Integrating a computer algebra software into the engineering curriculum: problems and benefits.

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This paper describes how **MAPLE,** a symbolic computation software, may be used as an educational tool in the computation of the eigenvalues and eigenvectors which arise, naturally, in the study of mechanical vibrations. Article 1 presents the rationale behind the choice of MAPLE over DERIVE, MACSYMA and MATHEMATICA. Article 2 describes the specific context where MAPLE was used in a first course of numerical analysis taught by the author. The simple programs used in solving a typical example is presented in Article 3, together with the results obtained at the end of each step. Finally, the advantages and disadvantages of using general purpose mathematical software with symbolic manipulation capabilities rather than programming languages such as Pascal, and C are discussed in the conclusion, Article 4.

1. Introduction

A careful review of the proceedings of the annual conference of the American Society for Engineering Education (ASEE) and of the Canadian Conference on Engineering Education (C2E2) of the last ten years, reveals that MAPLE enjoys a very good reputation in the academic community at large. This is also attested by the ever increasing number of textbooks which are geared to MAPLE.

An equally careful review of the qualifications of the professors in the Faculty of Engineering and Applied Science at Ryerson Polytechnic University reveals that a number of them are graduates of the University of Waterloo, Cambridge, Ontario, CANADA, where MAPLE originated. In addition, the fact that almost all the professors of the Mathematics, Physics and Computer Science department (MPCS) are very proficient with MAPLE, is probably another reason favouring its adoption. Indeed, several personal computers located in our offices carry MAPLE. Also, several personal computers in our microcomputer laboratories carry the full version of MAPLE. It is thus natural to think of integrating MAPLE in as many courses as possible in our undergraduate engineering curriculum.

2. **MAPLE** in the Engineering Curriculum

The introduction of MAPLE in the first year of the engineering curriculum, as part of the Calculus course, or as part of an introductory Computer programming course has been discussed by the faculty of the Mathematics and Computer Science department. The discussion led to the following pressing question: “Should the learning of MAPLE be left to the student, or should the instructor devote some lecture time to cover some of the salient features of MAPLE?” The answer to this question, already difficult in previous years, became even harder in view of the fact that the number of lecture hours in all the courses was reduced due to budgetary cuts.

During the discussions, several alternatives were considered. One of them was to implement MAPLE in the Calculus and Linear Algebra courses. However, as of the writing of this paper, no official policy emerged on this matter.

On the other hand, the author observed that a number of students, as early as 1989, were solving some problems in numerical analysis by means of an earlier version of MAPLE . The majority of students were still using Pascal, Fortran or BASIC. Some were even using spreadsheets. All of them, however, were required to use IMSL (International Mathematics and Statistics Library) to solve a specific number of problems. Subsequently, due to the relatively high cost of IMSL, the administration decided to discontinue it. It was at about that time that the author decided, as an alternative, to integrate MAPLE in numerical analysis, an effort that took him almost two semesters to complete. The net outcome of these changes is that, currently, students use spreadsheets and MAPLE. Almost none of them spends any time using a procedural programming language anymore.

The following article gives a detailed description of a typical example which exploits some of the more interesting features of MAPLE. This example is used as a vehicle for the introduction of eigenvalues and eigenvectors to mechanical engineering students.

3. An example involving MAPLE

Consider the following system of 5 particles, of mass m_1, m_2, m_3, m_4, m_5 placed at positions *2l, 3l, 6l, 8l, 11l,* from the fixed end A, respectively, along an elastic string of length 12 *l,* shown in Fig. 1. Assume that the system rests on a frictionless horizontal plane. Note that end B is also fixed.

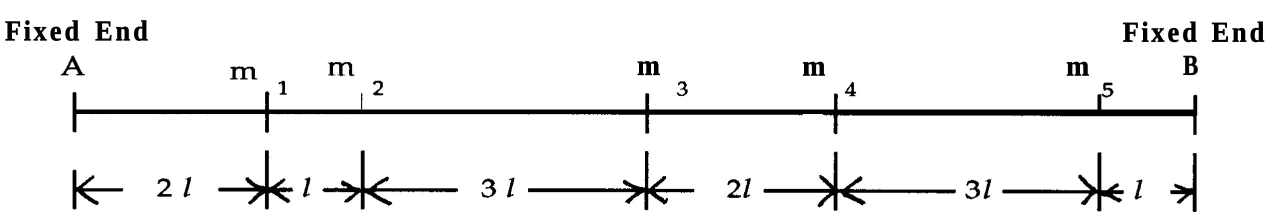

Fig.

