Different Approach in Design And Analysis of an Instrumentation Amplifier

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Abstract

This paper presents a different instrumentation amplifier design to minimize the magnitude and phase errors of conventional instrumentation amplifier using single-pole model of the operational amplifier. This analytical approach ensures maximum flat magnitude and phase responses over an extended frequency range. Simulation results are given to support the proposed technique.

Design And Analysis Using Single-Pole Model

There are numerous applications in which a differential signal needs to be amplified. These include low-level bridge measurements, balanced microphone lines, communication equipment, thermocouple amplifiers, data acquisition, and more. The immediate answer to these applications is the differential operational amplifier configuration. There are limitations to differential amplifiers, unfortunately. It is practically impossible to achieve matched high-impedance inputs while maintaining high gain and satisfactory offset and noise performance. For that matter, the input impedances are not isolated; indeed, the impedance of one input may very well be a function of the signal present on the other input. Thus, this is an unacceptable situation when a precision amplifier is needed, particularly if the source impedance is not very low.

An instrumentation amplifier (I-Amp) overcomes these problems. Instrumentation amplifiers offer very high impedance, isolated inputs along with high gain, and excellent common-mode rejection performance. Instrumentation amplifiers can be fashioned from separate Op-Amps. They are also available on a single IC for highest performance. The common structure of an I-Amp is given in figure 1. The first-order model of operational amplifier open-loop gain is given by

\[ A(j\omega) = \frac{A_0 \omega_c}{j\omega + \omega_c} \]  \hspace{1cm} (1)

where \( A_0 \) is the open-loop dc gain, \( \omega_c \) is the corner frequency, and \( \omega_u \) is the unity gain bandwidth of the operational amplifier. An approximated model at higher frequency can be written as

\[ A(j\omega) = \omega_u / j\omega \]  \hspace{1cm} (2)
Using Kirchhoff's Current Law to write a set of none equations in circuit of figure 1 gives the following

\[ (V_3 - V_{01})/R_2 + (V_3 - V_0)/R_1 = 0, \ (V_1 - V_{01})/R_3 + (V_1 - V_2)/R_c = 0 \]

\[ (V_2 - V_{02})/R_3 + (V_2 - V_1)/R_c = 0, \ (V_3 - V_{02})/R_2 + (V_3 - 0)/R_1 = 0 \]

Using (3) and the approximated model (2), the transfer function \( H(j\omega) = V_0/(V_2 - V_1) \) for the matched resistors \( R_1/R_2 = R_1'/R_2' = 1 \), yields

\[
H(j\omega) = \frac{0.5k\omega^2_u}{k(j\omega)^2 + \omega_u (1 + 0.5k)j\omega + 0.5\omega_u^2}
\]

where \( k = 1 + 2R_3/R_c \).

If \( H_0 \) and \( \angle H_0 \) are the ideal magnitude and phase angle of the transfer function \( H(j\omega) \), respectively, at zero frequency in (4), then it is possible to present the magnitude and phase angle of \( H(j\omega) \) as

\[
|H(j\omega)| = H_0 \ [1 - E_H(j\omega)], \ \angle H(j\omega) = \angle H_0 - E_\phi(j\omega)
\]

where \( E_H(j\omega) \) and \( E_\phi(j\omega) \) are the magnitude and phase errors. These error functions defined in (5) maybe approximately put in the following form

\[
E_H(j\omega) = (0.5k^2 + 2)(\omega/\omega_u)^2, \ E_\phi(j\omega) = -(k + 2)\omega/\omega_u
\]

It should be clear from equation (6) that the magnitude error is of a second-order, whereas the phase error is of a first-order magnitude.

The proposed I-Amp structure (figure 2) consists of five Op-Amps and eleven passive resistors that minimizes the magnitude and phase errors. Necessary conditions are derived to ensure the maximally flat magnitude and phase responses over an extended frequency range. A set of node equations for the matched resistors ratio \( R_1/R_2 = R_1'/R_2' = 1 \) and \( R_3/R_4 = R_3'/R_4' = M \), gives

\[
(V_3 - V_{01})/R_2 + (V_3 - V_0)/R_1 = 0, \ (V_4 - V_{01})/R_4 + (V_4 - V_{03})/R_5 = 0, \ (V_1 - V_{03})/R_3 + (V_1 - V_2)/R_c = 0
\]

\[
(V_2 - V_{02})/R_3 + (V_2 - V_{04})/R_4 = 0, \ (V_5 - V_{04})/R_5 + (V_5 - V_{02})/R_7 = 0, \ (V_3 - V_{02})/R_2 + (V_3 - 0)/R_1 = 0
\]

