was the incorporation of short mini-labs and demonstrations. The fin laboratory described herein was one of those mini-labs/demonstrations.
What the authors sought was an inexpensive educational experience for the students which challenged their intuition and initial perceptions, was also quick and easy to run, included and required comparison of data with theory, required that they interpret their results, and which was also fun (which the authors believe all learning should be). The authors believe that the mini-lab/demonstration described herein satisfies all these requirements.

PREVIOUS EFFORTS
A review of the literature shows that numerous approaches have been employed to help students better understand the phenomenon of steady-state heat transfer from extended surfaces. At one end of the spectrum, some were decidedly experimental in nature, while at the opposite extreme, others were entirely computer-based. These different approaches were undoubtedly driven by institutional, curricular, and/or hardware constraints.

- Abu-Mulaweh\(^1\), Abu-Mulaweh and Mueller\(^2\), and Mueller and Abu-Mulaweh\(^3\) devised a “design, build, test” heat transfer experiment for incorporation into a junior-level heat transfer
laboratory course; following the “build” phase, the experiment required students to measure temperatures along a heated aluminum fin rod, and to calculate the average convective heat transfer coefficient which brought the data into closest agreement with theory. Additionally, students were required to develop correlation equations for the average convective heat transfer coefficient.

- Somerton et al\(^4\) described the use of a family of props which were passed among students, allowing them to gain an appreciation for the fabrication details of fins in industry; further, they described an interactive Excel\(^\text{®}\) spreadsheet which allowed students to investigate various aspects of different fin arrangements in terms of not only dissipating a specified heat load, but also investigating cost, size, weight, choice of material, etc. While optimization was incorporated into the computer exercise assigned to the students, no heat transfer laboratory experience was involved.

- Kaminski\(^5\) discussed various heat transfer experiments appropriate at the technology level, including one which involved the measurement of temperatures along three different heated pin fin geometries, and comparison with the theoretical temperature distributions. An electric heater at the base of the fins was employed, and a small wind tunnel provided convective cooling. Actual fin heat transfer rates were ascertained through application of Fourier’s Law at the base of each fin, convective heat transfer coefficients were calculated using appropriate correlations, and fin performance was calculated and compared against published charts.

- An extended surface heat transfer experiment was described by Smith and Volino\(^6\), wherein students were required to estimate the convection coefficient from a single heated rectangular fin across which air was blown by a portable fan; the measured temperature distribution along the fin was used in these calculations. Heat transfer from a fin array was also calculated.

- Hinton et al\(^7\) discussed the data acquisition system for an undergraduate fin heat exchanger experiment; they incorporated acquisition of the temperatures along each of four fins, each with a different geometry. The data were then transferred to Excel\(^\text{®}\) for further analysis and interpretation. Aside from a short section discussing the theory of heat transfer along fins and another short section discussing the experimental results, the principal focus of the paper was on data acquisition.

- Finally, Karimi\(^8\) presents an Excel\(^\text{®}\) spreadsheet format for the solution of various fin heat transfer problems, including a variety of tip conditions and geometries; optimization methods are demonstrated.

No doubt there are many more approaches described in the literature, but these are representative and show the broad spectrum of approaches.
THEORY

The theory of steady state heat transfer from extended surfaces is well known, and its derivation will not be repeated here. Analytical equations for the analysis of fins operating under a variety of boundary conditions at steady state may be found in almost any basic text which covers conduction and convection heat transfer, and are shown in Table 1.

Table 16.4 Temperature Distribution and Heat Rate for Fins of Uniform Cross Section

<table>
<thead>
<tr>
<th>Case</th>
<th>Tip Condition ((x = L))</th>
<th>Temperature Distribution (\theta(x)/\theta_b)</th>
<th>Fin Heat Transfer Rate (q_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Infinite fin ((L \to \infty)): (\theta(L) = 0)</td>
<td>(e^{-\alpha x}) (\frac{e^{-\alpha x}}{\theta_b}) (M = \sqrt{\frac{h_l \rho c_\alpha}{k}}) (\theta_b)</td>
<td>(16.67) (\alpha x)</td>
</tr>
<tr>
<td>B</td>
<td>Adiabatic: (\frac{d\theta}{dx}\big</td>
<td>_{x=L} = 0)</td>
<td>(\cosh m(L - x) / \cosh mL) (\frac{\cosh mL}{\sinh mL})</td>
</tr>
<tr>
<td>C</td>
<td>Prescribed temperature: (\theta(L) = \theta_L)</td>
<td>(\frac{(\theta_b/\theta_L) \sinh mL + \sinh m(L - x)}{\cosh mL}) (\frac{\sinh mL}{\sinh mL})</td>
<td>(16.71) (\frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}) (M)</td>
</tr>
<tr>
<td>D</td>
<td>Active, convection heat transfer: (h\theta(L) = -\frac{d\theta}{dx}\big</td>
<td>_{x=L})</td>
<td>(\frac{\cosh mL - x + (h_l/\rho c_\alpha) \sinh mL}{\cosh mL + (h_l/\rho c_\alpha) \sinh mL}) (\frac{\cosh mL + (h_l/\rho c_\alpha) \sinh mL}{\cosh mL + (h_l/\rho c_\alpha) \sinh mL})</td>
</tr>
</tbody>
</table>

