The Use of Mathcad as a Lecture Aid for Compressible Flow

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Abstract

While Mathcad is primarily intended as a technical calculation and documentation aid, the use of this software for lectures in compressible flow was found to provide a number of advantages over traditional chalkboard approaches. For example, the students viewed much clearer and professional looking text, mathematics, figures and plots then would be possible using chalkboard based lecture and there was the additional bonus of the use of color. Handouts that duplicated the material presented were prepared and distributed thus giving students the opportunity to pay more attention to what was being said and less to copying notes from the blackboard. Lecture presentation was greatly facilitated by the Mathcad environment that made mathematical derivations simple through symbolic and arithmetical calculation, particularly those involving physical units. These same capabilities gave the instructor the ability to answer student "but what if...." questions during lectures by simply changing the calculation or derivation in real time and pursuing the line suggested by the student. As a bonus, it was easy to compile the lecture notes, examples, and quiz questions into an electronic book for the students to use in the future as the need to review compressible flow arises. While the time involved in preparing the initial set of lecture notes was much greater than the usual lecture preparation, the format that has been created will permit continuos future improvement with small additional expenditure of time. It may be possible in the future to further build on the electronic book as an aid or vehicle for distance learning.

Introduction

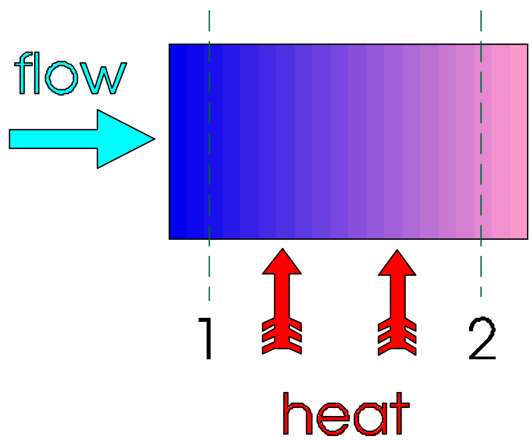
As is the case with most universities, Lamar University is experimenting with a number of alternate and multi-media mechanisms for the delivery or enhancement of education. All of these mechanisms involve substantial investment of capital resources at a time of constrained higher education funding. Therefore, the investments must be made in technology that has a direct educational pay off with very little room for failure. The ideal low risk path would involve an evolution of traditional instructional methodology into the emerging higher education environment of greater teaching obligations combined with students who may be time and/or location constrained.

Mathcad appeared to offer some assistance along this evolutionary path at least for quantitative engineering courses. A project of using Mathcad to prepare and distribute lecture notes for a senior/first level graduate course in compressible flow was initiated. Since the lecture notes existed in electronic form, the next logical step of using other software packages to display these notes to students was also initiated. A simple hardware system consisting of a notebook computer (486DX20 with a 100 meg hard drive and Windows 3.1), an LCD color "Boxlight", and an overhead projector all mounted on a caster equipped table was scrounged.

The initial lectures were presented using this hardware in conjunction with Powerpoint, a slide paradigm software package. Since the Powerpoint slides were prepared in Mathcad and pasted to the slides, digital storage quickly became a problem with a typical lecture requiring 10 megabytes. Therefore, it was decided to use Mathcad as the display software as well as the authoring software. This resulted in a loss of the slide paradigm, which was replaced by scrolling the Mathcad screen. An unexpected advantage was the retention of all the calculation tools previously available only during lecture preparation and not during the lecture.

The topics covered in our compressible flow course consist of simple area change, normal shocks, simple friction, simple heat addition, combined change flow (analytic and numerical solutions), linearized 2-D supersonic flow, Prandtl-Meyer Flow, and oblique shocks. The outline of each topic generally followed the same format: a sketch of the system to be analyzed, a list of the assumptions, development of the constitutive relations, plots of these relations, and examples to emphasize particular physical behavior of the flow type of interest. Mathcad provides no means of making sketches so CorelDRAW was used for this purpose and imported to Mathcad. Previously, traditional lecture was used to verbally list assumptions and it was left to students understand their importance and copy them into their notes. A sketch and list of assumptions for simple heat addition (simple T_0 change) is shown below.

