Abstract

Computer simulation and experimental testing play major roles in noise and vibration engineering. Modal analysis of structures, for instance, is regularly performed experimentally and with finite element analysis. Often the integration of simulations and experiments consists of nothing more than adjusting a fudge factor, like a material property, to get simulations to agree with test results. However, the current emphasis in industry and research laboratories is to more tightly couple testing and simulation-using test results to validate simulation models and simulation results to design experiments. For example, finite element analysis is used to identify how best to support and excite a structure to produce a particular vibration, and modal test results are used to establish “modal assurance criteria” on finite element simulations.

This paper presents two laboratory exercises that demonstrate the importance of coupling computer simulations with experiments for mutual validation. The exercises from a new course in “Acoustics, Noise and Vibration” at GMI Engineering & Management Institute also introduce students to tools and practices used extensively in noise and vibration engineering. The other six experiments in the course, like most undergraduate laboratory experiments, focus on demonstrating physical principles. These two exercises focus on the tools and methods employed in noise and vibration engineering. The first exercise comes near the beginning of the course and deals with the frequency domain analysis of signals using fast Fourier transforms (FFTs). The second exercise, near the end of the course, deals with structural modal analysis.

Introduction

Strong demand from students and their employers prompted the recent development of a new concentration in Acoustics, Noise and Vibration at GMI Engineering & Management Institute. Two courses in acoustics and vibration have been created and three more are planned as a cooperative effort between Mechanical Engineering and Applied Physics. The instructional laboratory used in the courses has been made possible with grants from the National Science Foundation and industry.

Eight laboratory exercises have been developed for the senior-level course “Acoustics, Noise and Vibration” to introduce students to the theory and application of important concepts in sound and vibration. These exercises introduce students to the concepts and equipment involved with sound radiation, frequency analysis, sound pressure and intensity measurements, sound absorption, room acoustics and reverberation time, acoustic filters and mufflers, and structural vibration. In another paper we describe the objective and learning outcome expected of each exercise, and how the sequence
Most of these experiments are designed to emphasize physical principles rather than the experimental tools or methods employed. However, the two experiments discussed in the present paper are designed to demonstrate the importance of coupling computer simulations with experiments to avoid mistakes, improve quality and enhance confidence in both the computer model and test result. The tools used in these exercises are used extensively in noise and vibration engineering for testing, design and analysis, and the techniques used are considered necessary practice for obtaining high quality results from either testing or computer modeling.

Frequency Domain Analysis of Signals

The first experiment (number 3 in the course sequence) is an exercise to expose students to frequency analyzers, both constant bandwidth (e.g. FFT) and constant percentage bandwidth (e.g. octave band) analyzers, and the use of computer programs such as Mathcad to obtain the frequency domain representation of a signal. Using simple periodic signals—sine, square, sawtooth and triangle waves—students discover that the amplitudes of the frequency domain components in the test and computer simulation appear to disagree.

The sine wave test consists of displaying a sinusoid from a function generator on an FFT Spectrum Analyzer. Results are verified by observing the same signal on an oscilloscope and comparing the frequencies and amplitudes reported by the function generator, analyzer and scope. In general this simple verification is not possible with complicated signals.

The computer-based FFT analysis simulates a situation where data are stored on disk or tape and are later fed to a computer program for Fourier analysis. This is a common test scenario, and the Mathcad† exercise shown in Fig. 1 demonstrates four complications which the dedicated FFT analyzer used in the experiment handled automatically: (1) sampling to avoid aliasing, (2) minimizing leakage, (3) associating entries in the signal and Fourier arrays with their respective times and frequencies, and (4) finding the scale factor to relate the FFT components with actual sine and cosine amplitudes.

The first issue introduces students to the sampling theorem—the sampling rate must be more than twice the highest frequency in the signal—and its consequence—any frequencies in the signal higher than half the sampling rate (called the Nyquist frequency) must be removed with an analog low-pass filter before taking the samples. Otherwise, frequencies above the Nyquist frequency re-appear as false low frequencies, or “aliases.”

