Dynamic Otto Cycle Analysis

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Abstract

Engineering students encounter the Otto cycle in their first course in thermodynamics (usually during the sophomore year). This cycle is the theoretical basis for the spark ignition (SI) internal combustion engine (ICE). The traditional analysis (the air-standard analysis) of the Otto cycle is a static thermodynamic analysis that cannot be used to predict the dynamic performance of a SI ICE. Given sufficient information, the work per cycle for a particular engine can be computed. However, by making three simple modifications, the air-standard analysis can be extended to include a computation of the dynamic performance of a SI ICE. The first of these modifications is the selection of representative values of specific heats and specific heat ratios for the working fluid during each process. This improves the accuracy of the analysis. The second is an equation relating the heat release during combustion to pertinent engine parameters (the fuel-air ratio and the compression ratio). The third is the inclusion of an equation for the volumetric efficiency of the engine as a function of engine speed. This incorporates into the analysis the single most significant loss and results in performance that is dependent on engine speed. The resulting analysis predicts the dynamic performance (power and torque as a function of engine speed) of contemporary SI ICE engines with reasonable accuracy. Most importantly, this analysis can be easily understood and conducted by engineering students in their first thermodynamics course. Students have used this analysis, with excellent results, to analyze typical engines for a variety of applications (various types of passenger cars, pick-up trucks, SUV's, Formula 1 vehicles and, even, "monster" trucks).

Background

The engine used for most contemporary motor vehicles is the four-stroke spark-ignition (SI) internal combustion engine (ICE). The engine typically has 4, 6 or 8 cylinders. The SI ICE combines non-flow and semiflow thermodynamic processes. The four strokes, which occur for each cylinder over two revolutions of the engine's crankshaft, are the intake stroke, the compression stroke, the power (expansion) stroke and the exhaust stroke. Combustion of fuel and air occurs as the compression stroke ends and the power stroke begins. These processes and their thermodynamic modeling are discussed in detail in books on thermodynamics[1] and internal combustion engines[2].

The theoretical thermodynamic model for the SI ICE is the Otto cycle. The Otto cycle is shown on pressure-volume coordinates in Figure 1. It is a stationary, closed thermodynamic cycle consisting of the following four internally reversible processes: isentropic compression (1-2), constant volume heat addition (2-3), isentropic expansion (3-4) and constant volume heat rejection (4-1). The *idealized* Otto cycle includes the following five assumptions (referred to as the “cold-air-standard” assumptions[1]) that are made to simplify the analysis:

1. the working fluid is air (at ambient temperature and pressure at state 1),
2. air behaves as an ideal gas,
3. air has constant specific heats, determined at 25°C,
4. the combustion process is replaced by external heating and
5. the exhaust/intake processes are replaced by external cooling.

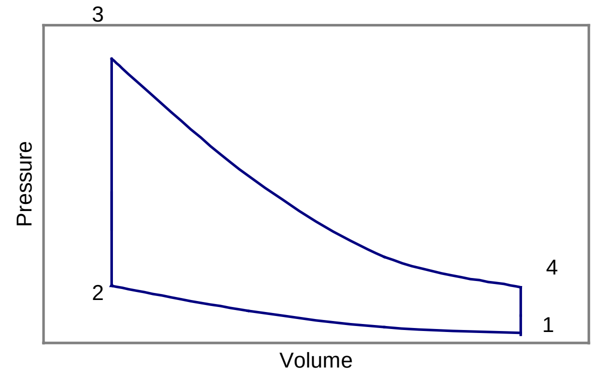

Figure 1. The Otto Cycle P-V Diagram

The net work produced by the idealized Otto cycle can be computed through a First Law of Thermodynamics analysis. The mass of air contained in a cylinder of an engine modeled by the idealized Otto cycle is a fixed value (independent of engine speed), dependent only on engine geometry and the ambient temperature and pressure. As a consequence, the work done per cycle is a constant and the power output of such an engine varies linearly with the engine speed (i.e., the number of cycles per second).

The Dynamic Otto Cycle Analysis

The goal of this study was to develop a simple First Law of Thermodynamics analysis that would predict, with reasonable accuracy, performance curves (power vs. engine speed and torque vs. engine speed) for contemporary automotive engines. The dynamic Otto cycle analysis developed in this study uses the assumptions of the idealized Otto cycle described above, with just two exceptions. First, and most importantly, the mass of air in the cylinder is dependent on the engine speed as well as engine geometry and ambient temperature and pressure. Second, the specific heats used in the analysis are assumed constant for each process but their numerical values are determined at the approximate mean temperatures for each process. In addition, the quantity of heat transferred during the heating process is related to engine parameters.