Let T be the tension everywhere along the string. Following the example described by Noble [7], suppose that the system is disturbed and that, as a consequence of this disturbance, the particles execute small transverse vibrations under no external forces. Also let X_1, X_2, X_3, X_4 and X_5 be the displacements of the particles m_1, m_2, m_3, m_4 and m_5, respectively, as shown in Fig. 2.

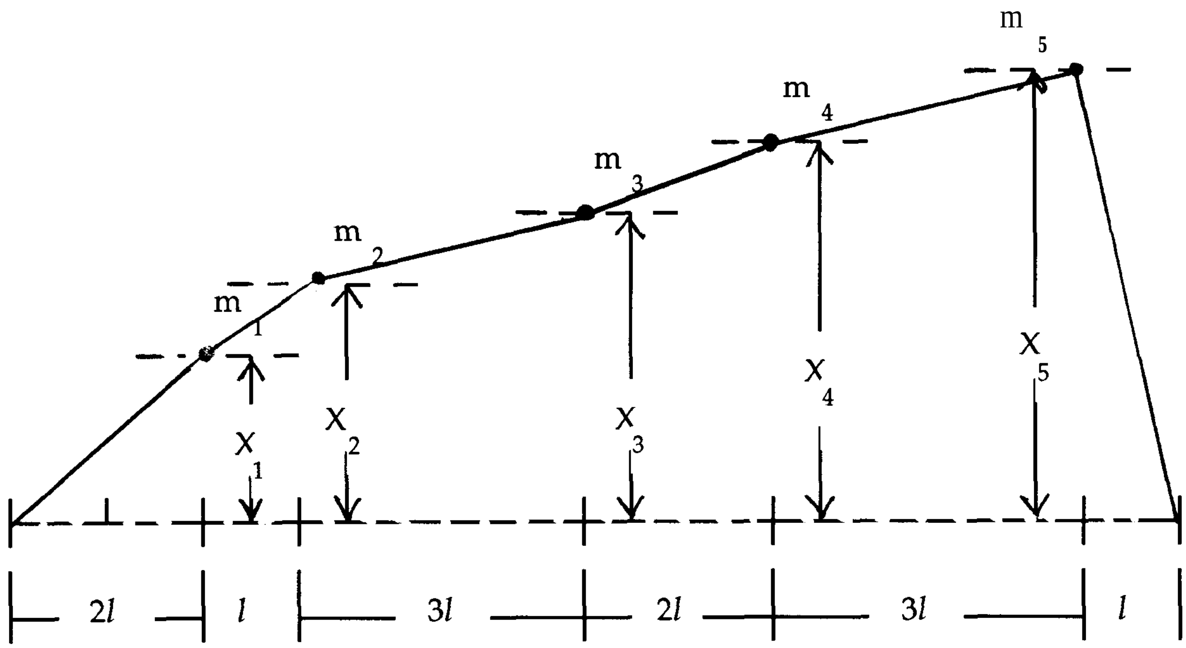

Fig. 2

The forces acting on mass m_1 are shown in Fig. 3. Since we assume that the displacements are small, the tension T can be considered to be constant. Furthermore, $\sin\theta_1$ and $\sin\theta_2$ can be approximated by $\tan\theta_1$ and $\tan\theta_2$ respectively.