The second issue is “leakage,” which occurs when the signal frequencies don’t coincide with the FFT frequencies. The FFT frequencies are multiples of the interval delta-f, the FFT resolution as shown in Fig. 1. Even if the signal is a sine wave, if its frequency is not a multiple of f it will “leak” to nearby FFT frequencies and appear to consist of those other frequency components. Leakage can only be avoided with periodic signals, and when the period is known a priori, by sampling an integer number of periods so that a period of the signal and a delta-f interval of the FFT coincide. This arises in the case of rotating equipment where a tachometer identifies the period, or in testing the response of a system using known excitation frequencies. With aperiodic signals, which have continuous frequency

† Mathcad’s FFT algorithm, taking advantage of the fact that the input array is real, produces an output array with N/2 + 1 entries. A general FFT takes N complex inputs and produces N complex outputs, but if the input is real the first and last (N/2 + 1) output entries are complex conjugates of each other.
Start with an analog sine wave with a frequency of 60 Hz:

\[ f = 60 \quad T = \frac{1}{60} \quad \text{The frequency and period of the signal} \]

\[ s(t) = 9 \cdot \cos(2 \cdot \pi \cdot f \cdot t) + 12 \cdot \sin(2 \cdot \pi \cdot f \cdot t) \]

The signal looks like this for \( t = 0, \frac{T}{100}, \ldots, 3 \cdot T \)

Create an array, "sa," with \( N = 64 \) samples taken over three periods of the signal.

\[ N = 64 \quad i = 0, \ldots, N - 1 \quad \text{"i" is the time index which ranges from 0 to 63} \]

\[ \Delta t = \frac{3 \cdot T}{N} \quad \text{"\( \Delta t \)" is the sampling period. We'll take 64 samples over 3 periods. To avoid aliasing we need more than two samples per period. We avoid leakage by taking samples over an integer number of periods.} \]

\[ sa_i = s(i \cdot \Delta t) \quad \text{The array "sa" consists of samples of the analog signal "s" taken at uniform \( \Delta t \) intervals.} \]

Use Mathcad's built-in fast Fourier transform algorithm, fft, to create the complex frequency array, S.

\[ S := \text{fft}(sa) \quad \text{"S" is the fast Fourier transform of the sa array} \]

\[ M := \text{last}(S) \quad M = 32 \]

\[ j = 0, \ldots, M \quad \text{"j" is the "frequency" array index ranging from 0 to 32, so S is a complex array with 33 components \( (N/2 + 1) \).} \]

The entries of S represent frequency components at uniform intervals of the frequency resolution, \( \Delta f \).

\[ \Delta f = \frac{1}{3 \cdot T} \quad \text{The frequency resolution is the inverse of the total sampling time} \]

Look at the real and imaginary components of the Fourier transform. To match the amplitudes of our original analog signal we need to scale S. Different FFT algorithms have different scale factors. For Mathcad the factor is

\[ c = \frac{2}{\sqrt{N}} \]

Note that the cosine appears as the real component and the sine appears at the imaginary component.

**Figure 1. Introduction to Fourier (FFT) Analysis using Mathcad**
spectra, leakage is unavoidable. Later in this exercise students are introduced to Hanning and Flat-Top "windows" which are used to minimize leakage in continuous signals. Leakage and windowing for transient signals are discussed in the second laboratory exercise, presented below.

