2006-1227: ACADEMIC DART GUNS – A REVEALING EXPERIMENT FOR 2-D KINEMATICS

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Academic Dart Guns – A Revealing Experiment for 2-D Kinematics

A small, portable kinematics experiment was developed for the purpose of improving the understanding of 2-dimensional (ballistic) kinematics for sophomore mechanical engineering technology students. Although this type of experiment is not unique, the equipment and accompanying laboratory exercise were used to provide students with important insights into the relative importance of the variables that can affect this and similar types of kinematics problems. In this case the precision of the measuring instruments was used to strengthen understanding of sources of variability and their impact on accuracy.

The Experiment

The experiment itself consisted of shooting a dart from a toy dart gun at a nearby vertical surface. Students made measurements with each test firing in order to determine the dart's initial velocity, using both chronometric and geometric methods. They then compared these two independent velocity results, noting trends in any differences obtained, and identifying the variables responsible. An initial classroom lecture of the experiment was done, including derivation of the final geometric and chronometric equations to be used, lab procedures to be followed, and potential results students might expect. Students were given credit on the assignment for; 1) using proper experimental methods in determining their results, 2) observation and interpretation of appropriate trends, and 3) deduction of the significance of potential sources of variability that affected their experiment. A subsequent classroom discussion was then done (after lab reports were turned in) to further explore these trends, in order to more clearly identify and understand the potential sources of variability as well as their relative importance.

The Equipment

Four types of laboratory equipment were used for this experiment: a projectile and its launcher, angular and linear measuring instruments, a mounting fixture for the launcher and chronometric equipment, and a custom-designed chronometric measuring instrument.

The projectile and its launcher were purchased from Wal-Mart. Simple children's dart guns were selected, consisting of a plastic air-charge gun and five urethane darts. Each gun was modified by removing the last 1.5 inches of the outer barrel in order to expose the dart tube for standardized installation in the mounting fixture. Then the dart with the lowest friction was selected and painted with flat black paint to enhance opacity through the chronometer beams.

Standard instruments were used for the angular and linear measurements. These consisted of a combination spirit level / protractor, and a measuring tape. The spirit level / protractor had a precision of $1/2^0$ (0.5 degrees) and provided a simple, common sense method for students to establish and verify the required initial gun angle (θ) of zero degrees. The tape measure provided horizontal (X) and vertical (Y) distance measurements between the first beam of the chronometer and the dart's impact point, as well as the distance between the chronometer beams (L), to a precision of $1/16''$ (0.0625 inches). See Figure 1.

Figure 1.

The dart gun mounting fixture consisted of two wooden blocks joined with a lag-bolt and wing nut. See Figure 2. The left block was solid and clamped to the edge of a table. The right block contained a precision-drilled hole to tightly contain the now-exposed dart tube of the gun. After alignment with the appropriate target surface and adjustment to the required zero-degree initial trajectory angle, the wing nut was tightened and checked for accuracy and rigidity.

Figure 2.

Chronometer Circuit Description

The projectile is propelled past two pairs of QRB 1134 reflective sensors. The QRB 1134 reflective sensor has an IR photo emitter and an IR photo detector on it. Both the photo emitter and photo detector are facing the same direction. The usual application of this part is to sense a light reflection from a surface. In this application two QRB 1134s are set up opposite each other with the opposing sensor reversed from its partner sensor so that the photo emitter on sensor "A" faces the photo reflector of sensor "B" and the photo emitter of sensor "B" faces the photo detector sensor "A". This method allows for two sets of detector- emitter pairs facing each other and only one is needed, so the second is disconnected. Connected in the manner described there is one IR beam which is used to detect the passing of the projectile. The first of these two QRB1134 pairs is used to detect the projectile at the first point, starting a counter circuit and the second pair of QRB 1134s is set-up identically to the first pair and is used to stop the counting circuit. The QRB 1134 was chosen for this application because of its availability and packaging.