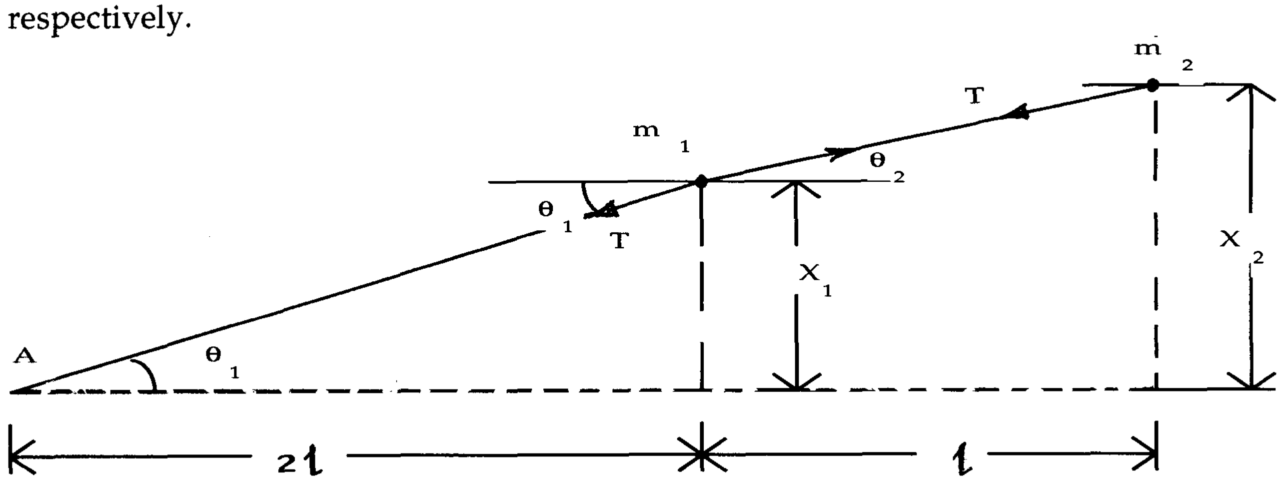

Fig. 3

Application of Newton's second law leads to the following second order ordinary differential equation

$$m_1 \frac{d^2 X_1}{dt^2} = -T\frac{X_1}{2\ell} + T\frac{X_2 - X_1}{\ell} \qquad (1\ a)$$

Similarly, for the motion of masses m_2, m_3, m_4 and m_5 we obtain

$$m_2 \frac{d^2 X_2}{dt^2} = -T\frac{X_2 - X_1}{\ell} + T\frac{X_3 - X_2}{3\ell} \qquad (1\ b)$$

$$m_3 \frac{d^2 X_3}{dt^2} = -T\frac{X_3 - X_2}{3\ell} + T\frac{X_4 - X_3}{2\ell} \qquad (1\ c)$$

$$m_4 \frac{d^2 X_4}{dt^2} = -T\frac{X_4 - X_3}{2\ell} + T\frac{X_5 - X_4}{3\ell} \qquad (1\ d)$$

$$m_5 \frac{d^2 X_5}{dt^2} = -T\frac{X_5 - X_4}{3\ell} - T\frac{X_5}{\ell} \qquad (1\ e)$$

Next, we assume that all the displacements vary sinusoidally with time. So let

$$X_k = a_k e^{i\omega t} \qquad k = 1, 2, 3, 4, 5$$

Substituting in equations (la) to (le) and simplifying we get

$$-\frac{6m_1 \ell\omega^2}{T} a_1 = 6a_2 - 9a_1 \qquad (2a)$$

$$-\frac{6m_2 \ell\omega^2}{T} a_2 = 2a_3 - 8a_2 + 6a_1 \qquad (2b)$$

$$-\frac{6m_3 \ell\omega^2}{T} a_3 = 3a_4 - 5a_3 + 2a_2 \qquad (2c)$$

$$-\frac{6m_4 \ell\omega^2}{T} a_4 = 2a_5 - 5a_4 + 3a_3 \qquad (2d)$$

$$-\frac{6m_5 \ell\omega^2}{T} a_5 = -8a_5 + 2a_4 \qquad (2e)$$

As a numerical example, consider the case where $m_1 = 2m$; $m_2 = m$; $m_3 = 3m$; $m_4 = 2m$; $m_5 = m$. If we let $\lambda = 36ml\omega^2/T$, then equations (2a) to (2e) take the following form:

