

## **Matlab Simulation Projects for a First Course in Linear Control Systems**

**Richard T. O'Brien, Jr.  
The United States Naval Academy**

### **Abstract**

Computer simulation is a vital part of a first course in linear control systems. At the United States Naval Academy, MATLAB simulation projects are used to reinforce the lecture material and present “real-world” design projects at an introductory level. In this paper, a series of three projects on the design of an automated steering controller are presented. The projects introduce the students to the “design process” and require the students to address open-ended problems. Furthermore, the students are generally intrigued by the idea of vehicle automation and do not require extensive motivation.

In the first project, the students investigated a simplified model of automated steering control system. The project emphasizes 2<sup>nd</sup> order system response and introduces the design process. Specifically, the students must compromise between competing objectives in selecting a proportional controller gain. The next project introduces a measurement device (an accelerometer). In this project, the students revisit 2<sup>nd</sup> order system response and investigate the effect of the accelerometer response on performance and stability. In the final project, a more realistic vehicle model is introduced and the students apply root locus techniques to the design of an automated steering controller.

### **I. Introduction**

The availability of computer simulation tools such as MATLAB has created both opportunities and challenges in engineering education. These tools allow the students to simulate, analyze, and design engineering systems quickly and easily, but the underlying concepts can be obscured. In this paper, a series of design projects for a first course in linear control systems are presented. In these projects, MATLAB is used to reinforce the course material and introduce the students open-ended design problems.

The problem of designing an automated steering controller is investigated in three design projects. Initially, the control problem is considered in a simplified form. As the semester

progresses, more realistic elements are added to the problem. In the first project, the students investigate a simplified model of automated steering control system. The project emphasizes 2<sup>nd</sup> order system response and introduces the students to engineering design. Specifically, the students must compromise between competing objectives in selecting a proportional controller gain. The next project focuses on the feedback process and measurement devices. In this project, the students investigate the effects of an accelerometer on the stability and performance of the automated steering control system. In the final project, the automated steering control problem is considered with accelerometer feedback and a wind gust disturbance. Using the results of the first two design projects, the students apply root locus techniques to the design of an automated steering controller.

The remainder of the paper is organized as follows. The automated steering problem is described in Section 2. In Section 3, the three design projects related to this problem are described and educational benefits are discussed. In Section 4, some conclusions are drawn from this experience.

## II. Automated steering control problem

One of the basic problems in automated steering control is a *lane change maneuver* as shown in Figure 1.

