A Hybrid Conceptual/Symbolic/Numerical Course of Mechanical Engineering Analysis

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Introduction

As an important part oft he recently re-vitalized Mechanical Engineering Curriculum at the University of Nebraska-Lincoln, the traditional computational course (using FOR-TRAN exclusively) for mechanical engineering juniors is replaced by a new one of ME-CHANICAL ENGINEERING ANALYSIS. This new course is updated (and upgraded) from the old in two ways: (1) Computerized symbolic manipulat ion (using MAPLE or the like) is incorporated, complementing the traditional numerical analysis via FORTRAN. (2) As an even more significant move, conceptual model-building and analysis (NOT at all computer-aided) are re-introduced.

While the inclusion of MAPLE (or other similar packages) is representative of the current trend, and to be expected; the re-emphasis of "PRE-COMPUTER" conceptual analysis (accounting for a major portion of the course content) may need further delibera-tion. This article, then, aims mainly at airing our views on this practice; at the same time we also argue for an INTEGRATED and fresh employment of conceptual, symbolic, and numerical analysis in the course. It is thus hoped that fellow instructors might be induced to share their opinions on these points. It is also hoped that, by reporting our experiences in the design, organization, and delivery of the course cent ent, we might encourage the inclusion of similarly designed courses in other curricula across the country.

In the more detailed description that follows, we do not wish to unduly draw attention to the specifics (such as exactly what topics are included, in what sequence, in what form, etc.). On the contrary, we wish to convey here the general spirit and philosophy behind the design. A detailed course outline or synopsis, although available from the author, will not be described here.

Mot iv-at ion

The original decision to devote a large portion of the class time to conceptual models and classical analysis, avoiding all computer aids in this phase, followed the belief that we are now acutely in need of a large dose of antidote in modern engineering education against over-application of artificial (or virtual) intelligence. While the present author himself has been constantly developing software packages with elements of artificial intel-ligence, he is keenly aware (as many others are) that KNOWLEDGE (in clear contrast against information) can not be transmitted in a totally virtual and passive fashion! Our student body lately is observed to manifest the following symptoms: They no longer cul-tivate the habit of thinking and reasoning; they lack the ability to formulate engineering problems in mathematical terms; they have a very limited vocabulary in both mathematical and engineering languages; they have retained surprisingly little conceptualizing and manipulative skills after sitting through so many courses in mathematics, physics, engineering mechanics, and other engineering sciences; and, finally, they lack the proficiency in interpreting, in engineering language, data generated (either in a laboratory or on a computer). To counteract these negative influences of the modern educational technology, we have chosen for our students a "survival kit" into which we put an absolutely minimum (barebone) selection of mathematical and engineering items. These items we develop, discuss, and apply in depth for about half of the course; and we are insistent in requiring the students to master this "kit" WITHOUT any contact with a computer, and to carry it into their career as a part of their second nat ure.

On the other hand, the modern trend of adding a software for symbolic manipulation in the students' learning environment may easily result in a separate layer being added t o the numerical platform, without any designed integration. The subject course addresses this possible lack of integration by consciously displaying its conceptual, symbolic, and numerical parts as three facets of the same object.

Consolidation

In assembling items for the conceptual part of the course, we do NOT review *per se* subjects from previous courses. (To be exact, at the Universit y of Nebraska, the prerequisites for the course are Calculus/Differential Equations, Engineering Mechanics, Thermodynamics, and FORTRAN.) Instead, we always (after being properly motivated by dire needs in mechanical engineering) re-introduce basic concepts from a generalized point of view, with a fresh perspective, or from an unusual angle. For example, variables always start with being vectors. This way, the discussion need not be repeated for scalars, components, or arrays; slight and natural modifications being all that are needed. In the same spirit, derivatives are first defined for vectorial functions of vectorial variables. These then lead to directional, partial, and ordinary derivatives as mere special cases. With a little extension, this generalized introduction also lend itself to differentiation in the complex domain (of analytical functions), total differentials, and material derivatives for moving continua. Also, with the complex notation, sine and cosine functions are but parts of the exponential function. Thus, all operations with trigonometric functions can be absorbed into those with the exponential function (of a complex variable). As another example, integration is introduced in such an all-embracing manner that volume-, surface-, plane-, curvilinear-, and (the definite) rectilinear-integrals all come out of the same definition. We have found this telescoping technique refreshing and exciting to the students; as well as time-saving for the lecturer. The students also found the approach efficient in putting many things in a few compact "boxes" , systematic in bringing together seemingly disjointed topics, economic in reducing the number of necessary concepts/procedures, and (therefore) easier to keep things clear in mind.

