AC 2009-947: CHALLENGES OF TEACHING EARTHQUAKE ENGINEERING TO UNDERGRADUATES

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Challenges of Teaching Earthquake Engineering to Undergraduates

Abstract

This paper presents a detailed discussion of challenges faced by instructors who teach earthquake engineering at the undergraduate level; particularly in an era of increased pressure to limit the number of credit hours required to complete a bachelor's degree in engineering (both civil and architectural). The challenges that we have faced in teaching the subject of earthquake engineering include: having only two semester credit hours allotted to the subject, having students with limited background in structural dynamics, a lack of a textbook that presents the subject matter at an undergraduate level, and the lack of available computational tools that can be used to solve the complex mathematics involved even in the simplest of earthquake engineering problems. To address these challenges, the instructors have developed a six topic course outline that includes the following: seismology, single-degree-of-freedom (SDOF) dynamic analysis, response spectrum analysis in the context of SDOF system analysis, generalized SDOF system analysis, multi-degree-of-freedom (MDOF) system analysis, and code based seismic analysis. This paper emphasizes the development of computational tools (in MS Excel and Mathcad) for Modal Response Spectrum analysis of MDOF systems. In addition, a commercially available computer program (RISA 3D) is used to compare the results. Discussion of the other five topics is included in order to properly place the computational tools in terms of the challenges in teaching this course. The course has been taught twice (once without the use of the tools and a second time with the tools). The course average exam scores on questions related to the six aforementioned topics (embedded indicators) and scores of relevant student evaluation questions are used to assess the effectiveness of the computation tools. The tools will be made available for other instructors via the internet.

I. Introduction

Earthquake engineering is widely taught in civil and architectural engineering graduate programs around the country. However, at the undergraduate level, there are few schools that offer a course in earthquake (or seismic) engineering; though most graduate programs allow qualified undergraduates to enroll in a graduate level course as an elective. On the other hand, the University of the Pacific requires all the undergraduate students in civil engineering to complete a course in earthquake engineering. The motivation for this requirement came from an assessment of the program educational objectives; one of which is to develop graduates who are capable of professional licensure. Based on the program's assessment process, which incorporates alumni surveys, performance on the California Civil Professional Engineering Special Seismic Principles Examination (henceforth, the seismic portion of the PE exam), the program faculty chose to enhance the preparation of all civil engineering graduates in the area of earthquake engineering. However, given the pressures to reduce the number of units required to complete a bachelors degree, it was decided that a new course in earthquake engineering would essentially be a two-unit semester based course, which is approximately 32 contact hours of instruction. A further challenge is that structural dynamics is not required as a prerequisite, so the instructor must present background material in structural dynamics in order to make the earthquake engineering principles comprehensible to the average student. Furthermore, available textbooks are either too advanced or dated. It is in this light that we present the material and learning tools that have been developed for a first course in earthquake engineering for undergraduate students.

The course has been taught twice and the instructor has made considerable efforts to improve delivery of content in order to ensure that students satisfy the following learning outcomes:

1) Demonstrate a basic understanding of earthquake engineering;
2) Identify and solve basic structural dynamics problems;
3) Perform basic equivalent static and dynamic seismic analyses of simple structural systems.

The first objective relates to the student's ability to operate in the first two cognitive domains of Bloom's Taxonomy (namely, knowledge of seismic engineering terms and comprehension of the overall area of earthquake engineering). The second and third objectives primarily concentrate on the next two cognitive domains (namely, application of structural dynamics in earthquake engineering and analysis of systems subjected to earthquake loading).

The first time the course was taught, the instructor conducted a detailed survey of online course descriptions. Most courses in the subject were offered at the gradate level, with few exceptions particularly in architectural engineering programs that focus on structures, such as Cal Poly San Louise Obispo. In many of the programs, earthquake engineering was presented as a topic in a structural dynamics course. The programs that include a course in earthquake engineering cover a balance of seismic analysis and design, with a prerequisite of structural dynamics. So, a decision was made to combine the prerequisite structural dynamics material into the earthquake engineering course.