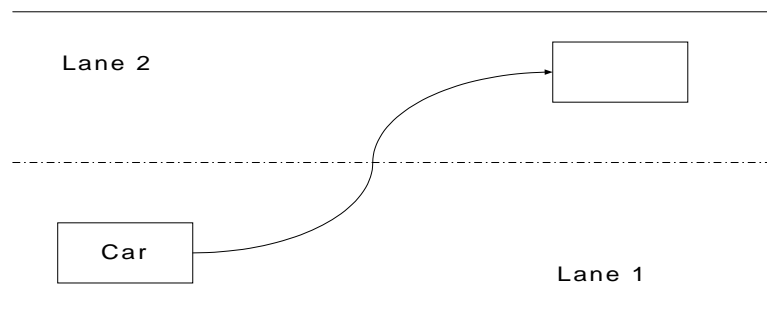


Figure 1: Lane change maneuver

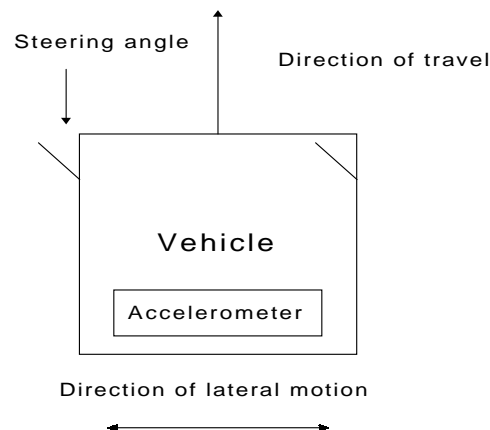


Figure 2: Vehicle with accelerometer

The steering input controls the lateral motion of the vehicle (see Figure 1). The automated vehicle steering control system uses information about the vehicle position relative to the center of the current lane to determine the steering wheel angle. A lateral force on the vehicle (and, hence, a lateral acceleration) is created as the wheels turn. The automated steering controller is designed to steer the vehicle from the center of the current lane to the center of an adjacent lane. Measurements of the vehicle's lateral position during the maneuver will be computed from the lateral acceleration measured by the accelerometer (see Figure 2).

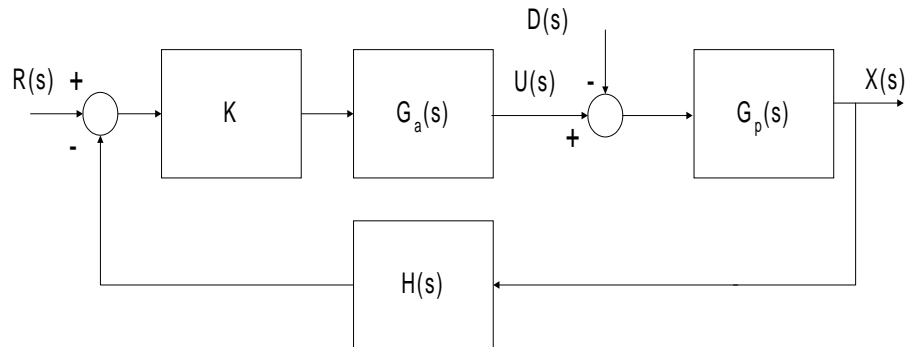


Figure 3: Closed loop steering control system

The closed loop steering control system with accelerometer feedback is shown in Figure 3. The vehicle has a transfer function  $G_p(s) = \frac{0.1}{s(s+1)}$ , the steering actuator has a transfer function

$G_a(s) = \frac{10}{s+5}$ , and the accelerometer has a transfer function  $H(s) = \frac{100}{s^2 + 20s + 100}$ . The signals in the Figure 2 are

- $x(t)$ : lateral position (units: lanes)
- $e(t)$ : lateral position error (units: lanes)
- $r(t)$ : desired lateral position (units: lanes)
- $u(t)$ : steering angle (units: degrees)
- $d(t)$ : wind gust disturbance (units: degrees)

The transfer functions are not based on realistic vehicle data but, they do provide a reasonable response.

The objective of the design project is to design an automated steering control system; that is, to choose a value for the control system gain  $K$ . The selection of  $K$  is based on the vehicle's motion during a lane change maneuver and on the effect of a lateral wind gust disturbance,  $d(t)$ . Note that there is a minus sign in the summing junction where  $d(t)$  enters because it is assumed to be acting against the motion of the car.

The specifications for the steering control design are that the vehicle completes the lane change maneuver quickly and safely without causing the passengers discomfort. From a systems engineering point of view, these specifications require that the step response of the vehicle's lateral position has a small rise and/or settling time and minimal overshoot. Furthermore, the comfort of the passengers is closely related to the lateral acceleration during the lane change maneuver. Specifically, passenger comfort requires that the lateral acceleration is small. Equivalently, it can be shown that the lateral acceleration is proportional to the steering input and, therefore, passenger comfort requires that the steering input is small.

The wind gust disturbance introduces a steady-state error which must be considered in the control system design. To understand the impact of the wind gust, recall that the steering input causes a lateral force on the vehicle. The wind gust disturbance creates a lateral acceleration acting against the motion of the vehicle and reduces the effect of the steering input.

The specifications on the control system design can be divided into three categories

1. Safety: The closed loop system must have less than 10 % overshoot in the unit step response.
2. Passenger comfort: The maximum steering input must be less than 4 degrees.
3. Disturbance rejection: The steady-state error for a unit disturbance must be minimized.