Simplification of equations (7) and employing the single-pole model (2) will result in the following transfer function \( H(j\omega) = V_0/(V_2 - V_1) \) for the proposed I-Amp

\[
H(j\omega) = \frac{0.5k\omega^3 + 0.5k\omega_u^2(1 + M)j\omega}{k(1 + M)(j\omega)^3 + 5k\omega_u (1 + M)(j\omega)^2 + \omega_u^2(1 + 0.5k)j\omega + 0.5\omega_u^3}
\]

where \( k = 1 + 2R_3/R_c \), and \( M = R_3/R_4 \). It should be noted that the transfer function in equation (8) satisfies the Routh Hurwitz stability criterion for all positive values of \( k \) and \( M \).
The optimization of equation (8) gives the condition for the maximally flat magnitude and phase responses, respectively, when

$$M = M_m = \sqrt{2k^2+4} - k, \quad M = M_o = k+1$$

(9)

Under condition (9) the magnitude and phase errors may be approximately put in form

$$E_m(j\omega) = [1.5k^2-(k^2+4)\sqrt{2k^2+4} + 8k](\omega/\omega_u)^4, \quad E_q(j\omega) = -k(k+2)^2(\omega/\omega_u)^3$$

(10)

The above expressions clearly indicate that the magnitude and phase errors are fourth-order and third-order as opposed to the second-order and first-order terms found in conventional instrumentation amplifiers.

Simulation Results

The transfer function for conventional instrumentation amplifier using the Op-Amp model parameters ($A_o = 10^5$, $\omega_c = 29 \text{ rad/sec}$, and $R_3/R_c = 5$) and equation (4) is

$$H(j\omega) = \frac{4205 \times 10^9}{(j\omega)^2 + 1713636.4(j\omega) + 3.823 \times 10^{11}}$$

(11)

The value of $M$ for proposed I-Amp is obtained from equation (9) to yield maximally flat magnitude and phase responses; $M_m = 3.7$, and $M_o = 12$. The corresponding transfer functions of proposed I-Amp for above values are

$$H(j\omega) = \frac{4205 \times 10^9 j\omega + 2.5946 \times 10^{18}}{(j\omega)^3 + 145 \times 10^4(j\omega)^2 + 1.05735 \times 10^{12} j\omega + 2.3587 \times 10^{17}}$$

(12)

$$H(j\omega) = \frac{4205 \times 10^9 j\omega + 9.38 \times 10^{17}}{(j\omega)^3 + 145 \times 10^4(j\omega)^2 + 3.823 \times 10^{11} j\omega + 8.5276 \times 10^{16}}$$

(13)

The frequency response of the proposed I-Amp for its magnitude and phase responses for $M_m = 3.7$ are illustrated in figure 3 and figure 4, respectively. The simulated magnitude and phase responses of conventional I-Amp are also plotted in figures 3 and 4 to facilitate performance comparison. Figures 5 and 6, on the other hand, illustrate the way in which magnitude and phase responses of the proposed I-Amp vary with frequency for $M_o = 12$ which is computed to yield maximally flat phase response.

Conclusion

A simple modification in the conventional instrumentation amplifier is proposed to minimize magnitude and phase errors at the expense of two additional Op-Amps and four passive resistors. Superior performance is accomplished in comparison to the conventional instrumentation amplifier. The necessary analytical conditions are obtained to ensure flat magnitude and phase responses over an extended frequency range. The resistor $R_c$
in the proposed I-Amp controls the gain whereas the resistors $R_3$ and $R'_5$ in relation to $R'_4$ and $R'_4$ control the flatness of the responses for magnitude and phase. The circuit is stable for all positive values of $k$ and $M$.

References


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Figure 3. Simulated magnitude compensated for maximally flat magnitude response

Figure 5. Simulated magnitude compensated for maximally flat phase response

Figure 4. Simulated phase angle compensated for maximally flat magnitude response

Figure 6. Simulated phase angle compensated for maximally flat phase response