A review of the literature did not yield much information about teaching difficult earthquake engineering concepts. The fundamental approach in published research on undergraduate earthquake engineering education has been the development of web-based simulation tools[1] and the development of bench-top physical modeling tools (such as shake tables)[2, 3]. The main objective of both approaches is to provide a conceptual understanding of a wide range of earthquake engineering related topics. One other objective is to use these tools for outreach to potential future earthquake engineering students. The current direction of undergraduate earthquake engineering education lacks the substance required for structural seismic design practice. While the web-based simulation and bench-top physical modeling tools are excellent approaches to develop intuitive experience in earthquake engineering, and for outreach for the discipline, they are not intended to develop confidence in performing practical routine seismic design calculations. The tools discussed in this paper are intended to help students gain confidence and understanding in performing practical design calculations.

II. Overview of the Course

The course is divided into six fundamental topics:
1) Seismology (terminology, etc.) – covered in one lecture period (two contact hours)
2) Single-degree-of-freedom (SDOF) dynamic analysis – covered in four lecture periods (eight contact hours)

3) Response spectrum analysis in the context of SDOF system analysis – covered in three lecture periods (six contact hours)
4) Generalized SDOF system analysis – covered in three lecture periods (six contact hours)
5) Multi-degree-of-freedom (MDOF) system analysis based on the Modal Response Spectrum Analysis Method – covered in three lecture periods (six contact hours)
6) Code based seismic analysis – covered in two lecture periods (four contact hours)

The material for the first topic was selected based on seismology material covered in the California seismic portion of the PE exam[4,5]. The second topic includes discussions of free and forced, damped and un-damped vibrations; with the aim of developing parameters to characterize a system mass, stiffness, and damping, which are used to calculate periods, frequencies, and other vibration properties. The discussion is primarily focused on the development of parameters that are relevant to seismic analysis. The third topic includes discussion of the computational processes used to develop elastic response spectra, which includes discussions of the Newmark-Hall method for developing a design response spectrum – both elastic and inelastic spectra. The fourth topic covers an introduction to MDOF building systems (or multistory shear buildings). It also covers the development of the generalized SDOF equation, which is easily completed with the selection of a shape function. With this formulation, the principles learned during the SDOF and response spectrum topics can be applied to analyze a multistory building as a SDOF system (i.e., find maximum shear and bending moment due to a seismic load). The process entails computing approximate participation factors and using a design response spectrum to get the story displacements, which are then used to obtain lateral story seismic forces. Topic five covers the analysis of a shear building modeled as MDOF system. This is conducted using a commercially available program, RISA 3D, which has a "black-box" effect. This is the primary topic covered in this paper. The sixth and final topic covers the equivalent static methods found in the International Building Code (IBC) formulations used to establish seismic loading. Coverage of relationships of the IBC based design parameters to the structural dynamics parameters is discussed to elucidate the intricate formulations presented in the IBC code.

Table 1. Average percent examination scores in problems related to the six topics covered in the course.

	year	
	2007	2008
Seismology topics	84%	76%
SDOF analysis	97%	89%
Response spectrum	78%	74%
Generalized SDOF	81%	71%
Code analysis	82%	90%
MDOF analysis	60%	74%
Total number of students	22	10

Table 1 presents a comparison of the average test scores obtained for the two years the course has been taught. The first year scores are higher in most areas, except for code analysis and MDOF system analysis areas. Note that in the second year, the class was significantly smaller and any outliers (two in 2008 vs. one in 2007) would have a larger effect in the results of the second year, which can partially explain the lower scores in 2008. However, the scores for the last two (and most important) topics improved in the second year. This improvement can be attributed to two things: 1) improved coverage of the material as a result of a second time teaching the course and 2) to the use of computational tools developed to teach code design calculations and the more complicated MDOF system analysis. In this paper, we concentrate on the computational MDOF system analysis tools.

One other item that we used to assess the effectiveness of the tools is the student evaluations; particularly the average score on a question regarding the usefulness of supportive materials such as programs. The score for this question went from 3.79/5.0 to 4.0/5.0 (5.0 indicates a high rating and 1.0 a low rating), or approximately 5% increase. Student comments for this question were positive regarding the effectiveness of the tools. However, these scores are low compared to those from other structural engineering classes the authors teach. This may reflect the fact that this material is more difficult to grasp compared to material covered in other structural engineering courses.

