

**AC 2009-1608: TEACHING CONTROL-VOLUME SELECTION TO AVOID  
ENTROPY ACCOUNTING “BOTTLENECKS” IN SECOND-LAW ANALYSES**

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## **Teaching control volume selection to avoid entropy accounting 'bottlenecks' in second law analyses.**

### **Abstract**

Application of the first law is invariably reduced to identifying energy departures from, and arrivals to, assorted discrete control volumes. Because, by the nature of the first law, energy is conserved, problems reduce to simple accounting exercises, only occasionally upset by the presence of energy generation terms (e.g. nuclear or possibly chemical reactions.) When such terms do arise they are usually well defined and located in discrete control volumes so that including them in the overall energy balance is generally straightforward.

When one wishes to instruct in the application of the second law, the accounting exercise and the definition of the control volume has to be undertaken with great care, as unlike energy, entropy is actually generated during transportation and transition processes, within the control volumes. It is not unusual therefore, for entropy generation terms to be allocated to the surrounding 'environmental control volume', as mixing, conduction, convection and radiation etc. These can of course occur well outside of the original problem focused control volume which has invariably been defined by simple consideration of the ensuing first law energy analysis and obvious geometry features. The 'bottleneck' referred to in the title of the paper is descriptive of what happens to the vast majority of Thermodynamics courses at this point. Students having obtained a solid understanding of the first law and its applications do not progress through to the same level of understanding with the second law. The vast majority learning to use the second law almost on autopilot without a real feel as to what the property is telling us.

The paper presents an analysis therefore of a simple hot water pipe to show specific issues and areas of confusion with second law control volume selection. It concludes emphasizing the importance of consistently listing assumptions both for the student solving, and the instructor setting an assignment. In summary, this paper highlights and gives an example of novel teaching methods that have been successfully used by the authors to overcome this 'bottleneck' in thermodynamic instruction.

### **Introduction**

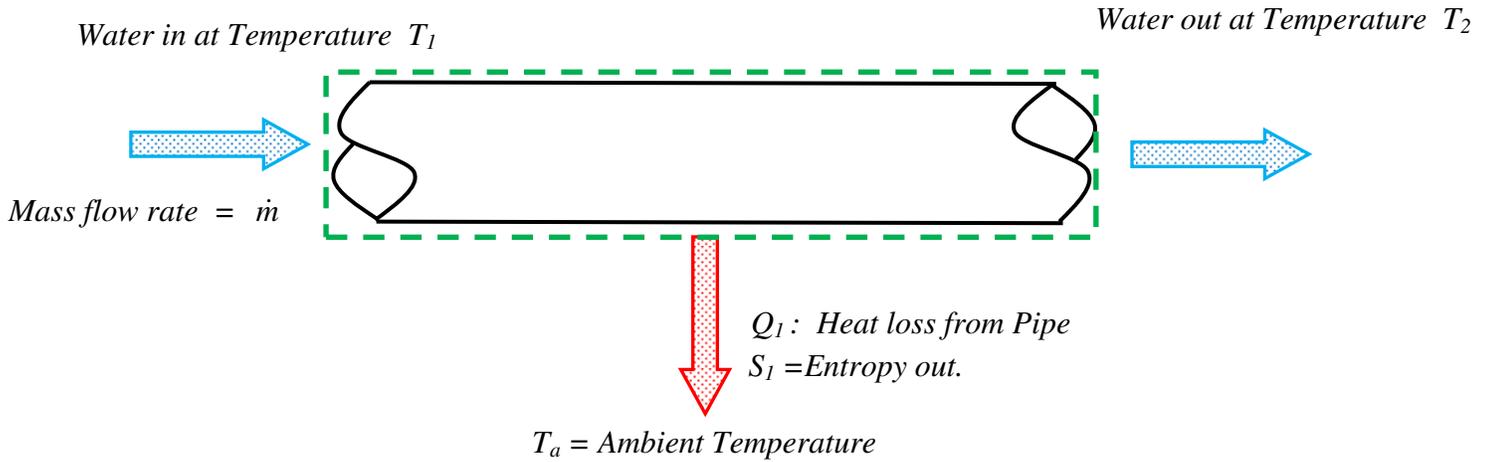
Foley [1] derived the property entropy from consideration of a simple orifice plate in a pipe. This derivation was unique in that the entropy term derived did not arise from any heat transfer. Instead, an uncontrolled expansion was used, and as such the concept of 'bad energy' as the numerator in the expression for entropy was introduced.