The power output of a four-stroke SI ICE is

$$\dot{W} = N_c \left(N/2\right) W_{net,c} \qquad (1)$$

where
$\dot{W}$ = the power output of engine, kW,
N_c = the number of cylinders in the engine,
N = the engine speed (crankshaft rotations per second), Hz, and
$W_{net,c}$ = the net work produced by one cylinder during two revolutions of the crankshaft (i.e., for one power stroke), kJ .

The net work per cylinder, $W_{net,c}$, is determined using a First Law of Thermodynamics analysis of the modified Otto cycle (as described above). The result is

$$W_{net,c} = -m_c c_{v,e}(T_4 - T_3) - m_c c_{v,c}(T_2 - T_1) \quad (2)$$

where m_c = the mass of air in the cylinder, kg,
T_i = the temperature of the air at each of the four terminal states, respectively, of the cycle (see Figure 1.), K,
$c_{v,e}$ = the mean specific heat at constant volume for the air during the expansion stroke (process 3-4), kJ/kg-K, and
$c_{v,c}$ = the mean specific heat at constant volume for the air during the compression stroke (process 1-2). KJ/kg-K.

The air temperature at state 1, T_1, is the ambient temperature and is known. T_2, T_3 and T_4 in Eq. (2) are determined using the following three equations:

for the isentropic compression process (1-2), $$\frac{T_2}{T_1} = r^{k_c - 1} \quad (3)$$

for the constant volume heating process (2-3), $$Q_H = m_c c_{v,h}(T_3 - T_2) \quad (4)$$

and for the isentropic expansion process (3-4), $$\frac{T_4}{T_3} = r^{k_e - 1} \quad (5)$$

where r = the compression ratio for the engine = V_1/V_2 ,
V_1 = the total volume of the cylinder, m^3,
V_2 = cylinder clearance volume, m^3,
k_c = mean ratio of specific heats, c_p/c_v, for air during the isentropic compression process,
k_e = mean ratio of specific heats, c_p/c_v, for air during the isentropic expansion process,
$c_{v,h}$= the mean specific heat at constant volume for the air during the constant volume heating process (process 2-3), kJ/kg-K, and
and Q_H = the quantity of heat added per cylinder during process 2-3 (and which must be known), kJ.

The recommended mean values of c_v and k for the processes of the dynamic Otto cycle are presented in Table 1. The quantity of heat added to the air in the cylinder during process 2-3 is

$$Q_H = m_c q_r \quad (6)$$

where q_r = the heat per unit mass of air added during during process 2-3, kJ/kg.

In the SI ICE, q_r results from the combustion of gasoline and air. A parametric study of the constant volume combustion of air and octane was conducted, using software[3] that simultaneously solves the First and Second Laws of Thermodynamics, to develop an approximate relationship for q_r in terms of pertinent engine parameters. The resulting equation is

Table 1. Representative Property Values[1]

Property	Temperature
$c_{v,c}$ = 0.736 kJ/kg-K	470 K
k_c = 1.39	470 K
$c_{v,h}$ = 0.800 kJ/kg-K	750 K
$c_{v,e}$ = 0.820 kJ/kg-K	840 K

$$q_r = 1700[(r-1)/r](f/0.0665)^{1/3} \tag{7}$$

where f = the fuel-air ratio, kg fuel/kg air.

The chemically correct value of the fuel-air ratio is f = 0.0665 and its practical range is

$$0.0600 < f < 0.0700$$

where the lower limit corresponds to a very fuel lean mixture ratio and the upper limit corresponds to a very rich mixture ratio. Lean mixture ratios (f < 0.0665) tend to result in better fuel economy and more oxides of nitrogen in the exhaust gases while rich mixture ratios (f > 0.0665) tend to result in greater power output and more carbon monoxide and hydrocarbons in the exhaust gases.