III. Overview of multi-degree-of-freedom (MDOF) system seismic analysis (Modal Response Spectrum Analysis Method)

The process for determining the maximum response of a MDOF system to a seismic load based on the modal response spectrum analysis can be readily performed using a canned structural analysis computer program such as RISA 3D. Unfortunately, to properly computationally model a structural system, engineers must understand the modeling tools. Also, an engineer who understands the steps the program takes in the modeling process is better prepared to resolve problems with the results of the analysis. Therefore, it is imperative for students in any earthquake engineering class to understand modal response spectrum analysis. For undergraduates with limited mathematical backgrounds (some have not even been exposed to linear algebra) it is important that the mathematical steps are presented in an unadulterated fashion. Before we discuss the programs developed in Mathcad and Excel, we will briefly review the steps in the response spectrum analysis method:

1. The mass matrix, [m] is obtained from the given floor weights

2. The stiffness matrix, [k] can be obtained from the column properties for a shear building(most undergraduate students are exposed to the stiffness method in a structural analysis course so obtaining a stiffness matrix is relatively straight forward)

3. With the stiffness and mass matrices, solve for eigenvalues, ω^2, which are used to determine the frequencies, ω, and periods, T, of the system: $\{\phi\}([k] - \omega^2[m]) = \{0\}$.
   a. Take the determinate of $([k] - \omega^2[m])$, i.e., $\det([k] - \omega^2[m])$ to get eigenvalues (in excel, we need to reduce the mass matrix to an identity matrix, so a limitation is that all the masses must be equal; the resulting operation is $\det([k]/m - \omega^2[I])$.)
      where,
      [I] is the identity matrix

   b. With the frequencies (which are the square root of the eigenvalues), we can obtain the periods, $T = 2\pi/\omega$.

4. Determine the spectral accelerations matrix from a design response spectrum by selecting the appropriate values for each period computed above, $[S_a]$.

5. With the eigenvalues, we can determine the eigenvectors and mode shapes.
   a. First, substitute each eigenvalue into $\{\phi\}([k] - \omega^2[m]) = \{0\}$ and solve for a $\{\phi\}$ vector for each of the eigenvalues.
   b. Then group the eigenvectors into a matrix [EV].
   c. Finally, normalize each vector by setting the first component to unity to get a second matrix $[\phi]$, the modal matrix.

6. Normalized the modal matrix to obtain the mode shape matrix, $[\Phi] = [\phi]/([\phi]^T[m][\phi])^{½}$.

7. Obtain the column vector of participation factors for all modes considered, $\{\Gamma\} = [\Phi]^T[m]\{1\}$.
   where,
   {1} is a column vector of ones

8. We can also obtain the specific participation factor associated with each mode shape (the contribution of each mode shape to the total response).

9. The displacements associated with each mode shape are obtained as:
   $$[x] = [\Phi][\Gamma][S_d] = [\Phi][\Gamma][S_a][\omega^2]^{-1}$$
   where,
   $[\Gamma]$ = diagonal matrix of participation factors
   $[S_d]$ = diagonal matrix of spectral displacements
   $[S_a]$ = diagonal matrix of spectral accelerations
   $[\omega^2]$ = diagonal matrix of squared modal frequencies

10. The resultant maximum displacement at each node is obtained from the square-root-of-the-sum-of-the-squares (SRSS) of the corresponding row vector: $x_{maxi} = (\Sigma x_i^2)^{½}$

11. The matrix of lateral forces at each node is: $[F] = [k][x]$

12. The resultant maximum lateral force at each node is obtained from the square-root-of-the-sum-of-the-squares of the corresponding row vector: $F_{maxi} = (\Sigma F_i^2)^{½}$

13. The column vector of total base shear forces is: $\{V\} = [F]^T\{1\}$

14. The maximum base shear force is obtained from SRSS as: $V_{max} = (\Sigma V_i^2)^{½}$

This process results in the maximum story displacements, maximum story forces, and maximum base shear. Note that ASCE 7[6] allows other methods for combining response parameters, such as the complete quadratic combination (CQC). This particular method along with the SRSS is also included in RISA 3D as a method for combining the contribution from each mode.

IV. Analysis Tools

To demonstrate the use of the analysis tools, we solve a simple two-degree-of-freedom shear building example. Figure 1 shows the frame used in the analysis, which we assume to have rigid beams. This problem is solved in the workbook by Williams[4]. The response spectrum is also given in Figure 1. Given that the beams are assumed to be rigid, we can model the shear building as a two degree-of-freedom system. The hand-calculations solution is rather tedious and most of our undergraduate students are not equipped with the mathematical skills to carry out the eigen solution.