While teaching a class an interesting discussion was raised as to where exactly entropy is generated in a heat transfer process, a brief discussion in Cengel & Boles [2] while touching on the topic did not answer the legitimate concerns raised by students. This was the motivation for this paper in which the focus has changed back to a heat transfer numerator in the entropy expression as the source of the 'bad energy'. (Unrecoverable energy transfer is another way of considering this term.) There is no subtle reason for this other than this turns out to be the easiest energy transfer to be considered in the example used, and is consistent

with the initial heat transfer problem that raised the issue. As in the earlier paper a simple pipe is used to demonstrate the points.

### The hot water pipe.

To illustrate the problem with second law analysis a simple hot water pipe is shown with a dotted line signifying the control volume for the system .( Figure 1) .



**Figure 1.** Simple pipe model for second law consideration.

Application of the energy accounting equation as described by Foley [3], gives :

$$\dot{Q}_1 - \dot{W} + (\dot{m}e)_{in} - (\dot{m}e)_{out} + \dot{E}_{gen} = \dot{E}_{CV} \quad . \quad (1)$$

Which for a steady flow, no work situation simplifies to :

$$\dot{Q}_1 = \dot{m} \cdot C_p (T_1 - T_2) \quad . \quad (2)$$

This is usually as far as we need to go in most practical problems, and hence any issues of entropy do not arise. However there is much to be learned by considering the entropy accounting equation for this case.

i.e.

$$\dot{S}_{in} - \dot{S}_{out} + (\dot{m}s)_{in} - (\dot{m}s)_{out} + \dot{S}_{gen} = \dot{S}_{CV} \quad . \quad (3)$$

Again, assuming a steady state condition has arisen, this reduces to:

$$\dot{S}_1 = \dot{m}(s_1 - s_2) + \dot{S}_{gen} \quad . \quad (4)$$

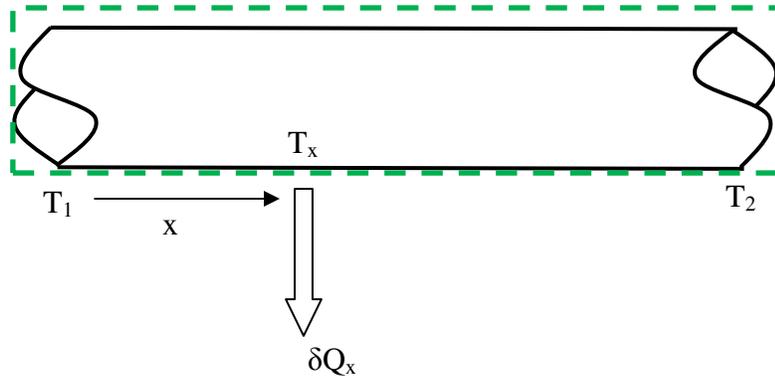
Where  $\dot{S}_1$  is the rate of entropy transport out of the control volume by virtue of the heat transfer  $\dot{Q}_1$  previously calculated in the energy equation (2).  $\dot{S}_{gen}$  is the entropy generated

within the control volume, and the  $s_1$  and  $s_2$  values are the specific entropies of the mass entering and exiting through the pipe.