Simple T_0 Change (Rayleigh Flow



ASSUMPTIONS: 1) steady flow
2) perfect gas ($P=\rho RT$ and c_p and c_v are constant)
3) no friction or other dissipative mechanisms
4) no mass injection or bleed
5) constant area
6) one-dimensional flow

Part of the development of property relations for the same case is shown below.

Development of Property Relations

Conservation of Mass

$$w_1 = w_2 \qquad \text{or} \qquad \rho_1 \cdot A_1 \cdot V_1 = \rho_2 \cdot A_2 \cdot V_2$$

$$\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} \tag{1}$$

Conservation of Momentum

$$\tau_1 = \tau_2$$

$$P_1 \cdot A_1 \cdot \left(1 + k \cdot M_1^{\,2}\right) = P_2 \cdot A_2 \cdot \left(1 + k \cdot M_2^{\,2}\right)$$

$$\frac{P_1}{P_2} = \frac{1 + k \cdot M_2^{\,2}}{\left(1 + k \cdot M_1^{\,2}\right)} \tag{2}$$

Conservation of Energy

$$q = w_1 \cdot c_p \cdot \left(T_{02} - T_{01}\right)$$

Rearranging equation (1) and then combining with equation (2) and the equation of sta

$$\frac{\rho_1}{\rho_2} \cdot \sqrt{\frac{T_1}{T_2}} = \frac{\dfrac{V_2}{\sqrt{T_2}}}{\left(\dfrac{V_1}{\sqrt{T_1}}\right)} = \frac{M_2}{M_1}$$

$$\frac{\dfrac{P_1}{R_{gas} \cdot T_1}}{\left(\dfrac{P_2}{R_{gas} \cdot T_2}\right)} \cdot \sqrt{\frac{T_1}{T_2}} = \frac{M_2}{M_1}$$

$$\frac{P_1}{P_2} \cdot \left(\frac{T_2}{T_1}\right)^{\frac{1}{2}} = \frac{M_2}{M_1}$$

$$\frac{1 + k \cdot M_2^{\,2}}{\left(1 + k \cdot M_1^{\,2}\right)} \cdot \left(\frac{T_2}{T_1}\right)^{\frac{1}{2}} = \frac{M_2}{M_1}$$

$$\frac{T_2}{T_1} = \left[\frac{M_2}{M_1}\cdot\frac{1+k\cdot M_1^{\,2}}{\left(1+k\cdot M_2^{\,2}\right)}\right]^2$$

Using the relation for the ratio of static to stagnation temperatur

$$\frac{T_2}{T_1}\cdot\frac{\frac{T_{02}}{\left(T_2\right)}}{\left(\frac{T_{01}}{T_1}\right)} = \left[\frac{M_2}{M_1}\cdot\frac{1+k\cdot M_1^{\,2}}{\left(1+k\cdot M_2^{\,2}\right)}\right]^2\cdot\frac{1+\frac{k-1}{2}\cdot M_2^{\,2}}{\left(1+\frac{k-1}{2}\cdot M_1^{\,2}\right)}$$

$$\frac{T_{01}}{T_{02}} = \left[\frac{M_1}{M_2}\cdot\frac{1+k\cdot M_2^{\,2}}{\left(1+k\cdot M_1^{\,2}\right)}\right]^2\cdot\frac{1+\frac{k-1}{2}\cdot M_1^{\,2}}{\left(1+\frac{k-1}{2}\cdot M_2^{\,2}\right)}$$

Taking the **M=1** condition at state 2 we have

$$\frac{T_0}{T_{0star}} = \frac{2\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)\cdot M^2\cdot(k+1)}{\left(1+k\cdot M^2\right)^2}$$

While Mathcad allows the symbolism shown above when working in symbolic mode, it would become confused in the arithmetic mode when we asked it to calculate a ratio instead of a certain variable name. For this reason ratios normally encountered in compressible flow were given variable names like T0overT0star in the case above and the relation was written in Mathcad functional notation as:

$$\mathrm{rT0overT0star}(M,k) := \frac{2\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)\cdot M^2\cdot(k+1)}{\left(1+k\cdot M^2\right)^2}$$

where the preceding small r is used to distinguish Rayleigh flow functions from other types of flow functions. A complete set of flow functions was developed in this manner for each type of flow.