In the initial offering of the course in 2007, it was decided that RISA 3D would be used to conduct a modal response spectrum analysis. All students taking the course had utilized RISA 3D as a tool for analysis of beams and frames in a first course in structural analysis. Therefore, the only new material students would have to learn would be carrying out dynamic analysis of similar beams and frames. This is a straight forward feature in RISA. However, the procedure for defining a response spectrum and performing a full modal response spectrum analysis is much more complicated. The students expressed frustration since results from the software were inconsistent with the results of hand calculations presented in class. The fundamental problem was the approach RISA takes to normalize eigenvalues and eigenvectors.

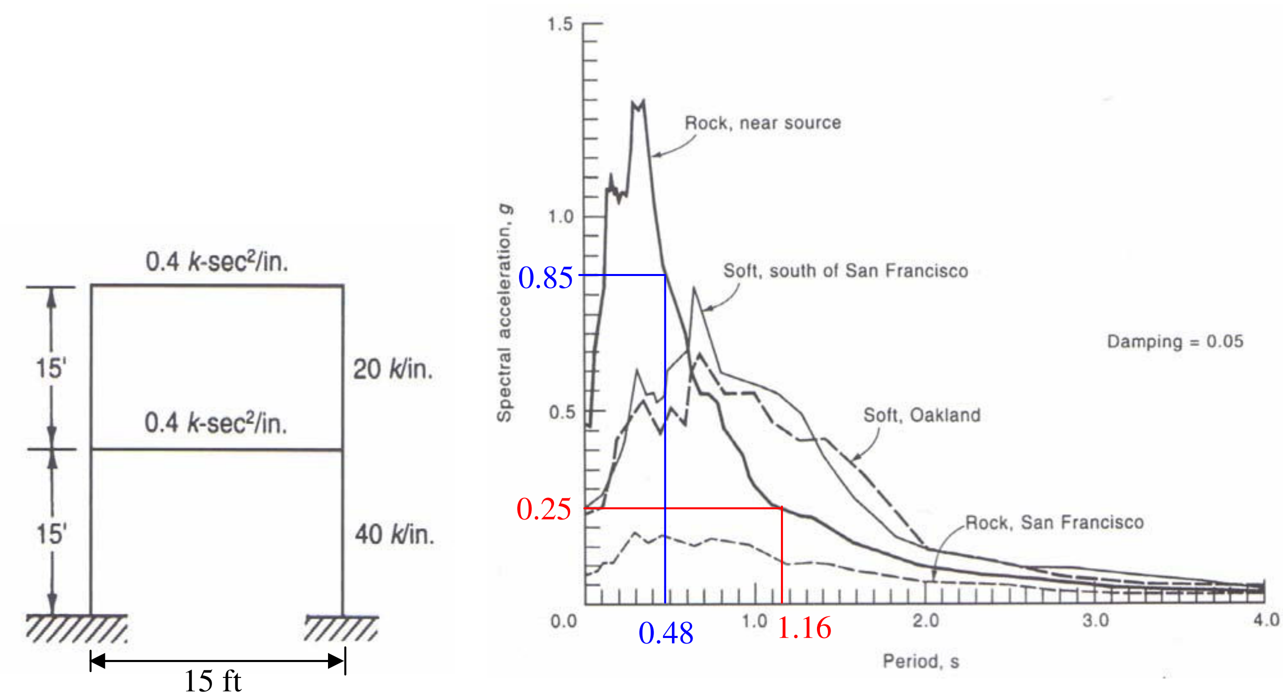


Figure 1. Two story shear building frame and Response Spectra for Loma Prieta Eathquake[4]

In the most recent offering of the course, a MS Excel spreadsheet was developed so that the students could be guided by well-defined steps and calculations in the analysis process. Table 2 depicts the spreadsheet calculations for the problem given in Figure 1. The main challenge in developing the spreadsheet was getting a macro that can solve for eigenvalues and eigenvectors.

Fortunately, there is such a macro in the form of a plug-in[7]. This MS Excel plug-in gives a pull down menu with several mathematical operations one of which is solving for the eigenvalues and another for eigenvectors, see the parts of Table 2 that are shaded in yellow. With the eigenvalues and vectors, the remaining matrix operations can be performed following Excel's standard matrix operations and implementing the steps outlined in Section III.