Each of the two photo emitters is connected to the +5 volt source through a 100 ohm resistor, the photo detectors are connected to the +5 volt source through a 10 k ohm resistor. When the light path is blocked between a photo emitter and photo sensor pair the voltage on the photo emitter goes up to approximately 3.8 volts. This voltage from each photo sensor then drives the input of an analog comparator. The analog comparator is on-board adjustable with a 10 k ohm potentiometer so that comparator's output will switch consistently with the passing of the projectile. Typically the comparator was set to switch at approximately 2.5 volts. The output of the analog comparator is open collector; this open collector output is connected to the +5volt supply through a 10k ohm resistor. The output of the first analog comparator is used as a start signal for the counter. The first sensor pair executes the start, and the second pair executes the stop. This start / stop is accomplished by the use of a J-K flip-flop. The start sensor was connected to the J input of the flip-flop and the stop sensor was connected to the K input. When the start sensor was triggered high it sends the J input high and sets the flip-flop output 'Q' to a logic high. When the flip-flop's output is high the 1 MHz clock signal is then passed through the AND gate to the modulus 10000 counter. When the stop sensor is triggered high, it sends the K input of the flip-flop high and resets the flip-flop's output to zero, disabling the clock input to the counter and thereby stopping the count. A manual reset button is used to reset the counter and the flip-flop simultaneously.

The two sensor pairs are precisely set at a distance of two inches. The counter circuit's clock rate and the distance between the start and stop sensors is taken into account in the displayed value. And the speed is calculated by the equation shown below in this paper.

The counter circuit is clocked at 1 MHz therefore incrementing at the rate of 1us. This count is accomplished with the use of the SG-51 series of integrated circuit crystal controlled oscillators. A 2 MHz oscillator was used and then a J-K flip was clocked and the output taken from the J-K flip-flop's output 'Q' to divide the frequency by two. The 1 MHz signal then clocked a modulus 10000 asynchronous counter.

The 74390 asynchronous decade counter was chosen for the modulus 10000 counter. Four decade counters are needed. Each 74390 contains two decade counters, therefore two 74390s are

used. Each of the counters is cascaded. This modulus 10000 counter is used to count the number of clock pulses between when the projectile passed the first IR emitter-detector pair and the second IR emitter-detector pair.

The count value is displayed on four seven segment displays. Each of the displays is common anode and were driven by the 7447 seven segment decoder/driver IC. The output of each of the 7447s outputs "a" through "g" are connected to the corresponding segment on the display through a 220 ohm resistor.

A separate reset circuit is provided that sets the displayed value back to zero by clearing and disabling the counter. The counter is not re-enabled until the 'start' IR photo sensor pair is triggered.

This circuit proved reliable and accurate. However the circuit was later simplified by the use of the red lion miniature display module unit (MDMU). The MDMU replaces the counter circuit, display drivers, and displays of the circuit described above. The sensors, analog comparators, and oscillator are identical. The red lion MDMU uses a start signal, a stop signal and a manual reset very similar to those described above. Both circuits gave accurate and reliable readings. The chronometer schematic is shown below.

Figure 3.

Introductory Lecture

Although this laboratory exercise was appropriately positioned in the course to immediately follow conventional ballistics exercises, students had not yet been exposed to the practice of

equation substitution for the purpose of creating a single governing equation appropriate to the task at hand. To this point students had only become familiar with the standard time-dependent vector equation for position ($r(t)$).

$$r(t) = (X_0 + V_0 \cos \theta t)_i + (Y_0 + V_0 \sin \theta t + a t^2 / 2)_j$$

At this point students were lead through the steps of a) substituting zero for X_0 & Y_0 based on judicious placement of the origin at the point where it has just exited the gun barrel (also the point where the dart crosses the first chronometer beam), b) rearranging the i component to get the identity for time t, c) substitution of this identity into the j component, and d) rearrangement to yield the kinematic equation for initial velocity (V_{0k}).