The power output of an actual SI ICE increases as engine speed increases, reaching a maximum and then decreasing as engine speed continues to increase (Figure 2). The deviation from the linear relationship (described above) for the idealized Otto cycle is primarily a consequence of the engine's inability to efficiently pump fresh air into the cylinder as the piston speed (engine speed) increases. This phenomenon can be incorporated into the Otto cycle analysis through the introduction of the volumetric efficiency of the engine[2]. For this analysis, the volumetric efficiency, e, is defined as

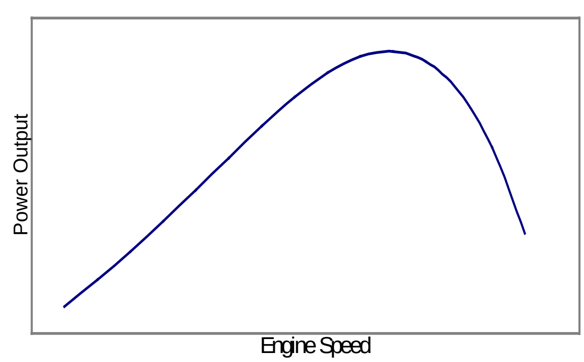

Figure 2. Power Output vs. Engine Speed

$$e = m_c / m_{ca} \tag{8}$$

where m_c = the actual mass of air contained in one cylinder, kg, and
m_{ca} = mass of air, at ambient temperature and pressure, contained in one cylinder, kg.

The mass of air at ambient temperature and pressure contained in one cylinder is

$$m_{ca} = \rho_\alpha V_1 \tag{9}$$

where ρ_a = the density of air at the ambient temperature, T_1, and pressure, P_1, (computed using the equation of state for an ideal gas), kg/m^3.

An appropriate relationship for the volumetric efficiency of engines for contemporary family-sized automobiles, found by studying the performance of this class of automobile, is

$$e = -7.67\times10^{-7} N^3 + 9.651\times10^{-5} N^2 - 1.719\times10^{-3} N + 0.897 \tag{10}$$

where $0 < N < 150$ Hz.

Equation (5) applies to engines that are neither supercharged nor turbocharged and that have their maximum power output occurring at 6600 rpm (110 Hz). This equation can be generalized to allow for turbocharging, supercharging and higher or lower operational speeds (characteristic of other classes of motor vehicles) as follows,

$$e = F_b(-7.67\times10^{-7} N_p^3 + 9.651\times10^{-5} N_p^2 - 1.719\times10^{-3} N_p + 0.897) \tag{11}$$

where $F_b = P_i/P_a$ = turbocharging boost factor,
P_a = ambient pressure, kPa,
P_i = engine inlet pressure after turbo/supercharging, kPa,
$N_p = 110\ (N/N_{max})$ = engine speed parameter, Hz, ($0 < N_p < 150$ Hz) and
N_{max} = engine speed at which maximum power output occurs, Hz

Combining Equations (8) and (9), the mass of air contained in one cylinder is

$$m_c = e\rho_\alpha V_1 \tag{12}$$

The volume of one cylinder, V_1, can be expressed in terms of engine parameters as follows,

$$V_1 = (\pi B^2/4)S\left[r/(r-1)\right] \tag{13}$$

where B = the cylinder bore (diameter), m, and
S = the piston stroke, m.

The displacement volume of one cylinder can be expressed as

$$V_{d,c} = V_1 - V_2 = V_1(r-1)/r \tag{14}$$

The displacement volume of the engine is

$$V_d = N_c V_{d,c} \ . \tag{15}$$

The engine torque is[2]

$$\tau = W / (2\pi N) \tag{16}$$

where τ = engine torque, m-N.

Analytical Procedure

For the purpose of this analysis, a unique engine design is defined by specifying the ambient temperature and pressure, T_1 and P_1; the bore, B, and stroke, S; the number of cylinders, N_c; the compression ratio, r; and the fuel-air ratio, f. The power output of the engine as a function of engine speed is computed using Eq. (1) where the net work produced by one cylinder is given by Eq. (2). The temperatures in Eq. (2) are computed using Eqs. (3), (4) and (5). The mass of air in one cylinder, as a function of engine speed, is computed using Eqs. (12), (10) or (11), (13) and the equation of state for an ideal gas. The engine displacement volume is computed using Eqs. (13), (14) and (15). The torque as a function of engine speed is computed using Eq. (16). A spreadsheet analysis is the best method for computing the desired results.