Students were more receptive to experimenting with RISA 3-D after studying the modal response spectrum analysis implemented in a spreadsheet and modifying the spreadsheet to accommodate a three-degree-of-freedom system. Following the presentation of the spreadsheet, students used RISA 3-D to conduct an analysis of a multi-bay, multi-story frame. Overall, the experience was very educational to both the students and the instructor.

After the topic had been covered with MS Excel, a decision was made to develop further tools using Mathcad. Mathcad was chosen as an option since many of the equations that are displayed in symbolic format are also simultaneously processing calculations. Perhaps equally important is the ability to easily provide text and explanations during the calculation process. In addition, built-in functions are readily available to solve for eigenvalues and eigenvectors in the software. This will be used the next time the course is taught. The students will be allowed to choose between Excel and Mathcad. In Appendix A, we present the new Mathcad formulation. A comparison of the results from hand calculations, RISA, Excel, and Mathcad are shown in Table 3. While the periods determined by all methods are identical; RISA deviates for all other results. This is reasonable given that we have modeled the frame slightly more accurately in RISA (essentially as a system of flexible columns and rigid beams as shown in Figure 1). In all cases, the difference is rather small. Also, RISA does not compute the equivalent story forces, which are not needed since the program gives the internal forces and moments for each member; which must still be computed by hand calculations in Excel, Mathcad, or other software!

Table 2. Spreadsheet Modal Response Spectrum Analysis[4]

k (k/in) =	20	[m] =	0.4	0	{1} =	1		
W(k) =	154.4		0	0.4		1		
m(K-sec^2/in) =	0.4	[S_a] =	96.6	0	[ω^2] =	29.289	0	
h (in) =	180		0	328.4		0	170.71	

Stiffness matrix for 2 DOF system

				Eigenvals, λ	sqrts of λ		ω (rad/sec)	T (sec)
[k] =	3	-1	x k/m	0.5858	0.7654	/sqrt(k/m)	5.41	1.16
	-1	1		3.4142	1.8478	/sqrt(k/m)	13.07	0.48

	eigen vectors			normalized eigen vectors			[EV] x [NOR]				
[EV] =	0.3827	0.9239	[NOR] =	2.6131	0	[ϕ] =	1	1	[ϕ]^T =	1	2.4142
	0.9239	-0.383		0	1.0824		2.4142	-0.414		1	-0.414

			sqrt([ϕ]^{T}x[m]x[ϕ])	
[ϕ]^{T}x[m]x[ϕ]=	2.7314	-6E-17	1.6527	0
	-6E-17	0.4686	0	0.6846

	[ϕ]/sqrt([ϕ]^{T}x[m]x[ϕ])			[[Φ]^T x [m]]x{1}	
[Φ] =	0.6051	1.4608	{Γ} =	0.8263	These are the participation factors
	1.4608	-0.605		0.3423	

Displacements:

			[[Φ] x [Γ]] x [S_a]	
[Γ] =	0.8263	0	48.3	164.2
	0	0.3423	116.61	-68.01

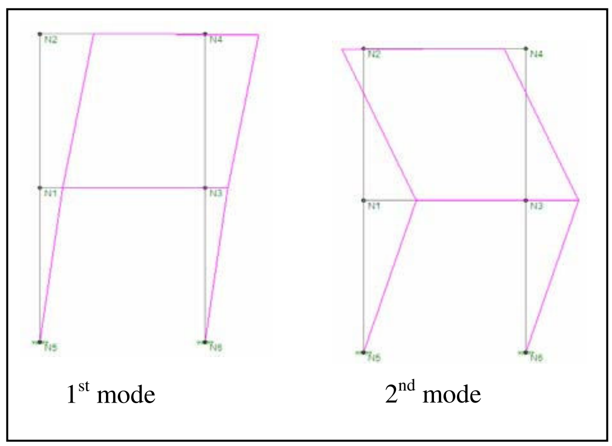
	[[Φ] x [Γ]] x [S_a]/[ω^2]		x_{max}=sqrt(Σx_i^2)	
displ, [x] =	1.6491	0.9619	max displ at each	1.9091
	3.9812	-0.398	node by SRSS	4.0011