Problems arise here as although the drop in entropy of the water is easily computed from :

$$s_1 - s_2 = C_v \cdot \ln\left(\frac{T_1}{T_2}\right) \quad (5)$$

The other two terms are not as straightforward. We have no value of  $\dot{S}_{gen}$  and as for  $\dot{S}_1$  we have the situation where heat lost along the pipe varies with  $x$ , as shown below in figure 2:



**Figure 2.** Variable heat transfer along a pipe.

Unlike the inlet and outlet cross sectional area, where constant temperature can be assumed to be a reasonable approximation, the temperature across the surface boundary is not constant. To find  $\dot{S}_1$  and hence  $\dot{S}_{gen}$  therefore requires integration along the pipe:

$$\dot{S}_1 = \int_1^2 \frac{\delta Q}{T_x} \quad (6)$$

The problem is compounded as not only must the temperature distribution and the local heat transfer be formulated, but we must also clearly define our control volume. Does it include the pipe or does it include some of the outer boundary layer? Is there convection, radiation, or conduction along the pipe etc? All in all, the problem expands immeasurably compared to the simple energy equation.

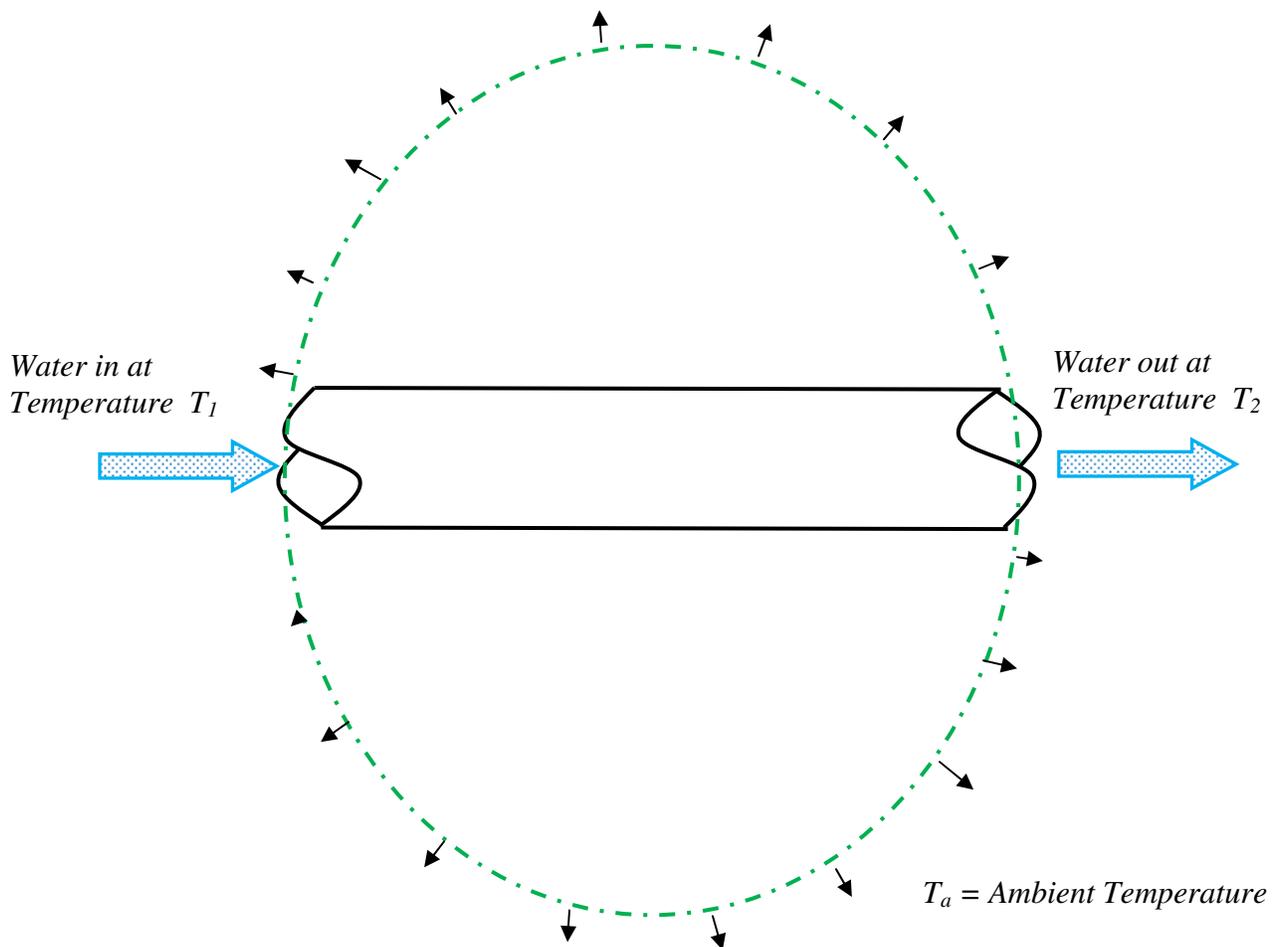
As a proverbial ‘wrench in the works’ even if the above is computed, there has been no allowance for the subsequent entropy generation in the surroundings as the heat transferred from the pipe dissipates into the ambient.