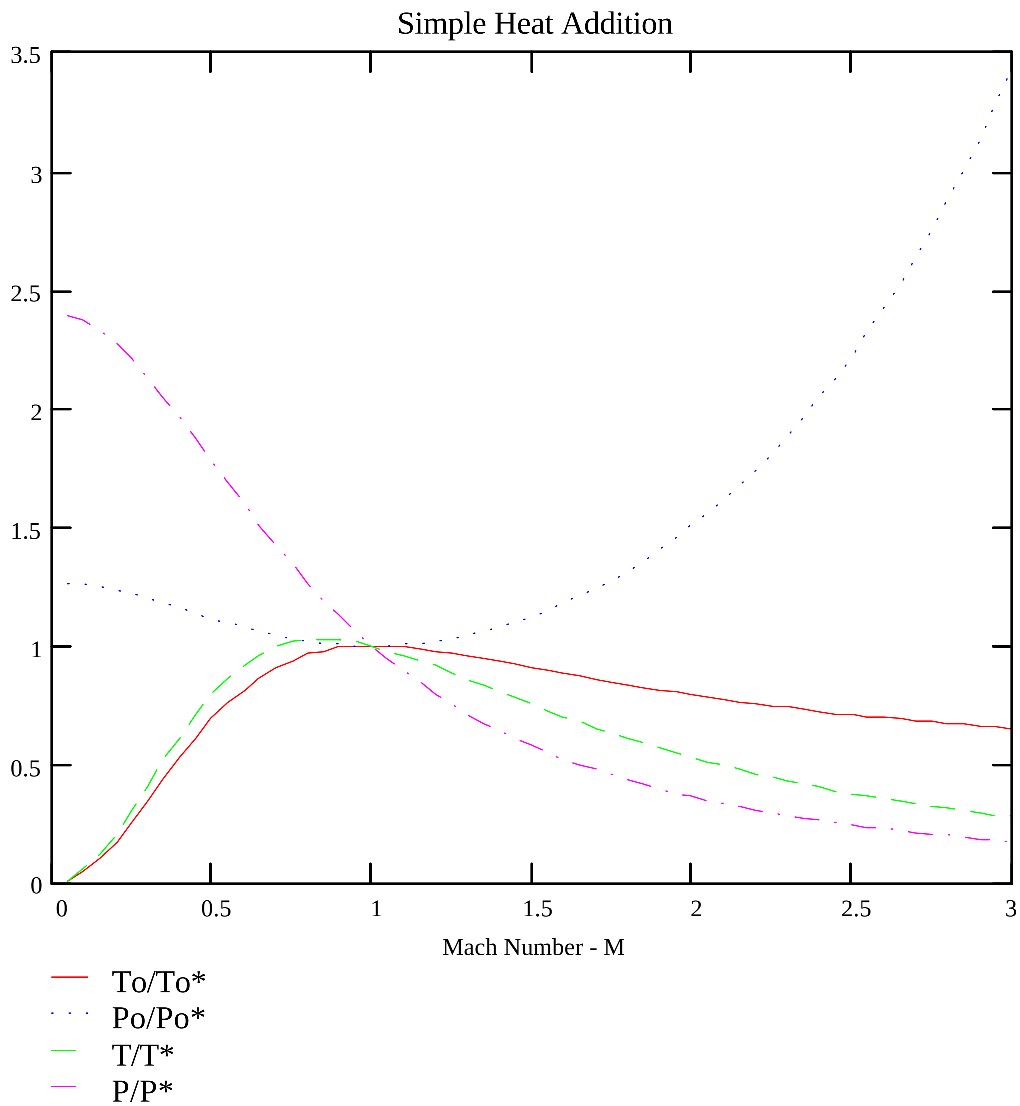
Each set of functional relations were plotted so students could begin to develop some understanding of how the functions varied, their orders of magnitude, and any maxima or minima. For simple T_0 change the plots are shown below. Unfortunately, this paper is in black and white so that the different color plots do not show up well. It is a good example of how color can be used to help clarify information for students.

Plots of Functional Relationships

Plot T_0/T_0^*, P_0/P_0^*, T/T^*, and P/P^* as functions of Mach Number over the range of 0.05 to 3 for a gas with a specific heat ratio of 1.4. Also, plot T/T^* vs $(s-s^*)/c_p$ for Mach Numbers between 0.5 and 2 where the (*) state is, as usual, the **M=1** state. This is the Rayleigh Line in a non-dimensional form.