Lateral forces:

	[k] x [x]		F_{max}=sqrt(ΣF_i^2)	
[F] =	19.32	65.68	max force at each	68.463
	46.643	-27.21	node by SRSS	53.997

Base Shears vector:

	([F]^T x {1})^T		V_{max}=sqrt(ΣV_i^2)	
{V} =	65.963	38.474	max V by SRSS =	76.363



indicates that we do: (yellow)	Matrix plug-in, then pull macros menu down, then click on eigen-solving, follow directions to do eigenvalues then eigenvectors.
indicates that we do: (green)	Matrix operations, so after entering =MMULT(B28:F32,H28:L32), select an appropriate range ??:?? starting with the formula cell. Press F2, and then press CTRL+SHIFT+ENTER.

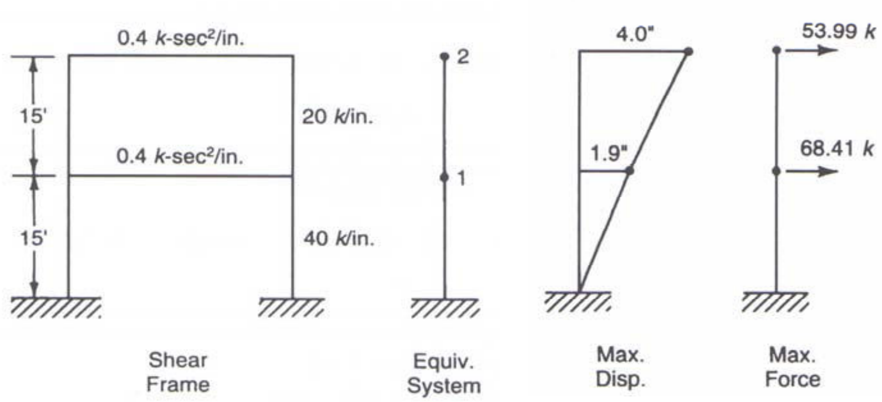


Table 3. Comparison of calculation results using, RISA, Excel, and Mathcad.

		Hand cals	RISA	Excel	MathCAD
Period (sec)	1st mode	1.16	1.16	1.16	1.16
	2nd mode	0.48	0.48	0.48	0.48
Max. Displs based on SRSS method, (in)	1st level	1.90	2.01	1.91	1.91
	2nd level	4.00	4.14	4.00	4.00
Max. forces based on SRSS method, (kips)	1st level	68.41	-	68.46	68.40
	2nd level	53.99	-	54.00	53.94
Max. base shear based on SRSS (kips)		76.35	80.40	76.36	76.29

V. Observations

From a pedagogical perspective, hand calculations can demonstrate in detail the fundamental steps needed to complete a modal response spectrum analysis; however, they do not facilitate the ability to compare and contrast structural systems with varying properties (i.e. stiffness, weights, degrees of freedom, etc.) On the other hand, commercially available structural analysis software, such as RISA 3D, allows students to evaluate the performance of various structural systems with practically any number of degrees of freedom. However, it is possible for students to easily lose sight of fundamental seismic design principles (i.e. learning outcomes) in the pursuit of complexity and increasing expertise with a particular brand of software.

We believe that the use of computational software, such as MS Excel and Mathcad, in earthquake engineering courses provides a learning tool that negotiates a valuable middle ground between hand calculations and commercial structural analysis software. More importantly it allows students to gain a fundamental understanding of the analytical process while affording the flexibility for students to examine and to evaluate structural systems by asking and answering questions such as: (1) how does increasing the stiffness of each story by 20% influence the maximum displacements and forces on the structural system?, (2) how does increasing the story height of the first floor alter design requirements?, (3) what aspects of a structural system can one alter to reduce the maximum base shear?

VI. Conclusions

Modal response spectrum analysis is a powerful tool for analyzing structures that may be subject to relatively high seismic loads; although it is readily available as an analysis tool in commercial structural analysis software, it is important for the engineering student to understand the fundamental steps. The spreadsheet and Mathcad programs presented in this paper are useful tools in helping students develop the skills necessary to implement the method, and understand the process involved in modern seismic analysis and design. These tools can be used to perform the arithmetic-intense calculations of the modal response spectrum analysis, and allow students to efficiently solve complex problems that would be otherwise prohibitively time-consuming. This is particularly challenging in a course of limited scope and class time. Furthermore, one of the outcomes in criterion 3 of ABET EC2000 is to demonstrate "*the students' ability to use techniques, skills and tools in engineering practice*". Our analysis tools allow instructors to assign problems that are more realistic without increasing the demand in students' time. It also gives the students a sense of confidence in solving rather complex structural analysis problems without using a "black box" type computer software.