$$(27-\lambda)a_1 - 18a_2 = 0 \quad (3a)$$

$$-36a_1 + (48-\lambda)a_2 - 12a_3 = 0 \quad (3b)$$

$$-4a_2 + (10-\lambda)a_3 - 6a_4 = 0 \quad (3c)$$

$$-9a_3 + (15-\lambda)a_4 - 6a_5 = 0 \quad (3d)$$

$$-12a_4 + (48-\lambda)a_5 = 0 \quad (3e)$$

Equations (3) can be written in matrix form as follows:

$$B\,A = \lambda A \qquad (4)$$

where:

$$B = \begin{bmatrix} 27 & -18 & 0 & 0 & 0 \\ -36 & 48 & -12 & 0 & 0 \\ 0 & -4 & 10 & -6 & 0 \\ 0 & 0 & -9 & 15 & -6 \\ 0 & 0 & 0 & -12 & 48 \end{bmatrix} \qquad A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$$

Thus, the solution to the problem consists of finding the eigenvalues and corresponding eigenvectors of the matrix B. This task is accomplished with Program 1 and Program 2. Program 1 yields the characteristic polynomial and the eigenvalues. Program 2 yields the eigenvectors corresponding to the eigenvalues obtained in the previous program. Furthermore, the program checks the validity of the equation

$$B\,A_k = \lambda_k A_k \qquad k = 1, 2, 3, 4, 5.$$

```
> with(linalg):
>   B   :=matrix(5,5,[27,-18,0,0,0,-36,48,-12,0,0,0,-4,10,-6,0,0,0,-9,15,-6,0,0,0,-12,48]);
> cpb  :=charpoly(B,lambda);
> fsolve(cpb=O);
```

Program 1

```
> with(linalg):
>  B  :=matrix(5,5,  [27,-18,0,0,0,-36,48,-12,0,0,0,-4,10,-6,0,0,0,-9,15,-6,0,0,0,-12,48]);
> Id :=&*();
> l1 :=2.519651457; l2 :=10.60656216; l3 :=19.09845917;
> l4 :=50.12757800; l5 :=65.64774921;
>  M  :=ffgausselim(x*Id-B);
> M[5,5] :=0:
>    subs(x=l1,evalm(M));nullspace(");v:="[l]:evalm(B&*v=l1*v);
>   subs(x=l2,evalm(M));nullspace(");v  :="[l]:evalm(B&*v=l2*v);
>    subs(x=l3,evalm(M));nullspace(");v:="[l]:evalm(B&*v=l3*v);
>   subs(x=l4,evalm(M));nullspace(");  v:="[l]:evalm(B&*v=l4*v);
>    subs(x=l5,evalm(M));nullspace(");v:="[l]:evalm(B&*v=l5*v);
```

Program 2

Having obtained the eigenvalues and corresponding eigenvectors, it is possible to obtain the graphical representation of the five modes of vibration by using Program 3.

```
> pi:=3.14159265:lambda1:=2.519651457:a1:=0.46414203641:a2:=0.6312421569:a3:
> =1:a4:=0.8258966528:a5:=0.2179130141:
> p1:=[0,0]:p2:=[2,a1*sin(t)]:p3:=[3,a2*sin(t)]:p4:=[6,a3*sin(t)]:p5:=[8,a4*sin(t)]:p6:=[
> 11,a5*sin(t)]:p7:=[12,0]:
> p11:=subs(t=pi/2,p1):p21:=subs(t=pi/2,p2):p31:=subs(t=pi/2,p3):p41:=subs(t=pi/2
> ,p4):p51:=subs(t=pi/2,p5):p61:=subs(t=pi/2,p6):p71:=subs(t=pi/2,p7):
> p12:=subs(t=pi/6,p1):p22:=subs(t=pi/6,p2):p32:=subs(t=pi/6,p3):p42:=subs(t=pi/6
> ,p4):p52:=subs(t=pi/6,p5):p62:=subs(t=pi/6,p6):p72:=subs(t=pi/6,p7):
> p13:=subs(t=-pi/8,p1):p23:=subs(t=-pi/8,p2):p33:=subs(t=-pi/8,p3):p43:=subs(t=-p
> i/8,p4):p53:=subs(t=-pi/8,p5):p63:=subs(t=-pi/8,p6):p73:=subs(t=-pi/8,p7):
> p14:=subs(t=-pi/3,p1):p24:=subs(t=-pi/3,p2):p34:=subs(t=-pi/3,p3):p44:=subs(t=-p
> i/3,p4):p54:=subs(t=-pi/3,p5):p64:=subs(t=-pi/3,p6):p74:=subs(t=-pi/3,p7):
> with(plots):
> G1  :=plot([p11,p21,p31,p41,p51,p61,p71],linestyle=1):
> G2  :=plot([p12,p22,p32,p42,p52,p62,p72],linestyle=2):
> G3  :=plot([p13,p23,p33,p43,p53,p63,p73],linestyle=3):
> G4  :=plot([p14,p24,p34,p44,p54,p64,p74],linestyle=4):
> plots[display]({G1,G2,G3,G4});
```