The course also employs a few main logical THREADS to go through the various topics. This is especially helpful in strengthening retention for a student after his/her having reasoned through the steps along such a thread. Power series is one of these main threads. Our exposition actually starts with power terms, which are eventually linked into power series. Then, after the introduction of generalized derivatives, we formally construct the exponential function (in the complex domain) via a power series, by defining it as the function proportional to its own derivative. This definition opens up the wide use of the exponential function (together with its real and imaginary parts) in the solution of differential equations; as well as in the establishment of Fourier series, Fourier transforms, and Laplace transform. The development, and the direct use of Taylor's series in numerical analysis, finally terminate this thread. We can limit ourselves, in this manner, to the minimal number of concepts; while developing each concept to the maximum extent,

On the engineering side, we also manage to avoid repetition by introducing old concepts in a generalized or fresh way. For example, kinematics and dynamics of particles in a plane are coached in the language of complex analysis wherever feasible. Actually, the course introduces four-bar linkages, efficiently, in exactly this form. Also, vectors are always employed in their coordinate-invariant form (as they ought to be) until decomposition can no longer be postponed; in contrast to the common practice of treating them always as collections of components (and thereby defeating the very purpose of introducing a vector). Main t breads that permeate the entire realm of mechanical engineering are duly maintained. One such thread emphasized throughout is the formulation of models based on the laws of balance (of mass, of force-momentum, of torque-angular momentum, and of energy). The procedure of non-dimensionalization is also constantly applied. The concept ual model-building is done on both systems and control-volumes. In the introductory part of the course, the balance laws are stated in two versions, one for systems and one for control-volumes. The two versions are shown to be equivalent, in the more advanced part, by way of the Reynolds transport theorem. The system approach is always used for particles and solid bodies; the control-volume approach, for moving cent inua. The distinction between the particle description and the field description is also maintained throughout.

In addition, the course strives to CONSOLIDATE knowledge from both fronts, engineering and mathematics, in a micro-mixture. It does not delineate mathematical and engineering subjects in dichotomy. For instance, by referring to the simple quenching (or other engineering tasks) as a paradigm, the course touches upon rate of cooling, Newton's law of cooling, differential equations, initial/boundary condit ions, non-dirnensionalization, etc.; all in one continuous sweep. This scheme keeps mathematical topics very close to their applications; there are no separate chapters on mathematical items.

Resurrection

In treating all three hybrid aspects of the course, topics are selected and presented in such a way that the conceptual, the symbolic, and the numeric are INTEGRATED into one entity whenever possible and appropriate. As an illustration, the Taylor's series is an inherently powerful method of solving differential equations. It is, however, traditionally bypassed (and declared dead) as a numerical scheme; since it calls for higher and higher derivatives in its implement at ion. Such derivatives are cert airily cumbersome to carry out without computer aid. But, wit h MAPLE, this is no longer an obstacle. In this course, we choose to revive the use of the classical method of Taylor's series as a numerical procedure. In this respect, we essentially discuss the general procedure first; and then, for each specific problem, wc employ MAPLE to yield derivatives symbolically to the order desired. The result from MAPLE can be automatically translated (converted) into its FORTRAN form, which is then built into a complete program for the numerical solution of that problem. This unique procedure is again fully integrated to take advantage of all three kinds of analysis. As to be expected, if higher-order derivatives can be included (thanks to MAPLE), the Taylor algorithm can be very successful with rather large (and not necessarily equal) marching steps.