$$V_{0k} = \text{sqrt} [(a X^2) / (2 (\cos \theta)^2 (X \tan \theta - Y))]$$

Then by inserting $a = 32.2 \text{ ft/s}^2$ and $\theta = 0$, and using units of feet for the measurements of X & Y, students arrived at the final usable form of the kinematic velocity equation

$$V_{0k} = \text{sqrt} [16.1 * X^2 / Y] \quad \text{with units of feet per second. See Figure 1.}$$

Next students were guided through the process of determining initial velocity using a double-beam chronometer, where L represented the distance (in inches) between two parallel beams, and N represented the number of counts of the chronometer between interruptions of each beam as a dart passed through. Since the counter designed for this exercise (see above) had a clock rate of 10^6 counts per second, the final chronometer-based velocity equation became

$$V_{0c} = [(L * 10^6) / (12 * N)] = 13,889 / N \quad \text{with units of feet per second.}$$

Throughout this presentation, hints were provided regarding the impact precision of the various substitutions (for example if theta varied by a degree from zero, or if L happened to be a thirty-second of an inch more or less than measured) might have on the velocities they would achieve during experimentation.

Laboratory Exercise

For the exercise itself four three-person teams of students first installed the fixtured dart gun on a convenient table approximately five feet from a vertical surface that darts would adhere to, such as a glass display case or wood cabinet. Next the initial gun angle (θ) was adjusted to zero, the wing nut tightened, and the angle (θ) verified. Students then measured the chronometer beam separation distance (L) and the horizontal distance (X) between the first beam of the chronometer and the target surface, to the available precision of one-sixteenth of an inch. Finally each group of students divided the tasks of a) controlling the dart gun, b) measuring the vertical drop of each dart (Y), and c) controlling the chronometer, and began their testing. After fifteen shots all data was collected and entered into a spreadsheet, where the independently-determined velocities (V_{0k} & V_{0c}) were calculated. The average, range, and standard deviations (using n) of each data set were then determined. Finally the difference ($\Delta = V_{0c} - V_{0k}$) was determined and students began the task of explaining what difference (if any) they achieved, and identifying underlying causes.

Results

Analysis of these data yielded dart velocities between 30.7 and 27.0 feet per second, depending on the team and method of determination. Velocity differences ($\Delta = V_{0c} - V_{0k}$) ranged from +3.7 to -2.7 fps, with an average absolute difference for the four teams of 2.23 fps. Two teams had higher average chronometric velocities while two teams had higher average kinematic velocities.

Clearly students recognized that the two methods did yield differing results for initial velocity, contrary to their initial assumption. All groups also recognized that variations in initial gun angle (θ), vertical impact displacement (Y), horizontal impact displacement (X), and chronometer beam separation distance (L) could account for their discrepancies. Some students also identified wind resistance of the dart during flight, the low level of precision (1/16") of the measuring tape, chronometer position relative to the initial position of the dart, and personal variability or attention to detail during the measurement process as potential contributing factors. The aspect still lacking from their perception of the experiment, however, was an appreciation for the relative magnitude of their influence. A more in-depth examination of the relative influence of these potential factors on resulting velocities was then the subject of the follow-up lecture.

Follow-up lecture

With all reports submitted, the subsequent lecture was designed to further strengthen student understanding of kinematics situations and provide insight into the factors involved that should receive the most attention in any future work they encountered. First, the kinematics side of the problem was addressed.

Factors involved in the kinematics analysis included measurement deviations of the horizontal impact distance (X), the vertical impact distance (Y), and the initial gun angle (θ). Air drag was mentioned for completeness but not included in a rigorous analysis since the (intentionally) short travel distance meant its contribution would be minimal, especially at the speeds involved.