\(\theta\) See Fig. 16.17b for related surface energy balances.
\(\theta\) Temperature excess definitions: \(\theta = T - T_w\) and \(\theta_b = \theta(\theta) - T_s - T_w\).

Table 1: Table 16.4 from Reference10. Used with permission.

The four steady state boundary conditions considered included an infinite fin, adiabatic tip, active (convective) tip, and a tip whose temperature is prescribed; for each of these conditions, course objectives at USAFA require that students be able to calculate the fin temperature distribution, heat transfer rate, efficiency, and effectiveness.

USAFA APPROACH

While many other approaches have implemented teaching tactics focused principally on either experimental or computer methods, the approach adopted at USAFA was hybrid in nature using both. It may be accomplished as either a demonstration in the normal sequence of class lectures, or as a more formal laboratory. No separate heat transfer laboratory course exists in the curriculum at USAFA, so these laboratories experiences for our students are conducted within the allotted lesson times for each course; many are, in fact, performed as demonstrations, with students performing data reduction on the experimental results. Still, the authors recognize that it is always desirable that students have a hands-on experience whenever possible, rather than having them either perform simulations on a computer or merely reduce data.
The apparatus was extremely simple, and consisted of a VWR® Dylatherm® hot plate (Figure 1: Hot plate and fin apparatus.), atop which was placed a block of aluminum, and into which were mounted (either threaded with heat tape, or press-fitted) two circular rods, each of 0.95 cm diameter. The first
rod, manufactured of aluminum, was 20 cm in length; the second rod was manufactured of stainless steel, and was 32 cm long. Each of the rods was slightly notched at intervals, to provide detents for temperature measurement. Natural convection heat transfer to the ambient air could be readily obtained, and a small fan was used to achieve forced convection; air velocity was measured with a LaCrosse® hand-held anemometer (Figure 2), obviating the need for a more sophisticated wind tunnel arrangement.

Initial consideration was given to the incorporation of manufacturing a wiring harness with data recording accomplished automatically with LabView®, but this was abandoned in order to allow maximum student involvement [Students do, however, receive instruction in LabView® in another laboratory course that is oriented principally towards structural mechanics]. In fact, as will be discussed later, there was a certain amount of suspense associated with the data-taking portion of the demonstration, and this helped to keep students involved throughout the entire laboratory. A type K thermocouple and Ωmega® model HH23 microprocessor thermometer were employed (Figure 3: Digital thermocouple readout.). Initially, it was thought that an infrared gun could be used to ascertain the temperature at various points on the fins, but parallax issues (between the sensing point and the pointing laser) and background temperature readings resulted in inconsistent readings. The next method of taking temperature data entailed students outfitted with an insulated glove, and holding the thermocouple against each fin; unfortunately, temperature readings for each point would vary, depending upon the force with which each student held the thermocouple against the fin. A second arrangement had the thermocouple mounted at the end of a stick to remove the requirement for an insulated glove; still, the readings varied depending upon the force applied. Finally, the clothespin-arrangement shown in Figure 4: Close-up of clothespin-mounted thermocouple. provided consistent readings. The thermocouple wire was led in through a hole drilled into one side of the clothespin and held in place with a dab of glue; the spring on the clothespin assured consistent pressure (and, therefore, readings). Additionally, to move from one detent to the next, it was only necessary to pinch the clothespin sufficiently to allow the device to ride along the fin to the next detent; with a sufficiently-deep detent, the thermocouple could be felt to slide into place at the next measurement point, allowing rapid data collection. The total cost of the entire apparatus (materials, thermocouples, fan, anemometer, and digital thermocouple readout) was $550, and roughly 3 man-hours were required to manufacture it.
Only 15 minutes were required prior to class for the apparatus to achieve steady state with roughly a half-power setting on the hot plate rheostat. Students were not provided the laboratory instructions in advance, and had been notified merely that there would be an in-class demonstration. Prior to taking data, several small fins from computer CPUs and other similar devices were passed around the class. The students were then queried regarding which fin in the experiment would experience a higher heat transfer rate: the shorter or the longer? A tally was taken, with the usual outcome being that three-quarters of the class anticipated that the longer fin would provide a higher rate of heat transfer. More often than not, the students would not ask any questions regarding the material from which each fin was constructed—most of them tacitly assumed they were of the same material. The students were