To be more concrete, let us quote one (simple) numerical example:

For an initial-condition problem

$$\begin{aligned} \frac{d^2y}{dt^2} &= f\langle t, y, \frac{dy}{dt}\rangle \\ @\, t &= 0 : y = \alpha,\ \frac{dy}{dt} = \beta \end{aligned}$$

we divide the interval of interest [0, Υ] into segments by markings $t_\iota = \iota.\varepsilon$, where $\iota =$ $0,1,2, \ldots \nu$; with $\nu.\varepsilon = \Upsilon$. (We have chosen equal segments for ease of writing.) We then have, consecutively:

$$\begin{aligned} y_{\iota+1} &\approx y_\iota + \varepsilon \cdot \frac{dy}{dt}\rfloor_\iota + \frac{\varepsilon^2}{2!} \cdot \frac{d^2y}{dt^2}\rfloor_\iota + \ldots \\ \frac{dy}{dt}\rfloor_{\iota+1} &\approx \frac{dy}{dt}\rfloor_\iota + \varepsilon \cdot \frac{d^2y}{dt^2}\rfloor_\iota + \frac{\varepsilon^2}{2!} \cdot \frac{d^3y}{dt^3}\rfloor_\iota + \ldots \\ &\ldots\ldots\ldots\ldots \end{aligned}$$

Now, if $f\,(t, y, \frac{dy}{dt}\rangle = 5t^2 + 3 - y - t. \frac{dy}{dt}$, *we can* display (using MAPLE to avoid human error):

$$\begin{aligned} \frac{d^2y}{dt^2} &= 5t^2 + 3 - y - t \cdot \frac{dy}{dt} \\ \frac{d^3y}{dt^3} &= 10t - 2 \cdot \frac{dy}{dt} - t \cdot \frac{dy}{dt} \\ \frac{d^4y}{dt^4} &= 10 - 3 \cdot \frac{d^2y}{dt^2} - t \cdot \frac{d^3y}{dt^3} \\ \frac{d^5y}{dt^5} &= -4 \cdot \frac{d^3y}{dt^3} - t \cdot \frac{d^4y}{dt^4} \\ &\ldots\ldots\ldots \end{aligned}$$

From here, the simple and straight forward implement at ion in FORTRAN follows. When the procedure is truncated at (as early as) the fifth term of the Taylor's series, the numerical solution compares favorably (identical to five decimal places) with those generated by other methods, with a marching step $\varepsilon = 0.1$. (It may be of interest to note that, in the previous revival of the method around 1980, some researchers deemed it necessary to supply *ad hoc* programs for the machine-generation of the needed derivatives. Their programs may very well have been the fore-runners or mot ivators of the current crop of symbolic manipulators!)

Integration

As another illustration, let us also describe the introduction of the Runge-Kutta method in this course. In a traditional course on numerical analysis, it is customary to avoid presenting the detailed justification of the classical Runge-Kutta procedure (especially the well-used fourth-order formulation) as too messy. The procedure itself is usually presented after a brief introduction. Although the concept behind the procedure is rather simple, the unfortunate omission oft he (messy) justification of the claimed small order of error does not at all invite confidence; and actually makes the procedure work somewhat like black magic! In the subject course, we present the entire topic this way: After prescribing the Runge-Kutta scheme with undetermined coefficients, it is explained that the aim here is to compare the Runge-Kutta formula (still with undetermined coefficients) with the Taylor series of the (unknown) solution, and to demand that the two agree to the fourth order. A set of nonlinear algebraic equations for the unknown Runge-Kutta coefficients will then result. BUT, the actual comparison and construction are done on a computer symbolically, employing MAPLE! The result of this session with MAPLE (the set of equations linking the coefficients to be determined) can be displayed explicitly and clearly on about two screens (or printed on one sheet of paper); all the messy differentiations and algebra are handled internally (but symbolically) by MAPLE. Our discussion now continues. It is first pointed out that, since there are more unknowns than equations, various arbitrary choices of values of certain unknown coefficients are possible. One convenient choice yields the classical Runge-Kutta rule of the fourth order; one choice yields the sometimes-used three-eighth rule; still another yields the often-used Heun method; etc. (Incidentally, this session also gives us opportunity to dig deeper into MAPLE and teach some more advanced and subtle commands.) At this point, the course enters its FORTRAN part,; and programs realizing these rules or methods are developed and discussed; making the topic of the Runge-Kutt a met hod a truly integrated conceptual, symbolic, and numerical exercise,