When determining deviation it was important to first establish a baseline from which to work. For the presentation the author chose values very close to the teams' initial setups and results. Using initial values of; a horizontal distance (X) of 5.000 feet, a vertical distance (Y) of -0.4471 feet (negative representing a final position lower than the initial position), and an initial gun angle (θ) of 0.00 degrees together provided an initial velocity of 30.00 feet per second. Increments of deviation were then based on the precision of the measuring instruments used. Therefore, distances (X, Y, and L) varied by 1/16" (0.0625 inches or 0.0052 feet) while angles (θ) varied by $1/2^\circ$ (0.50 degrees). Negative angles were also defined as moving in the counter clock-wise direction (up). Distances and angles were varied by up to four times the measurement precision (a total of ± 0.25 inches & ± 2.0 degrees).

When shown Figure 4, students began to realize how much impact their instruments' precision could have on apparent kinematic velocity. In the case of measured distances, both velocities would gradually increase with increasing distance, and decrease with decreasing distance.

Conversely, apparent velocity would change rather dramatically with changes in initial gun angle (θ), decreasing as initial angle increased (rotated clock-wise) and increasing as angle decreased (rotated counter clock-wise) from zero. This made even more sense to students when a typical trajectory diagram was presented, with its characteristic parabolic shape, and overlaying the horizontal (X) and vertical (Y) distances required for impact. Next the chronometric method was analyzed.

Figure 4.

In analyzing apparent velocity associated with the chronometric method, the baseline values for a velocity of 30.00 feet per second included a beam separation distance (L) of 2.0000 inches and a chronometer count (N) of 5555. Precision for the beam measurement was again $1/16''$ (0.0625 inches) since the same tape measure was used for its measurement. Precision for the chronometer counter was 28 counts, chosen to represent approximately 0.5% of the baseline value. Apparent velocities were then graphically shown to decrease as count (N) increased but to increase as beam separation distance (L) increased. This produced apparent velocity profiles having opposite slopes but the same order of magnitude. Changes in beam distance (L) of ± 0.25 inches (or $\pm 13\%$) and count (N) of ± 112 (or $\pm 2\%$) produced deviations of approximately 5% or less.

Figure 5.

Finally, all forms of potential error were presented together, in order to reinforce awareness of relative importance. See Figure 6.

Here all variables were graphed together, and two trends were presented; order of magnitude and linearity. By using the graphical form both of these features become immediately evident. Deviations caused by errors in the precision (1/16") of the linear measuring instrument (X, Y, and L) were all minor and linear within the bounds of the initial experiment. Likewise, deviations in the counter (N) were also minor and linear.

However, deviations caused by linear errors in measurement of initial gun angle (θ), for instance between one-quarter and one-half degree, caused apparent velocity to change much more dramatically, and in an obviously non-linear manner. Magnitude was evident to students from the groupings of the graph lines, and by showing an overlay of the variables involved onto the parabolic trajectory shape. Non-linearity was further demonstrated with simple calculations of how the values for sine and cosine change as an angle approaches 90° . In the context of this experiment, angular deviations of only 2 degrees produced a negative change (decreasing apparent velocity) of over 10 percent and a positive change (increasing apparent velocity) of roughly 30 percent.

Figure 6.

Summary

This experiment provided a simple, yet realistic example of projectile kinematics. It required only low cost components, straightforward measurements, and algebraic calculations of dart velocity. As each team presented its results, however, important trends began to appear. Some teams achieved consistently low kinematic velocities compared to chronographic velocities, while others achieved consistently high kinematic velocities.

The subsequent classroom lecture and discussion were used to explore and make sense of this discrepancy. By exploring the contribution of each variable present in the experiment, students were led to the fact that all variable contributions are not equal. In this case deviations in the initial gun angle proved to be much more important in determining accurate kinematic results than the other variables. Additionally, the aspect of measurement precision was shown to have strong practical implications as well.

Student learning of these common-sense aspects to experimental work appeared to be good, with most recognizing important lecture points and contributing to the discussion in meaningful ways. It remains to be seen whether retention will carry into their professional lives.