To make progress in this problem the recommended solution is to simply calculate the  $\dot{S}_1$  term as follows :-

$$\dot{S}_1 = \frac{Q_1}{T_a} \quad (7)$$

The rationale being that all the heat transfer ultimately ends up as energy stored at the ambient temperature,  $T_A$ . This logic appears sound and the resulting  $\dot{S}_{gen}$  for the process can now be calculated from equation (4).

The problem with this approach is that it is now very easy to assign all of the entropy generation to the original control volume, as this is still the only one that has been defined. This is incorrect and to avoid this it is useful to encourage the application of a different control volume for the entropy accounting exercise. i.e.



**Figure 3.** Heat diffusion from an expanding control volume.

At the boundary of this control volume the temperature difference has diminished so much that in the limit the heat is being transferred at the ambient temperature  $T_a$  and no entropy generation is taking place in the surroundings. (i.e. External to the control volume shown.)

Application of the accounting equation to the surroundings gives :-

$$\dot{S}_{SURR} = \frac{\dot{Q}_1}{T_a} + \dot{m}(s_2 - s_1)$$

$$\dot{S}_{SURR} = \frac{\dot{Q}_1}{T_a} + \dot{m}.C_v.\ln\left(\frac{T_2}{T_1}\right) \quad (8)$$

Application of the accounting equation to the control volume itself now gives :-

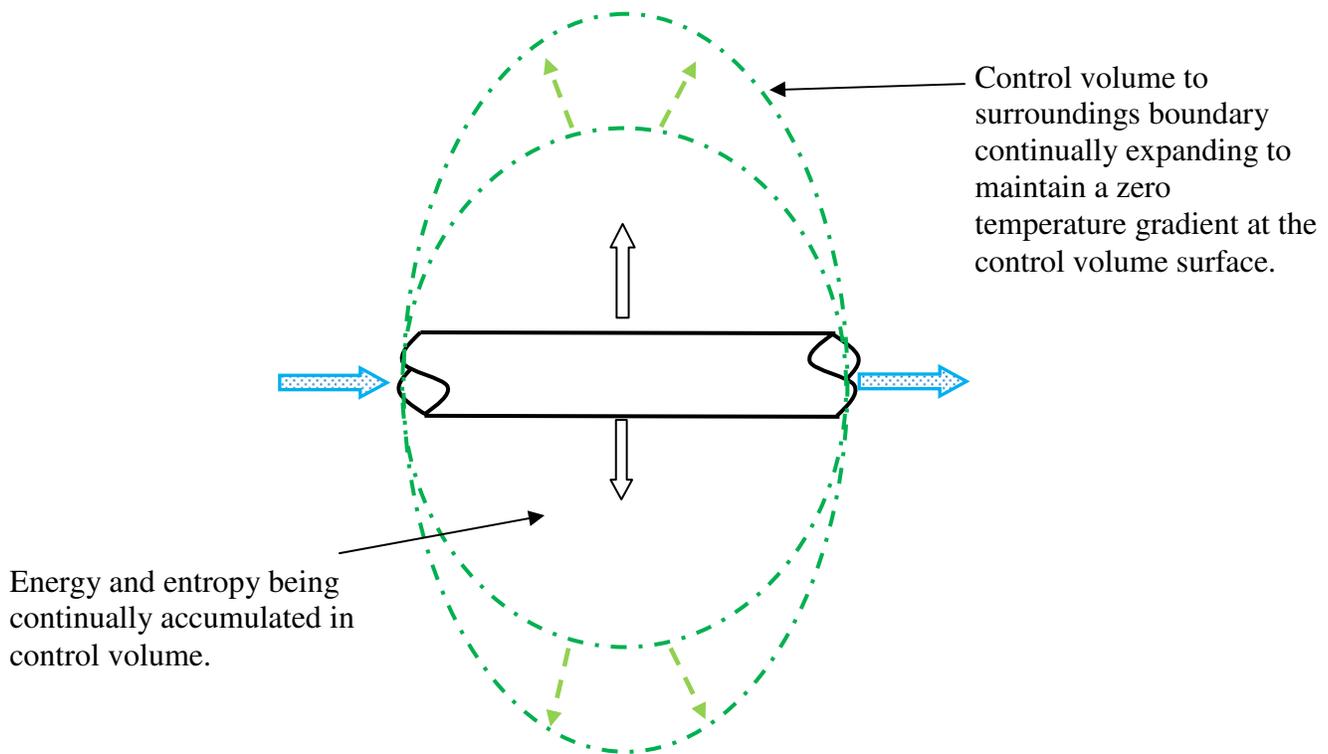
$$\dot{S}_{CV} = \frac{-\dot{Q}_1}{T_a} + \dot{m}(s_1 - s_2) + \dot{S}_{gen} \quad (9)$$

