PROBLEM SOLVING IN TWO DIMENSIONS

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ABSTRACT

Incoming freshmen are notoriously deficient in problem solving skills. In our freshman problem solving course we are primarily concerned with enhancing students' skills in solving word problems, problems which yield to logical analysis, and finally problems requiring a good deal of imagination for their solution.

An excellent collection of word problems requiring only algebra in their solution can be found in Mildred Johnson's " How to Solve Word Problems in Algebra".

The second type of problem, those requiring logical analysis for their solution, is easily obtained from a plethora of books on puzzles.

Certain types of puzzles lend themselves to be used as problems of the third type. But a really challenging problem can be design in Abbott's "Flatland". Abbott postulates a two dimensional world and in less than 100 pages describes this world, its inhabitants, some of its history and workings. Much is left unsaid and leaves room for postulating problems to be solved. Some examples are transportation, utilities, written communications, etc. These topics lend themselves to team based term projects.

COURSE OBJECTIVES

Incoming college freshmen are notoriously deficient in problem solving skills. They are compulsive memorizers and users of formulas and of teacher-certified algorithms. When confronted with a new type of problem, they tend to use a seemingly related algorithm without ever wondering if their solution makes sense. So, for example, in a pretest the problem statement,

> Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors

elicits as an answer “P=6S” as often as “S=6P”.

Another pretest problem,

> A can do a certain job in 3 days. The same job takes B 6 days to do.
> If they work together, how long before the job is done?

gets the average, 4-1/2 days, as the answer in the majority of cases.

The purpose of our freshman problem solving course is to convert these beginning engineering students, so utterly devoid of fundamental problem solving skills, and so unaccustomed to using common sense in connection with school subjects into efficient, logical, and creative problem solvers.

The skills we wish our student to have developed by the end of the course are:

1. The ability to transform a verbal problem statement into appropriate equations and/or algorithms;

2. The use of logic in problem solutions which do not involve equations; and

3. The resort to the imagination and to creative thinking in the solution of open-ended problems.

ALGEBRA WORD PROBLEMS

The first of these skills is developed by the use of algebra word problems, preferably tied to basic physics and chemistry. An excellent source for one-step problems of this type is Mildred Johnson’s “How to Solve Word Problems in Algebra” [1]. The book has chapters on time-speed-distance, mixtures, levers, work, finance, etc. Unfortunately the book is not suitable as a textbook since it works out the algorithm for each type of problem and has complete solutions to all assignable problems. Nor does Johnson go far beyond the single step problem. The expansion to multi-step problems is however imperative and must be supplied by the instructor. An example of this type of expansion is shown in Appendix A.

LOGIC PROBLEMS

To develop the use of logic as a problem solving skill we resort to puzzles and underspecified problems. Examples of the former can be found in a plethora of puzzle books [2-8]. Problems number 1,3,5, & 6...in the Problem Set attached as Appendix B are examples of these.

As an example of the second type of problem, consider the following coin problem:

> I have 9 coins, consisting of pennies, nickels, dimes, and quarters. There
> are more pennies than any one other category, and there are fewer nickels

than quarters. How many of each coin are there? Trial and error solutions are not accepted. Use logical analysis. Explain your reasoning.

PROBLEM SOLVING IN TWO DIMENSIONS

We stimulate the creative thinking processes in our students with word-position puzzles of the type shown in Appendix C, play dice games of the type described in Appendix D, where the instructor announces the score and the students have to use inductive logic to arrive at the rules of the game. And we end up with a design project in "FLATLAND"[9]. Abbot postulates a two dimensional world populated by two dimensional beings. The book's main character and narrator is a "Square" who describes the inhabitants, their world and ways, and ends up, after a visit by a sphere , speculating on higher dimensional worlds. Written in fewer than 100 pages, "Flatland" leaves much to the imagination and therefore is a rich lode of imaginative design problems in such areas as transportation, communication systems, etc. The two most overwhelming design restrictions are that everything in FLATLAND looks like a line to a Flatlander, and there is no way of crossing anything or getting over anything. So there can be no bridges, no underground continuous supply lines or wires, etc.

COURSE LOGISTICS

The course is taught on a cooperative learning basis. Students are assigned to teams of 3 or 4 and are responsible for each other's learning. In a typical classroom problem, the first team arriving at a solution receives recitation credit. However, as soon as the team announces that it has solved the problem, the instructor selects randomly a team member to present the solution. Only if that team member can present the solution clearly and answer questions does the team get credit.

We use the design project also to improve the students' presentation skills. Each team must present its design project solution and each team member must participate in the presentation.

Although the primary objective of the course is to improve and extend the students' problem solving skills, one ancillary but invaluable outcome is the creation of a small learning community the members of which tend to continue supporting each other in subsequent semesters, and thereby improve their chances of success.