Program 3

The third and fourth line of program 3, contains the five elements of the eigenvector A_1 corresponding to the eigenvalue $\lambda_1 = 2.519651457$. This mode of vibration is shown in Fig. 4a for four values of the time t. By substituting the values of a_1, a_2, a_3, a_4, a_5 corresponding to $\lambda_2, \lambda_3, \lambda_4$ and λ_5, we obtain the graphical representation of the remaining modes, as shown in Fig. 4b, 4c, 4d and 4e.

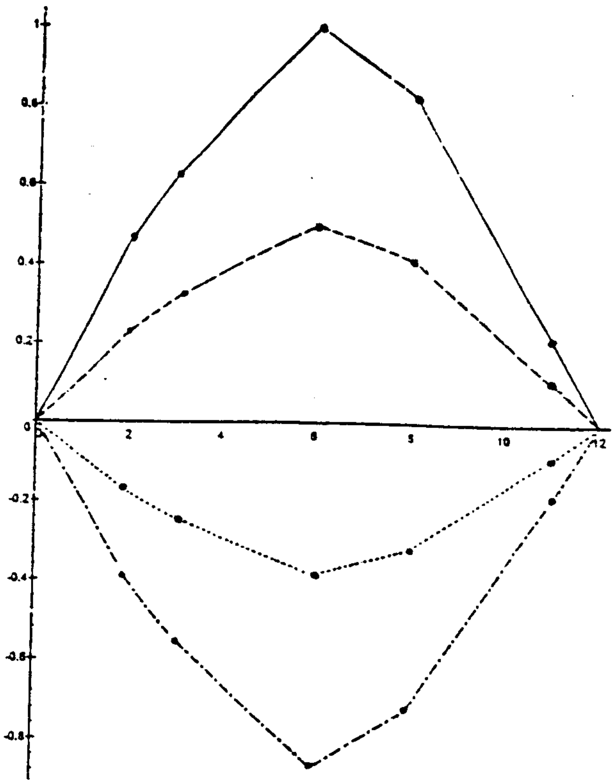

Fig. 4a

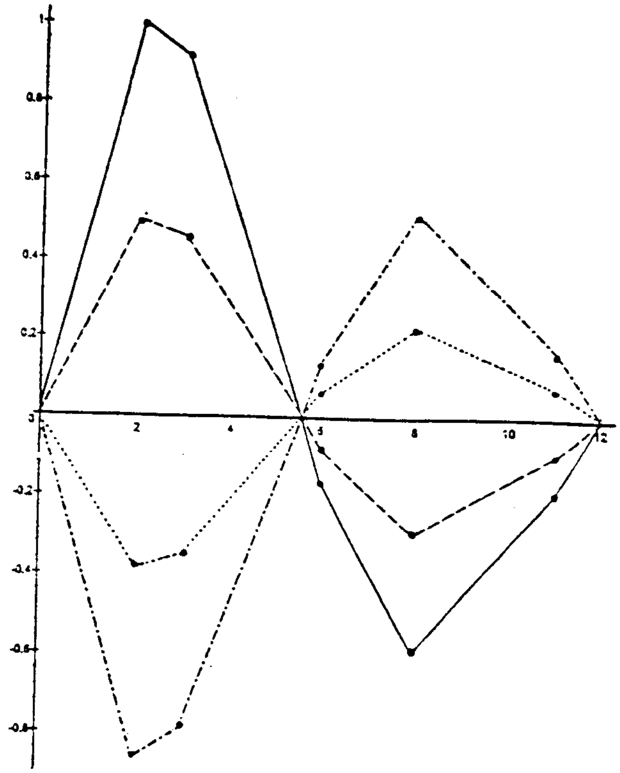

Fig. 4b

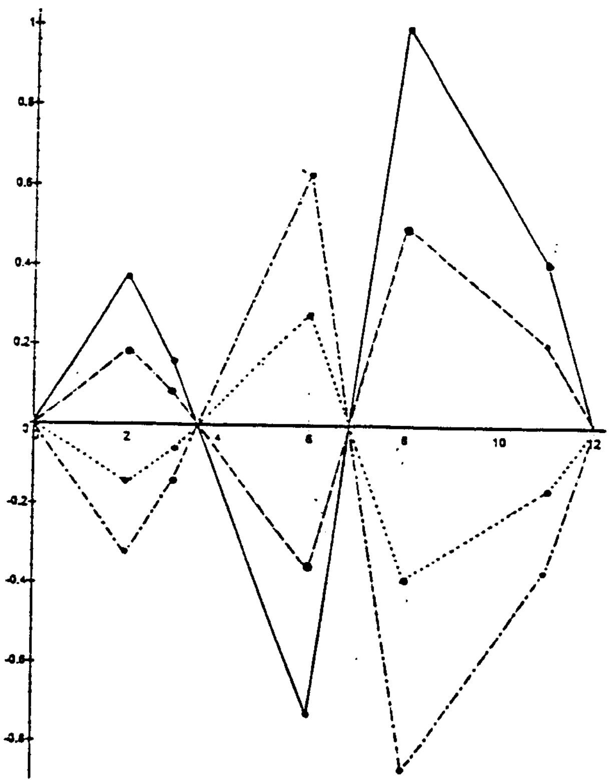

Fig. 4c

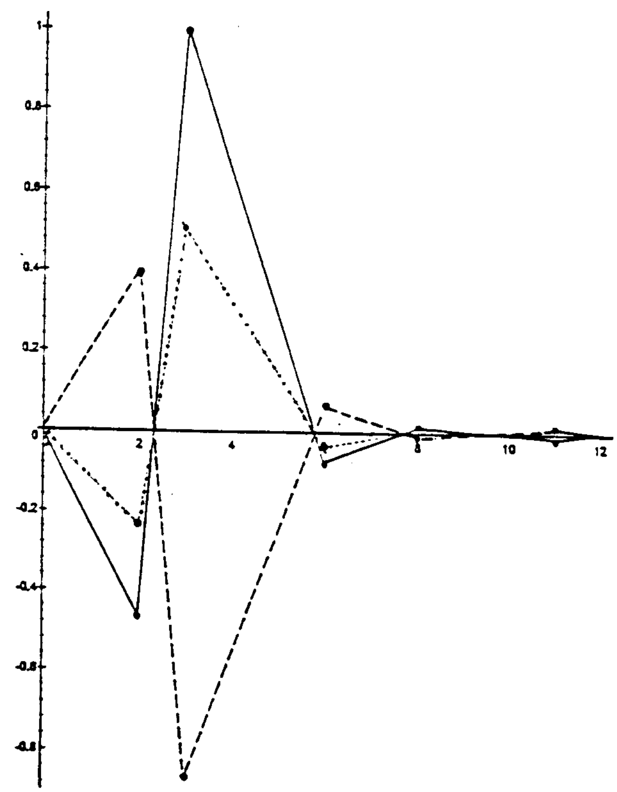

Fig. 4d

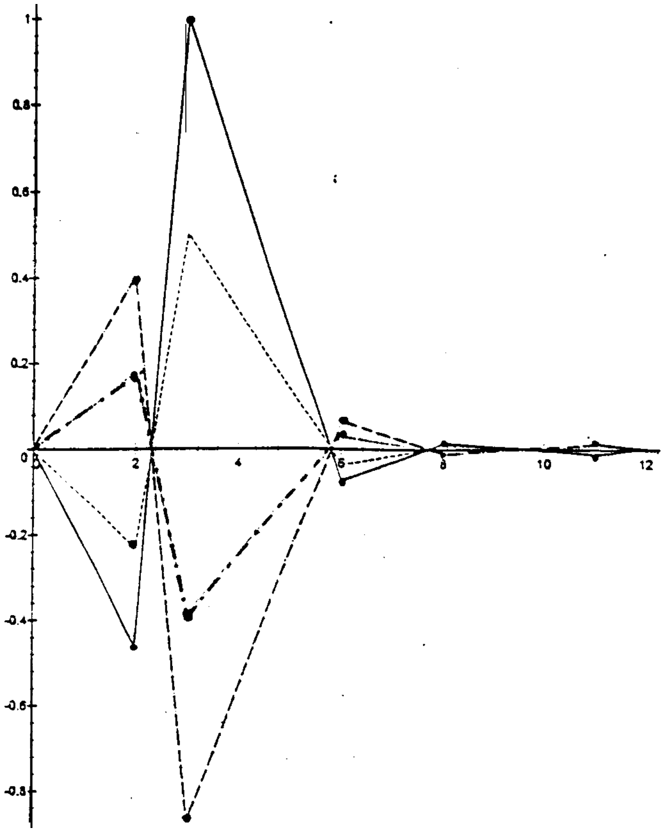

Fig. 4e

It should be emphasized that all the students were asked to find the roots of the characteristic equation earlier in the course using several methods (Bisection, Fixed-Point Iteration, Newton-Raphson, False-Position, Bairstow's Method, Quotient-Difference algorithm). Subsequently, they were also asked to use the power method to find the largest eigenvalue and corresponding eigenvector. As a bonus, they were also asked to use deflation to find the remaining eigenvalues and corresponding eigenvectors.