And as the control volume is still in steady state ,  $\dot{S}_{CV} = 0$  and hence :-

$$\dot{S}_{gen} = \frac{\dot{Q}_1}{T_a} + \dot{m}.C_v.\ln\left(\frac{T_2}{T_1}\right) \quad (10)$$

This is the same as  $\dot{S}_{SURR}$ , which should of course be no surprise. While the answer is no different from that which would be arrived at with the initial assumption of all the heat transfer occurring at ambient temperature, it is important that the second control volume, although somewhat arbitrarily drawn, highlights that there is physically something different occurring here. Indeed, the paradox is that the boundary is so defined that if all the heat transfer does occur at ambient where there is effectively no temperature gradient, then there should in fact be no heat transfer. There is an argument therefore to say that all the entropy generated, and indeed the energy in the heat transferred, is stored in the arbitrary control volume. If this was so then the logical explanation is that in order to maintain the effective zero temperature gradient at the control volume boundary, the control volume would have to continually grow. See figure 4 below.

Luckily for us, the surroundings are expansive enough to cope, although with the advent of global warming, some may argue that as engineers we are too quick to make this assumption, and that our atmosphere and oceans are not the infinite reservoirs we take them to be.



**Figure 4.** Zero temperature gradient expanding control volume..

## Conclusion

Although a simple hot water pipe is used, the complexities and subtleties of entropy accounting are clearly highlighted. Taking some of the modeling assumptions to the extreme illustrates some of the paradoxes that are frequently overlooked. i.e. Assuming that all the heat transfer out of our system ultimately gets transferred to the surroundings at ambient temperature where no temperature gradient and hence no heat transfer should occur ?

The usefulness of the examples shown are in reinforcing a comprehension of entropy accounting and most importantly that the control volume suitable for an energy accounting analysis is not necessarily the optimum control volume for an entropy accounting analysis. Although the above discussions of entropy and 2<sup>nd</sup> law control volume selection are generally well received the author still sees the projection of a physical meaning to entropy as one of the greatest challenges in Thermodynamic instruction.

## References

- [1] *A.C.Foley(2007)*Escape from Carnot : A new way to introduce the mysterious property, entropy. ASEE 2007 National convention, Hawaii AC 2007-233
- [2] *Cengel. Y.A, Boles M.A (2008)* Thermodynamics an engineering approach. Chapter 7, p395. 6<sup>th</sup> Ed. Mcgraw Hill. ISBN 978-0-07-352921-9
- [3] *A.C.Foley (2006)* Applying the ‘catch all’ general control volume and the Reynolds transport equation to improve thermodynamic instruction.. ASEE National Convention, Chicago 2006.AC 2006-862