Ideally, one would prefer to use a summer bridge program to ease the transition from high school to college. In an urban environment where high school graduates can find summer employment relatively easily, and with financial aid starting only in the fall, bridge programs at the University of Houston cannot reach those students most in need of them. Our program for disadvantaged students (PROMES- PROgram for Mastery in Engineering Studies) therefore built the bridge program concept into the freshman year. In the fall semester we teach this problem solving course and a student development course based on Ray Landis's book "Studying Engineering : A Road Map to a Rewarding Career" [10] In the spring, these courses are followed by a one credit hour laboratory where the student has a choice of a single discipline or the smorgasbord approach. PROMES students are also counseled into collaborative workshops in all required math and science courses.

Since PROMES students participate in the whole package, the effects of an individual course cannot be separated out. Freshman retention in PROMES, however, is very good and has been as high as 98%. Overall retention for those students normally considered at risk has been commensurate with that of the college as a whole. Before and after tests in the problem solving course, however, document the students intellectual growth. Moreover, the time-rate-distance problems, and problems dealing with moments, mixtures, and per unit concepts are good preparation for the first courses in physics and chemistry. Student ratings in the course are invariably high and always significantly higher than those for other courses at that level.

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BIOGRAPHICAL SKETCH

Gerhard F. Paskusz

Dr. Paskusz received his B.S. and Ph.D. in Engineering at UCLA in 1949 and 1961 respectively. He was a full-time member of the instructional staff of UCLA from 1952 to 1961 and has been on the Electrical Engineering faculty at the University of Houston since then. He is the author of numerous journal articles and seven books, including the tradition breaking Linear Circuit Analysis, published by Prentice-Hall in 1964. In 1973 he founded and assumed the directorship of PROMES, the very successful minority engineering program at the University of Houston. He has been directing yearly residential summer engineering programs for underrepresented high school students since 1973. He is interested in and experiments with teaching methods. He received the Reginald H. Jones Distinguished Service Award in 1995.

APPENDIX A

The single- step problem

There is a single track railroad line between Exville in the West and Wyeburg in the East. The two cities are 100 miles apart. Two trains enter the track at 2:00 p.m. from opposite ends. The train from Exville travels at 60 miles per hour, and the train from Wyeburg has a speed of 40 mph. When does the accident occur?

First escalation

There is a section of double track starting at Zeetown, which is located 59 miles East of Exville, and ending 1-1/2 miles East of Zeetown. All westbound trains are automatically switched to the double track. Assuming both trains to be single engines, will there be an accident?

Second escalation

Same problem, but now the train out of Wyeburg is a half-mile long freight train.

APPENDIX B

PROBLEM SET

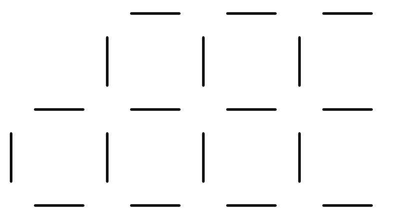
1. A bricklayer has 8 bricks. Seven of the bricks weigh the same amount and 1 is a little heavier than the others. If the man has a balance scale, how can he find the heaviest brick in only 2 weightings?

2. It was cold and they sat in front of the fireplace. The logs were burning. "I thought you put both logs on at the same time", she said, " but look, one of them has still twice as much left as the other." "No wonder," he replied, "one is a six hour log, the other one is only good for four hours." How long have the logs been burning?

3. Two squares are removed from opposite corners of a checkerboard leaving 62 squares. Can the checkerboard be filled with 31 dominoes, each domino covering two adjacent squares?

4. Find a way to get 100 using the least number of 4's and the standard arithmetic symbols: + - x ( ) [] ! ÷ √

5. Consider the following figure made of matches. Remove 3 matches to get exactly 5 squares with no loose matches sticking out.



6. You have 5 pieces of chain 3 links each. Find a way to get one 15 link chain with only 3 cuts and welds.

7. Write a true equation that uses all ten digits exactly once. Nothing else may be used in the equation except the equal sign (i.e., no signs of operation such as +, -, x, ÷, •, !, etc.).

8. Alonzo, Bat, Cid, and Elmo were alone in a room where a jar of peanut butter was broken. Having dealt with the gentlemen before, I recalled that three of them were liars and one was a truar; but I didn't recall which. These were the statements I received on entering the room.

Bat:	Elmo broke the jar.
Elmo:	Alonzo broke it.
Cid:	I didn't break it.
Alonzo:	Elmo lied when he said I broke it.

   Who is what and who broke the jar?

APPENDIX C

WARDS/LONG

SPAPARKCE

NOONGOOD

SIJOBDE

0134 ALIKE

ITS
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HEAD
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N

HNOECHALNCEL

APPENDIX D

DICE GAME

1. <u>Double Jeopardy I</u>

   Only one of the two counts.

2. <u>Double Jeopardy II</u>

   Every double halves the total score

3. <u>Straight Shooter</u>

   Only sequences of 4 or more count.