The third issue awakens students to fundamental differences between a continuous analog signal and its digital representation. Once the analog signal has been sampled it becomes an array of numbers, and the FFT just manipulates one array of numbers into another array of numbers. The burden is on the user to know how to associate physical meaning to those arrays. It is clear to students that the "time" array, \( S_a \) in Fig. 1, represents samples of the continuous signal \( s(t) \) taken at uniform time intervals \( \Delta t \). But it comes as a surprise that the "frequency" array \( S_j \), represents cosines (real components) and sines (imaginary components) at uniform frequency intervals \( \Delta f \). Interpreting the frequency array requires knowing \( \Delta f \), which is determined by the total sampling period. By themselves, without knowing \( \Delta t \) or \( \Delta f \), the arrays are relatively meaningless.

Issue four can be a cause of great frustration in trying to interpret FFT results. Typically the amplitudes of the real and imaginary components of the FFT do not equal the amplitudes of the original cosine and sine components of the signal, even when there is no leakage. This is an artifact of the FFT's numerical implementation. As Fig. 1 shows, however, there is a scale factor which recovers the original sine and cosine amplitudes. This scale factor varies depending on the FFT algorithm and frequently must be found by trial and error using a known input signal as in Fig. 1. To use the Inverse-FFT (IFFT) it is important to remove the scale factor, since the IFFT will only reconstruct the original time array from the frequency array produced by its FFT counterpart. Students are also cautioned to exercise care and contrive test cases when combining analytically derived frequency response functions with FFTs in doing linear system convolutions.

After clarifying these issues, students still find a discrepancy between Mathcad and the physical experiment using the dedicated analyzer. Following laboratory instructions, students discover that the analyzer is reporting root-mean-square (rms) amplitudes while in Mathcad they obtained peak amplitudes. Applying another scale factor of 0.707 in Mathcad (to convert from peak to rms) produces the desired matching results.

The effect of these complications is to impress on the students the many ways in which errors of interpretation may arise in the use of computer tools, and how important it is to experiment-computationally and physically-to make sure the process is fully understood.

This laboratory exercise goes on to introduce windowing, octave band analyzers, the effects of linear and logarithmic frequency axes, and the representation of white and pink noise on FFT and octave band analyzers. In each case, students are introduced to techniques for testing the equipment and computer algorithms so that they understand what the tools are doing.

### Structural Modal Analysis

The second experiment (8 in the course sequence) focuses on structural modal analysis. In this experiment students develop a finite element model of a simple beam and predict the natural frequencies and mode shapes. The results of the finite element analysis are used to select locations for holding the plate and hitting it with an instrumented hammer in order to excite specific modes of vibration. The test and simulation results typically disagree, and students are guided to suspect mass loading of the structure by the accelerometer as the reason for the disagreement. Then computer software is used to factor in the mass loading effect, and the results typically show good agreement.
A cantilever beam (simple thin beam clamped at one end and free at the other end) was chosen for analysis for several reasons. First, the natural frequencies and mode shapes may be determined analytically from the differential equations of motion. Students are able to calculate the natural frequencies and mode shapes for bending and torsional modes of vibration. These analytical results provide a nice comparison for the computational and experimental data which would not be available for a more complicated, albeit more realistic, structure.

Secondly, the purpose of this laboratory exercise is primarily to introduce the students to the techniques involved in experimental and finite element modal analysis, not to confuse them with an analysis of a complicated structure. Thirdly, students are able to see the limitations of analytical results which usually involve important simplifying assumptions. For example, the differential equations used to derive the eigenfrequencies and eigenmodes for a cantilever beam make the Euler-Bernoulli thin beam approximation which ignores shear stress and rotational inertia. The resulting frequencies are quite accurate for the lowest, few modes, but higher order modes in a real beam diverge from analytical predictions as shown in Fig. 2. Having the students compare analytical calculations with computational and experimental results brings out the limitations of a theoretical approach.