A further goal of this paper is to begin a dialog with other educators (as well as engineers who routinely take advantage of the capability of commercially available seismic loading analysis programs) to advance the discipline of earthquake engineering education in a direction of developing *abilities* of undergraduate students to perform practical routine seismic design calculations; consistent with current accreditation standards.

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Appendix A. Mathcad Modal Response Spectrum Analysis

EARTHQUAKE ENGINEERING: TWO STORY SHEAR BUILDING

PROBLEM STATEMENT. A two story shear building with given properties and a damping ratio of five percent is located on a rock site near the source of the Loma Prieta earthquake. Determine the lateral forces, displacements, and base shear.

GIVEN:

Column stiffness, $k := 20\dfrac{kip}{in}$ Height of each story, $h := 15ft$

Weight of each floor, $W := 154.4kip$ Mass of each floor, $m := \dfrac{W}{g} = 0.4 \cdot kip \cdot \dfrac{s^2}{in}$

FIND: Apply matrix methods and seismic principles to determine lateral forces and displacements. Calculate maximum displacements, lateral forces, and base shear.

APPROACH: Solve the eigenvalue problem to determine natural frequencies and mode shapes. Use natural periods to determine spectral acceleration from response spectrum curves. Calculate participation factors, corresponding lateral forces and displacements. As well as the maximum lateral forces and displacements.

1) The mass matrix is obtained from the given floor weights.

Mass matrix, $M := \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix} \cdot kip \cdot \dfrac{s^2}{in}$

2) The stiffness matrix is obtained from the column properties of the shear building.

Stiffness matrix, $K := \begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix} \cdot k = \begin{pmatrix} 60 & -20 \\ -20 & 20 \end{pmatrix} \cdot \dfrac{kip}{in}$

3) Calculate natural frequencies and mode shapes from the eigenvalue problem.

Eigenvalue equation, $(K - \lambda_n \cdot M) \cdot \varphi = 0$

where λ_n are eigenvalues; ω_n, are circular natural frequencies and $\lambda_n = \omega_n^2$. The eigenvalues are determined from the frequency determinant shown below.

$$\left| K - \lambda_n \cdot M \right| = 0$$

$$\lambda_n := \text{sort}\left(\text{eigenvals}\left(K \cdot M^{-1}\right)\right) = \begin{pmatrix} 29.296 \\ 170.75 \end{pmatrix} \cdot \frac{rad}{s^2}$$

Note: the command "eigenvals" solves for the eigenvalues and "sort" orders the result from lowest to highest values.

The circular natural frequencies and periods for the two modes of vibration,

$$\omega_n := \sqrt{\lambda_n} = \begin{pmatrix} 5.413 \\ 13.067 \end{pmatrix} \cdot \frac{rad}{s} \qquad T_n := \frac{2\pi}{\omega_n} = \begin{pmatrix} 1.161 \\ 0.481 \end{pmatrix} s$$

4) Determine spectral accelerations from the design response spectrum.

Using the response spectrum and natural periods, T_n, spectral accelerations are determined,

$$S_{a_{1,1}} := 0.25g = 96.522 \cdot \frac{in}{s^2} \qquad S_{a_{2,2}} := 0.85g = 328.175 \cdot \frac{in}{s^2} \qquad S_a = \begin{pmatrix} 96.522 & 0 \\ 0 & 328.175 \end{pmatrix} \cdot \frac{in}{s^2}$$

5) Determine mode shapes by substituting eigenvalues into the eigenvalue equation.

e.g. $\left(K - \lambda_{n_1} \cdot M\right) \cdot \varphi^{\langle 1 \rangle} = 0$ solving for $\varphi^{<1>}$ will yield the first mode shape.

$$\varphi^{\langle 1 \rangle} := \text{eigenvec}\left(K \cdot M^{-1}, \lambda_{n_1}\right)$$

$$\varphi^{\langle 2 \rangle} := \text{eigenvec}\left(K \cdot M^{-1}, \lambda_{n_2}\right)$$

Note: the command "eigenvec" solves for eigenvectors or modeshapes corresponding to the calculated eigenvalue, λ_n.