To be more specific, we would like to quote some of the MAPLE commands used in developing a third-order Runge-Kutta method:

$m := 3;$
$taylor(y(x+h), h=0, m+1);$
$taylorform := normal((convert('', polynom) - y(x))/h);$
$k1 := taylor(f(x, y(x)), h=0, m);$
$k2 := taylor(f(x+h*c2, y(x)+h*(a21*k1)), h=0, m);$
$k3 := taylor(f(x+h*c3, y(x)+h*(a31*k1+a32*k2)), h=0, m);$
$rkform := convert(series(b1*k1+b2*k2+b3*k3, h, m), polynom);$
$devi := expand(taylorform - rkform);$
$eqns := \{coeffs(devi, [h, F, Fx, Fy, Fxx, Fyy])\};$
$vars := indets(eqns);$
$sols := solve(eqns, vars);$

From the solutions displayed, we can easily arrange various options. (We display as an example here the case for the third order, fully aware that it is never employed in practice. The popular fourth-order scheme can be also justified; but the details and the story-line are no longer as obvious as the above display.)

Incidentally, as special cases, numerical schemes for the solution of ordinary differential equations double also as those for quadrature. And, Taylor's series can provide the foundation for the numerical evaluation of transcendental functions, differentiation, interpolation, and curve-fitting. These are all taken advantage of in this course, revealing the true power of Taylor's series a multi-purpose tool.

Conclusion

In conclusion, we have designed, organized, and taught (for two cycles) a course of mechanical engineering analysis, which is new, all-embracing, and logically structured. We have emphasized the development and retention of a cluster of must-have terminology, concepts, and analytical skills; while letting students take full advantage of modern technology in the form of computer-aided symbolic and numerical manipulations. It is to be hoped that we have succeeded in pointing out the need for integration and balance.

Further refinements are constantly being sought. A possible addition being contemplated now is the introduction of the method of weighted residue which may serve as a unifying vehicle for the preliminary discussion of finite-element and finite-difference methods. On the graduate level, our new curriculum also includes a sequel: ADVANCED ANALYSIS OF MECHANICAL ENGINEERING SYSTEMS, developed by another faculty member.

Appendix: Paragraphs from the First Lecture

TO THE MEMORY OF KNOW – WHY,
A VICTIM OF KNOW – HOW!

AN OVERVIEW

In this Part which we treat as our SURVIVAL KIT, we will start with the mathematical translation of basic concepts in mechanical engineering, up to and including the formulation of ent ire problems or models. Simpler manipulation techniques selected from various branches of mathematics will then be applied to draw useful conclusions regarding the problems/models. Finally, these mathematical conclusions will be translated back into engineering jargons again.

In this sense, this survival kit will be optimally constrained to be small and low-levelled; it will contain only the bare minimum, the rock bottom, the absolutely necessary. Any potent ial engineer in the mechanical field will be expected to master this kit, as a matter of course, without any aid from a computer.