**Figure 5:** Spreadsheet, as mailed to students following experiment.

then notified that they would have a chance to change their minds prior to a second vote which would be conducted following the experiment.
The data sheet depicted in Figure 5: Spreadsheet, as mailed to students following experiment. was then projected onto a screen. Ambient temperature was recorded from the thermocouple prior to reading fin temperatures, and then the temperature at the base of each fin was recorded using a thermocouple attached to a wooden stick. A data recorder was assigned, and students were then sequenced through the apparatus to move the clothespin up each fin; another student was assigned to verify that the clothespin was in the appropriate detent. With the fin lengths and rheostat setting employed, the aluminum (shorter) fin achieved ambient temperature in a much shorter distance than will the stainless
steel (longer) fin, and this will normally provoke a question from the class regarding the materials of which each fin is constructed. With prompts from the instructor, suspense grew regarding how temperatures would vary on each fin.

After the data are recorded, another vote was taken, and around one-quarter to one-third of the class still maintained that the longer (stainless steel) fin would provide the greater rate of heat transfer. A sample set of reduced data is shown in Figure 6: Completed spreadsheet, including VBA control for variation of average heat convection coefficient, including a VBA scrollbar (to change the value of the average convection coefficient) and plot to allow immediate feedback. Depending on the heater rheostat setting and the fin lengths, it is indeed possible that one (or both) fin(s) is (are) excessively long—it is recommended that the lab be conducted beforehand to ensure that this situation exists to ensure students will have to comment on how this impacts fin efficiency and effectiveness. In fact, it is the relative magnitude product of which will determine which fin is more efficient—for the conductivities and lengths employed in this lab, the smaller value of mL for aluminum means that most of the aluminum fin is hot, making it the one which will provide a greater overall rate of heat transfer.

Students were then paired off to begin data analysis. The spreadsheet was emailed to the students, and contained the data, conductivities, ambient temperature, fin lengths, diameters, etc.; several cells were named to facilitate formula troubleshooting, both by students AND instructors! The following deliverables were required to be submitted on the third lesson following the experiment:

- A completed Excel® spreadsheet in two formats, each depicting grids and row and column headings:
  - One format depicting cell calculation results
  - Another format displaying embedded formulas. If named cells appeared in any formula, students were required to use the cell names instead of referring to a cell by its row and column.

- A “mini-lab” report, not to exceed 1-1/3 pages, was required from each group, and was to include
  - For each fin
    - Comparison of actual temperature distribution to the theoretical
    - Appropriate analytical equations (have to choose which of the boundary conditions is most appropriate)
    - Fin heat transfer rate, using both
      - Fourier’s Law at base
      - Calculation of convection along entire length, assuming each section between measurement points is at the same temperature
    - Fin effectiveness and efficiency
  - Comments on the value of the average convection coefficient which brings actual and theoretical temperature distributions into closest agreement. Comment on the magnitude of this value. [Students enrolled in the course had either completed or were currently enrolled in EM305 Engineering Tools Seminar11, a component of which entailed rudimentary programming instruction in Visual Basic for Applications (VBA).]
One of the topics covered in that course was the incorporation of VBA controls; it was suggested that students might include a scroll bar in order to vary the average convection coefficient in order to see how well the processed data compared with theory.

- Additionally, folded into their reports, they were required to provide
  - Sample hand calculations for each part of the spreadsheet analysis.
  - Comments regarding how their initial perceptions agreed or disagreed with their experimental results.
  - The ratio of the heat transfer of the long fin to the short fin, and a discussion regarding this value.
  - The significance of the product mL of these fins, as opposed to length alone.
  - The effect of the tip boundary condition used.
  - A prediction of what would happen if both fins were shortened by 4.0 cm (students were instructed to use the spreadsheet to do this).
  - Comments on the meaning of the values obtained for fin effectiveness and efficiency.