The resulting matrix of eigenvectors, $\varphi = \begin{pmatrix} 0.383 & -0.924 \\ 0.924 & 0.383 \end{pmatrix}$

The matrix of relative modeshapes is determined by setting the first components of the modeshape factors to unity.

$$\varphi := \text{unity}(\varphi) = \begin{pmatrix} 1 & 1 \\ 2.414 & -0.414 \end{pmatrix}$$

Note: the "unity" function sets the first components of the mode shape factors to unity and outputs a matrix of relative mode shapes.

6) Determine the mass normalized modal matrix, Φ, from

$$\Phi_{i,j} = \frac{\varphi_{i,j}}{\sqrt{\sum M_{i,j}\left(\varphi_{i,j}\right)^2}}$$

$$\Phi := \varphi \cdot \left(\text{sqrt}\left(\varphi^T \cdot M \cdot \varphi\right)\right)^{-1} = \begin{pmatrix} 0.605 & 1.461 \\ 1.461 & -0.605 \end{pmatrix} \cdot \sqrt{\frac{in}{kip \cdot s^2}}$$

Note: the "sqrt" function calculates the square root of each component of a m x n matrix and returns an m x n matrix.

7) Define a column vector of participation factors as follows:

$$\Gamma := \Phi^T \cdot M \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.826 \\ 0.342 \end{pmatrix} \cdot \sqrt{\frac{kip \cdot s^2}{in}}$$

where, **Γ**, is a column vector of participation factors for all mode shapes considered, and, **Φ**, is the normalized modal matrix

8) The participation factors can be associated with each mode shape

Placing each component of the n x 1 column vector of participation factors into an n x n diagonal matrix .

$$\Gamma_{diag} := \text{diag}(\Gamma) = \begin{pmatrix} 0.826 & 0 \\ 0 & 0.342 \end{pmatrix} \cdot \sqrt{\frac{kip \cdot s^2}{in}}$$

9) Determine the matrix of maximum node displacements for each mode. This is represented symbolically as,

$$x = \Phi \cdot \Gamma \cdot S_d = \Phi \cdot \Gamma \cdot S_a \cdot \left(\omega^2\right)^{-1} = \Phi \cdot \Gamma \cdot S_a \cdot \left(\lambda_n\right)^{-1}$$

In order to conduct the appropriate matrix algebra, expand the m x 1 column vector of eigenvalues into a m x m diagonal matrix of eigenvalues,

$$\lambda_{diag} := \text{diag}\left(\lambda_n\right) = \begin{pmatrix} 29.296 & 0 \\ 0 & 170.75 \end{pmatrix} \cdot \frac{rad}{s^2}$$

Matrix of maximum node displacements $x := \Phi \cdot \Gamma_{diag} \cdot S_a \cdot \lambda_{diag}^{-1} = \begin{pmatrix} 1.647 & 0.961 \\ 3.977 & -0.398 \end{pmatrix} \cdot in$

10) Calculate the maximum displacement at each node from SRSS of the row vector.

$$x_{max} := SRSS(x) = \begin{pmatrix} 1.907 \\ 3.997 \end{pmatrix} \cdot in$$

Note: The "SRSS" function calculates the square-root-of-sum-squares of each row in a m x n matrix and returns a m x 1 vector.

11 + 12) Determine the matrix of lateral forces associated with each mode and resultant maximum lateral force at each node from SRSS

$$F := K \cdot x = \begin{pmatrix} 19.3 & 65.62 \\ 46.594 & -27.181 \end{pmatrix} \cdot kip$$

$$F_{max} := SRSS(F) = \begin{pmatrix} 68.399 \\ 53.943 \end{pmatrix} \cdot kip$$

13) Determine the column vector of total base shear forces, $\mathbf{V} = \mathbf{F}^T\{1\}$, ***maximum base shear from SRSS.***

$$V := F^T \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 65.894 \\ 38.439 \end{pmatrix} \cdot kip$$

$$V_{max} := SRSS\left(V^T\right) = (76.287) \cdot kip$$