Program 4 exploits the animation facility of MAPLE. It animates the mode of vibration which corresponds to the smallest eigenvalue (and as a consequence, to the smallest angular frequency).

```
> f :=proc(x);
> if O<=x and x<2 then 0.4641420364*x/2;
> elif 2<=x and x<3 then (0.6312421569-0.4641420364)*(x-2) + 0.4641420364;
> elif 3<=x and x<6 then (1-0.6312421569)*(x-3)/3 + 0.6312421569;
> elif 6<=x and x<8 then (0.8258966528-I) *(x-6)/2 + 1;
> eiif 8<=x and x<11 then (0.2179130141-0.8258966528)*(x-8)/3 + 0.8258966528;
> elif 11<=x and x<=12 then -0.2179130141*(x-11) + 0.2179130141;
> else 0;fi;end;
> with(plots):
> animate('f(x)'*sin(t), 'x'=0.,12, t=-Pi..Pi, frames=40);
```

Program 4

4. Conclusion

The integration of a symbolic computation software such as MAPLE in the mathematics curriculum is a goal which is pursued vigorously in our department. It is a powerful tool and its judicious use in teaching elementary and/or advance calculus, differential equations, linear algebra and numerical analysis can greatly enhance the learning and teaching process. Although students should be exposed to a procedural language like C, this endeavour should not detract from the fact that they should also be exposed to additional software such as spreadsheets, statistical systems, and symbolic computation software. By exploiting those features which cannot be found in calculators or are very tedious to program, this type of software allows students to explore and see situations which could not be dealt with otherwise. Furthermore, the availability of the software on personal computers, coupled with their low price, are definite advantages, and should be taken into account. The author of this paper found that MAPLE greatly enhanced the delivery of a first course in numerical analysis, without adding any burden on the students. He also found that the availability of many colleagues with a strong knowledge of MAPLE favored the adoption of this software. Finally, the author is also exploring the possibility of exploiting the statistics features of MAPLE in the Statistics course offered to engineering students.

References

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