**STUDENT FEEDBACK**

Students performed exceptionally well on this experiment (92.9 %). In general, almost all students were enthusiastic about the laboratory, and commented favorably on it in their end-of-course critiques. Since this apparatus was being tested to see how it would be received, only one unit (device or apparatus) was used when the experiment was performed as a demonstration; unfortunately, most of the students involved in the actual conduct of the experiment were merely moving the thermocouple up and down each fin, and were somewhat chagrined that they were not more directly in charge of the experiment. Accordingly, given the favorable feedback from students, three additional units will be fabricated for the next offering of the course, allowing students to perform this as a _bona-fide_ laboratory; although each apparatus will have four students performing the laboratory, they will still be required to perform the data reduction and lab report in groups of two.

Additionally, comments from the students in their reports indicated that they needed to constantly question their initial perceptions, since they weren’t necessarily as they might originally have seemed. Unfortunately, no control data exist from previous course offerings on student assessment on the topic of heat transfer from extended surfaces, so that the efficacy of this new teaching tool could be compared to previous performance.

**FUTURE MODIFICATIONS AND OTHER OPTIONS**

The following additional options may present themselves for future course offerings, and may be readily incorporated into this laboratory:

- Vary rheostat settings on the hot plate
- Employ different fin geometries (i.e., cross section, taper, etc.)
- Use fins of different lengths, but of the same material
- Vary fan speeds (for either calculation of different convection coefficients, or the establishment of a mathematical correlation) (see Reference 7)
• Compare convection coefficient obtained to that calculated via Hilpert or Churchill-Bernstein correlations (N.B.: this could be incorporated into the lab, or saved for use as an exam).
• Since no teaching assistants are available at USAFA, the use of a standard Excel® spreadsheet format, named cells, etc. greatly facilitated grading of the laboratory reports. If, however, grading assistance is available, a more “free form” data reduction could be allowed.
• Incorporation of the Excel® “Goal Seek” or “Solver” options in the calculation of optimal fin design.

CONCLUSIONS
A method of allowing students to learn about heat transfer from extended surfaces in conjunction with the normal lecture sequence was a hybrid between a total laboratory experience and a complete computer presentation of analytical equations. Perhaps one of the most interesting ways in which the demonstration/laboratory was conducted was that it was in a modified “Socratic method” which caused the students to question their own perceptions (or MIS-perceptions). Analysis of the data allowed students practice with Fourier’s Law, Newton’s Law of Cooling, and analytical calculation of heat transfer rates, and caused them to compare and contrast the values obtained by each method. Additionally, it exercised them in various fin performance measures, and caused them to critically analyze how the values of m, M, and mL affected the performance of each fin. In short, there was a lot of “bang for the buck” with this laboratory, which may also be used strictly as a classroom demo.

ACKNOWLEDGEMENT
Special thanks are due Mr. Rob Lotz, technician in the Department of Engineering Mechanics at USAFA, for his superior craftsmanship, expertise, and common sense in the fabrication of this laboratory apparatus.

DISCLAIMER
The views expressed are those of the authors and do not reflect the official policy or position of the U.S. Air Force, Department of Defense, or the U.S. Government.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area (m²)</td>
</tr>
<tr>
<td>D</td>
<td>Diameter (m)</td>
</tr>
<tr>
<td>h</td>
<td>Average convective heat transfer coefficient (W/m²·K)</td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity (W/m·K)</td>
</tr>
<tr>
<td>L</td>
<td>Fin length (m)</td>
</tr>
</tbody>
</table>
m Fin parameter, $m = \sqrt{\frac{hP}{kA_c}}$ (1/m)

$M = \sqrt{hP kA_c} \theta_b$ (W)

P Fin perimeter (m)

q Heat transfer rate (W)

$R_{t,f}$ Fin resistance, $R_{t,f} = \theta_b / q_f$ (K/W)

t Fin thickness (m)

T Temperature ($^\circ$C, K)

x Distance along fin, measured from base (m)

**Greek**

Θ Temperature difference between fin and ambient temperature, $\Theta = T - T_\infty$ ($^\circ$C, K)

$\mathcal{E}$ Fin effectiveness, $\mathcal{E} = q_f / hA_c \theta_b$

$\eta$ Fin efficiency, $\eta = q_f / q_{max}$

**Subscripts**

b Base of fin
c Cross-section, or corrected length
f Fin
L Condition at fin tip
max Maximum possible
$\infty$ Condition at infinity (i.e., ambient conditions)

**REFERENCES**