$k := 1.4 \qquad i := 1..60 \qquad M_i := .05 \cdot i \qquad T_i := rTover Tstar(M_i, k) \qquad P_i := rPoverPstar(M_i, k)$

$P0_i := rP0overP0star(M_i, k) \qquad T0_i := rT0overT0star(M_i, k)$



Examples were chosen to illustrate particular behavior of each type flow. In the case of simple heat addition an example was used to locate the Mach Number at which the maximum static temperature was reached in a Rayleigh flow.

Determination of the Mach Number Associated withthe Maximum Value of T/T*

Start with the relation between Mach Number and **T/T***:

$$\frac{T}{Tstar} = \frac{(k+1)^2 \cdot M^2}{\left(1+k\cdot M^2\right)^2}$$

then take the derivative with respect to Mach Number and set it to zer

$$0 = 2\cdot(k+1)^2\cdot\frac{M}{\left(1+k\cdot M^2\right)^2} - 4\cdot(k+1)^2\cdot\frac{M^3}{\left(1+k\cdot M^2\right)^3}\cdot k$$

$$M = \begin{bmatrix} 0 \\ \dfrac{1}{\sqrt{k}} \\ \dfrac{-1}{\sqrt{k}} \end{bmatrix}$$

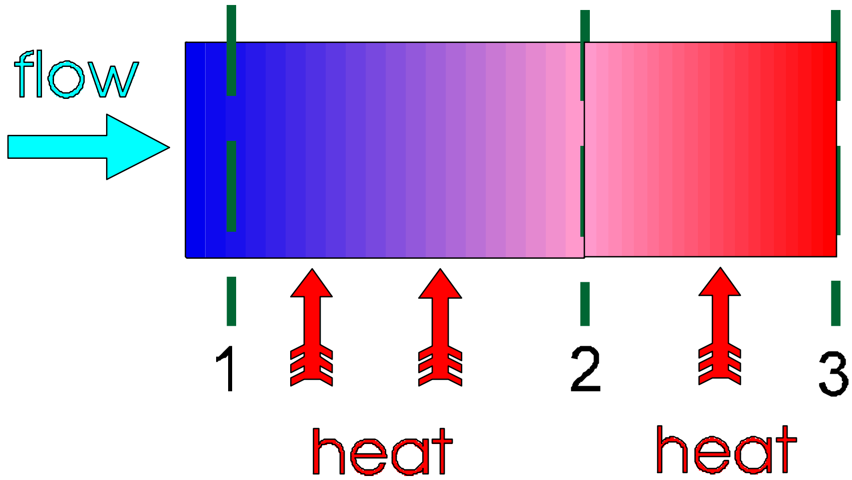
The first and last solutions do not correspond to physically real cases and therefore the only significant solution is:

$$M = \frac{1}{\sqrt{k}}$$

The case above demonstrates some of the many powerful symbolic features available in Mathcad which bridge the gap between using large amounts of class time on routine algebraic manipulation and asking students to accept results on faith. A more numerical example building on the results above to include heat transfer induced "choking" is shown below.

Simple Heat Addition

Air flows in a constant area frictionless duct at an initial Mach Number of 0.5 and stagnation temperature of 25 degrees C. Heat is added between sections 1 and 2 such that $\mathbf{T_2/T_1}$=1.30175. **a)** What is the Mach Number at section 2 and how much heat is added per unit mass? **b)** Additional heat is added between section 2 and section 3 such that $\mathbf{M_3=1}$. What is the ratio of $\mathbf{T_3/T_2}$? **c)** If more heat is added than in part **(b)** such that $\mathbf{T_{03}}$ is increased by 20% over the former value, what happens?



Since the star state is constant for simple heat addition, $\mathbf{T_1/T^*}$ can be found from $\mathbf{M_1}$ and then multiplied by $\mathbf{T_2/T_1}$ to get $\mathbf{T_2/T^*}$.