The survival kit will cover basic definitions of mechanical engineering entities, a systemat ic and general review of some fundamental topics in mathematics (presented from a fresh angle of perspective, and in a fully generalized and compact manner, whenever feasible), fundamental laws of physics as applied to mechanical engineering, both general and *ad hoc* st rat egies for the formulation of problems or models, as well as associated examples and exercises. As mentioned before, a student is expected to keep all these tools of survival firmly in mind, clear and straight, WITHOUT RESORT TO ANY COMPUTER AID. Instead of trying to commit everything to memory, the student is urged to think through a concept or statement, to ponder on the unwritten implications, to probe the hidden lines, to ask oneself questions (and to answer them on one's own), to provide examples (and, even more important, COUNTER-exarnples). It is then the duty of the teacher to prevent wrong concepts or misleading statements from ever forming in anybody's mind—a formidable task indeed for any teacher! This course will try to offer WARNINGS, CAUTIONS, NOTES and DEBUNKERS preemptively to shield the student against forming misconceptions, jumping to wrong conclusions, or misapplying formulas.

Another unique style adopted in presenting this course needs to be emphasized also: The mathematical content is "micro" -mixed with its engineering subject-matter, and the various mathematical topics are discussed ONLY after proper engineering motivation. The two worlds are presented as an intermingled, combined, and integrated (as much as possible) universe! In this spirit, the reader probably will notice that integration appears rather late, while solving (simpler) differential equations comes in early (see the following paradigm), in the discussion.

PARADIGM OF QUENCHING

Consider as a paradigm of mechanical engineering analysis the following task:

A red-hot iron ball is submerged in a tank of cooling oil maintained at a constant ternperat ure TO. We would like to be able to predict when the ball will reach To, within a margin of 1 %. Let us say the ball is small enough to show a uniform temperature 'T at every stage of the quenching process. The first thing we realize from this task description is obviously the object of the study: the iron ball; or more specifically its temperature 'T. With the time t as the independent variable.

The evolution of T (as the dependent variable) would eventually show up as a function of t: T = T(t). Our task will be to find this functional form T(t), and to solve inversely for t when $T = 0.99T_0$. To this end, we are probably helped by some experimental observations which establish the empirical relation (known as Newton's law of cooling) that the (time) rate of change of *T* is actually proportional to *(T – TO),* or

$$(\text{Rate of change of } T) \propto (T - 'TO)$$

which we interpret mathematically as

$$(\text{Rate of change of } T) = C(T - To)$$

where C is the corresponding proportionality const ant. The proport ionalit y const ant can be experimentally determined. It is in fact a negative number, indicating that the ball DECREASES its temperature whenever it is hotter than the quenching oil.

To calculate the (instantaneous) rate of change, we obviously need the mathematical operation of limit or differentiation; thus,

$$(\text{Rate of' change of } T) = \lim_{\Delta t \to 0} \frac{\Delta T}{\Delta t} = \frac{dT}{dt}$$

We, then, have a mathematical relation (called a differential equation) governing the +engineering process of quenching:

$$\frac{dT}{dt} = C(T - T_0)$$

To construct the evolution of *T vs. t, we* may chose the following procedure: On the *T-t* diagram, starting with the point $T = T_i$ (the known initial ball temperature at the beginning of quenching) at $t = O$ (known as the initial condition), draw a short straight line with slope $C(T_i - T_0)$. Following this line for a very short interval *At, we* march to a new point (T_1, At). From there, we march for another Δt, along the new slope $C(T_1 — T_0)$; and arrive at the point $(T_2, 2\Delta t)$. By repeatedly marching like this, we will soon see a curve represent ing graphically the evolution *T = T(t)* on the diagram. (Incidentally, we have just solved the differential equation, subject to the given initial condition, by the marching met hod.) Finally, as an engineer, we can predict the instant when $T = (1.\ 99T_0$ by reading off the t-value against $0.99T_0$ from the constructed diagram.

In the above brief description, first of all, we notice the following general scheme:

Engineering Entity/Relation/Operation

⇓

TRANSLATED INTO ⇓

(FORMULATION) ⇓

⇓

Mathematical Entity/Relation/Operation

⇓

ANALYZED ⇓

(ANALYSIS) ⇓

⇓

I Mathematical Conclusion

⇓

TRANSLATED BACK ⇓

(INTERPRETATION) ⇓

⇓

Engineering Conclusion