Once the students have theoretically predicted the natural frequencies and mode shapes of the beam they create a finite element model. Of the many finite element programs available we have chosen to use Can for Windows primarily because it runs on a desktop PC, and is therefore easily accessible to students, and it has a rather quick learning curve. For a simple structure like a cantilever beam the students are able to create the model, perform the analysis, produce a list of the first 20 natural frequencies, and plot and animate the corresponding mode shapes in about
20 minutes on a 33 MHz 486 with 8 MB of RAM. Being able to observe animated mode shapes of the structure helps the students visualize the vibrational behavior. In addition, knowing a priori where points of maximum and minimum vibration are located on the structure allows the students to determine optimum locations for excitation and measurement of the beam response. Obtaining useful information from an experimental modal analysis requires placement of the response transducers at points of maximum vibration. If an accelerometer is placed at a structural node then no experimental information is obtained for that particular mode of vibration.

The experimental modal analysis is carried out using the fixed response technique in which a single accelerometer is placed at a desirable response location, and the structure is excited with an impact hammer at various locations over the structure surface. The students choose the location of the accelerometer after analyzing the finite element results. To minimize leakage for the transient excitation and response, students use "Force" and "Exponential" windows while obtaining the modal data, which builds on the leakage and windowing lessons learned in laboratory 3. The frequency response spectra are analyzed using the software package Star Modal.

Once the students have curve-fit the frequency response data, they are able to observe experimentally generated natural frequencies and mode shape animations for the cantilever beam. Mode shapes obtained by experimental modal analysis usually agree quite well with those predicted by finite element analysis. A finite element model, however, usually produces modes which are not observed experimentally. For example, a finite element model of a cantilever beam predicts in-plane bending modes which cannot be observed experimentally by a uniaxial accelerometer positioned to pick up transverse bending and torsional modes. For comparison, Fig. 3 shows the first few mode shapes found experimentally and by finite element analysis. Note that the third finite element mode, in-plane/lateral bending, was not detected experimentally. In addition, the experimentally obtained frequencies are usually somewhat lower than predicted by the finite element model. The students are guided to suspect that this frequency shift is due to the mass loading of the structure by the accelerometer, and they are shown how to remove the mass loading effect using the Star Modal software. Once the mass loading effect has been eliminated the experimental and computational results agree very closely.

The three-fold approach to a structural vibration problem with analytical, computational, and experimental results provides a very nice comparison of different methods of studying a simple structure. Students are introduced to the basic techniques involved in all three methods, and to the limitations and strengths of each method. From this laboratory, students acquire the basic tools they need to be able to effectively analyze a more complicated structure.

Summary
Two laboratory exercises are presented that demonstrate the importance of coupling computer simulations with experiments to validate results obtained by each method. The first exercise deals with the frequency domain analysis of signals using fast Fourier transforms (FFTs). Students are introduced to the complications of aliasing and leakage, and issues related to interpreting both the abscissas (time and frequency) and ordinates (amplitudes) of the digital time and frequency arrays in computer-based FFTs. These complications impress on the students the many ways in which errors of interpretation may arise in the use of computer tools, and how important it is to experiment—computationally and physically—to make sure they understand what their tools are doing.
Figure 3: Comparison of experimental and finite element modes for a cantilever beam

(a) First four experimentally determined modes

(b) First five modes from finite element analysis

Figure 3: Comparison of experimental and finite element modes for a cantilever beam
The second exercise introduces students to structural modal analysis from three vantage points: theoretical/analytical, computational, and experimental. Obtaining results by all three methods introduces students to the basic techniques of the methods, provides a good comparison of different methods for studying structures, and demonstrates the limitations and strengths of each method. Students also learn how to use finite element results to design the experiment, and what to look for when there are discrepancies between theoretical, computational, and experimental results. From this laboratory, students acquire the basic skills they need to be able to effectively analyze more complicated structures.

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References
2. Full descriptions of laboratory exercises may be downloaded from the WWW URL address: http://www.gmi.edu/~drussell/anvlabs.html.
5. StarModal, part of the StarStruct suite, from GenRad Structural Test Products, 2855 Bowers Ave., Santa Clara, CA 95051-0917. Tel. 408-970-1600.

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