$$M_1 := 0.5 \qquad k := 1.4 \qquad c_p := .24\frac{BTU}{lb \cdot R}$$

$$T_{01} := (273 + 25) \cdot K \qquad Mn := .5$$

$$T2overTstar := 1.30175 rToverTstar(0.5, k)$$

$$T2overTstar = 1.029$$

a)

$$M_2 := root(rToverTstar(Mn, k) - T2overTstar, Mn) \qquad M_2 = 0.824$$

$$T0star := \frac{T_{01}}{rT0overT0star(M_1, k)} \qquad T0star = 431.036 \cdot K$$

$$T_{02} := T0star \cdot rT0overT0star(M_2, k) \qquad T_{02} = 419.302 \cdot K$$

Then the heat addition per unit mass can be found from $\mathbf{q/w = c_p * (T_{02} - T_{01})}$.

$$qoverw := c_p \cdot (T_{02} - T_{01}) \qquad qoverw = 1.219 \cdot 10^5 \cdot \frac{joule}{kg}$$

At $\mathbf{M_3 = 1}$ $\mathbf{T_3 = T^*}$, therefore $\mathbf{T_3/T_2 = T^*/T_2}$:

$$T3overT2 := \frac{1}{T2overTstar} \qquad T3overT2 = 0.972$$

This problem presents the counter-intuitive case in which heat is added and the static temperature goes up as expected but at some point additional heat is added and the static temperature goes down which is not expected. Close examination of the plots in the previous example clearly show this phenomenon but the implications demonstrated by this problem are often over looked.

Parts (b-c) of this problem have been deleted to save space.

Quizzes were also prepared using Mathcad and hence students were not faced with different formats or nomenclature at this critical time. Since the quizzes were prepared using Mathcad, it was very easy for the instructor to simply insert space between quiz questions and insert a solution which was passed out to the students during the next class period when the quiz was discussed.

Since the initial purpose of this experimental project was directed at lecture presentation only, some of the potential uses of Mathcad were not recognized. For example, while the lectures, class notes and quizzes were prepared using Mathcad, the students were given the option of using whatever calculation vehicles with which they were comfortable. Most chose to use a compressible flow function calculator supplied by the instructor (available from ASME over Internet at ftp://192.217.237.100/pub/fluid/compflow.zip). Plans are currently being developed to equip a lab with Mathcad so that students can receive "live" electronic copies of course notes, particularly all of the flow functions. This would extend many of the advantages offered the instructor in preparing class materials to students in doing homework and taking tests, i.e. numerical, unit and symbolic calculation capability integrated into a self documenting work environment.

At the end of the semester all of the course notes and quizzes were integrated into a Mathcad electronic book for the students to use in the future as a review of the course material as it is needed in professional practice, advanced courses, or research. A copy of the table of contents of this electronic book is shown below.

Lamar University Compressible Flow Review

Contents

Mathsoft, Inc. has been kind enough to permit distribution of a Mathcad Engine to these students so that they can access these review materials. This engine while permitting the display of all the material contained in a previously compressed file, does not permit changing the contents of these files or other new calculation or plotting of data.

Future Development

The author hopes that the large expenditure of time in the development of the course materials in Mathcad format will pay dividends in the future. Future lecture preparation will involve updating subject content and improving the clarity of presentation and ease of student learning. The former will be the instructor's responsibility while the latter two will involve student and instructor critique. It is hoped that the portability of the lecture documentation (from instructor to student and back) as well as the ease of modification will lead to a more effective quality improvement process than the older style of handwritten lecture notes (usually on 3x5 note cards).

The portability of the lecture notes also offers opportunities to better meet other types of student needs. For example, even in the present form the lecture notes seemed to be very effective in bringing students back up to speed after they had been forced to miss class. The audio and video extensions offered in Mathcad 6.0 may permit new paradigms for the lecture-homework-quiz models we often now employ in teaching engineering. The author asserts that students will always need the interaction with a learning mentor for efficient learning. This new technology may give teachers the ability to control when and where information flow, development of problem solving skills, and synthesis takes place. Hopefully, more time can be utilized in interaction with students, which does require instructor presence, and less in information flow, which may now be augmented by means other than textbooks.

Summary

The use of Mathcad as a lecture presentation aid required substantially more time than the usual lecture preparation time. However, the following benefits were found:

clearer more interesting presentations,
ease of numerical or symbolic modification to answer student questions,
increased student attention due to the elimination of the need for students to take notes,
increased ability to respond to the needs of students who are forced to miss class,
increased ability to apply quality improvement techniques to lectures,
sufficient class time was saved by the electronic presentation method that all homework problems could be displayed and discussed,
and
potential to increase time spent mentoring students since information is conveyed more efficiently.