Table 2. Comparison of Manufacturers' and Computed Results

Vehicle	Nissan Maxima SE[4]	Homda Accord LX[5]	Mazda 626 LX[5]	Toyota Camry LE[5]
Model Year	2000	1998	1998	1997
B (m)	0.0930	0.0860	0.0845	0.0874
S (m)	0.0733	0.0860	0.0742	0.0831
N_c	6	6	6	6
r	10	9.4	9.5	10.5
f	0.0685	0.0685	0.0685	0.0685
Power[a]	222 hp at 6400 rpm	200 hp at 5500 rpm	170 hp at 6000 rpm	192 hp at 5200 rpm
Torque[a]	217 lbf-ft at 4000 rpm	195 lbf-ft at 4700 rpm	163 lbf-ft at 5000 rpm	207 lbf-ft at 4400 rpm
Power[b]	220 hp at 6400 rpm	203 hp at 5500 rpm	178 hp at 6000 rpm	200 hp at 5200 rpm
Torque[b]	200 lbf-ft at 4000 rpm	198 lbf-ft at 4700 rpm	165 lbf-ft at 5000 rpm	205 lbf-ft at 4400 rpm

a Manufacturers data[4,5]
b Prediction using the Modified Otto Cycle analysis.

The specifications for the engines of four contemporary family-sized automobiles are presented in Table 2 along with the manufacturers' performance data at specific engine speeds. An ambient temperature and pressure of 298.2 K and 101.3 kPa, respectively, were used for each case. The dynamic Otto cycle analysis, presented above, was used to analyze these engines and the predicted performance at the specified engine speeds is also shown in Table 2. The predicted performance agrees with the manufacturers' stated performance within 10%. Detailed results for the Nissan Maxima SE engine are presented in Tables 3 and 4 and Figures 3 and 4. The air temperatures at each thermodynamic state are shown in Table 3. The volumetric efficiency, power output and torque are presented as a function of engine speed in Table 4. The power output and torque are shown graphically as a function of engine speed in Figures 3 and 4 respectively.

Table 3. Temperatures Nissan Maxima

Point	T (K)
1	298.2
2	731.9
3	2663.4
4	1189.7

Table 4. Predicted Engine Performance Data for the Nissan Maxima

N (Hz)	N (rpm)	e	Power (kW)	Power (hp)	Torque (m-N)	Torque) (lbf-ft)
10	600	0.889	16	21	247	182
20	1200	0.895	31	42	249	183
30	1800	0.912	48	64	254	186
40	2400	0.934	65	88	260	191
50	3000	0.956	84	112	266	196
60	3600	0.976	102	137	271	200
70	4200	0.986	121	162	274	202
80	4800	0.984	138	185	274	201
90	5400	0.965	152	204	268	197
100	6000	0.923	161	216	257	189
110	6600	0.855	164	220	238	175
120	7200	0.755	158	212	210	154
130	7800	0.619	141	189	172	127
140	8400	0.443	108	145	123	91
150	9000	0.222	58	78	62	45

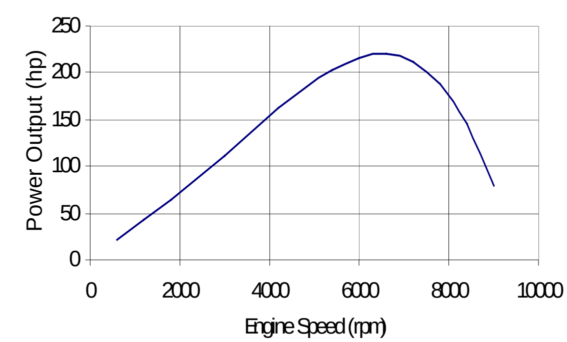

Figure 3. Horsepower vs. Engine Speed for the Nissan Maxima

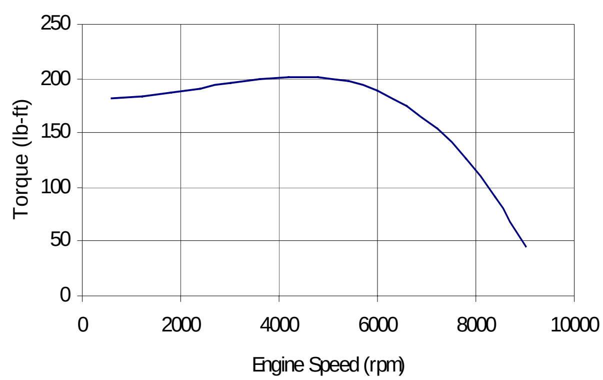

Figure 4. Torque vs. Engine Speed for the Nissan Maxima

Recapitulation

The cold air standard Otto cycle analysis was modified so that performance curves for a particular SI ICE could be determined with reasonable accuracy. This analysis allows students to investigate the effect of varying engine parameters on the performance of the SI ICE. The analysis can also be used for the preliminary design of a SI ICE, which is required to produce a particular power output at a specified engine speed. This is an open-ended problem and a number of design decisions must be made.

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