This students will attempt the control system design in the third design project after obtaining preliminary results in the first two design projects.

### III. Design Projects

The problem of designing an automated steering controller is investigated in three design projects. The projects are intended to enhance the students understanding of the course material. To accomplish this end, the automated steering control problem is simplified initially and, as the semester progresses, more complexity is added to the problem.

#### Design Project # 1

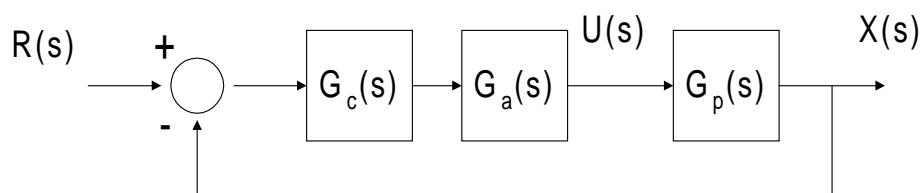


Figure 4: Closed loop block diagram for design project # 1

The first project reinforces the classroom material on 2<sup>nd</sup> order systems and provides an introduction to proportional feedback control. In this case, the disturbance is not included, ideal feedback of the lateral position is assumed, and the actuator dynamics are neglected (i.e.  $G_a(s) = 2$ ). The resulting closed loop system is a standard 2<sup>nd</sup> order system and is shown in Figure 4. In this project, no performance specifications are given. Instead, the students must explore a variety of closed loop responses and determine the best response.

In the project, the students are asked to examine the closed loop step response (lane change response) for various values of the proportional control gain  $K$  through MATLAB simulations and by calculating the damping ratio and undamped natural frequency as a function of  $K$ . The objective is for the students to make a connection between the response of the vehicle during the lane change maneuver and the theoretical results on the step response of a standard 2<sup>nd</sup> order system.

The project introduces the concept of proportional feedback control and provides an example of a basic engineering design. Although this problem has been simplified, there are two aspects that are present in an design problem: the system must be modeled and simulated to understand and characterize the system's response; and the selection of control system parameters requires a trade-off between competing specifications or objectives. Specifically, the response becomes faster as the gain increases, but the overshoot (i.e. the potential for interference with vehicles in other lanes) and the steering input increase as well. The students must balance these competing objectives in their selection of the gain.

### Design Project # 2

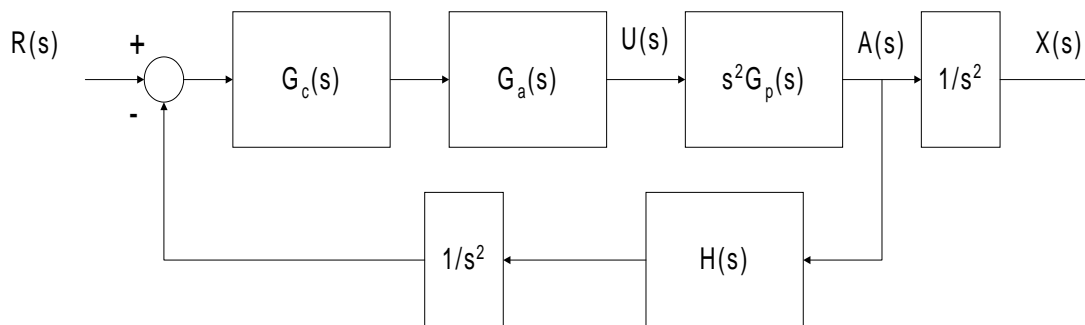


Figure 5: Closed loop steering control system with accelerometer

The second project builds upon the results on the first design project. In particular, the ideal feedback (measurement of position) is replaced by accelerometer feedback. Using the control gain determined in design project # 1, the effect of the accelerometer is investigated by observing the response of the closed loop system. The closed loop system is shown in Figure 5. To analyze this closed loop system, the system is redrawn in a more convenient form (Figure 6) using block diagram algebra. This exercise provides motivation for block diagram algebra.

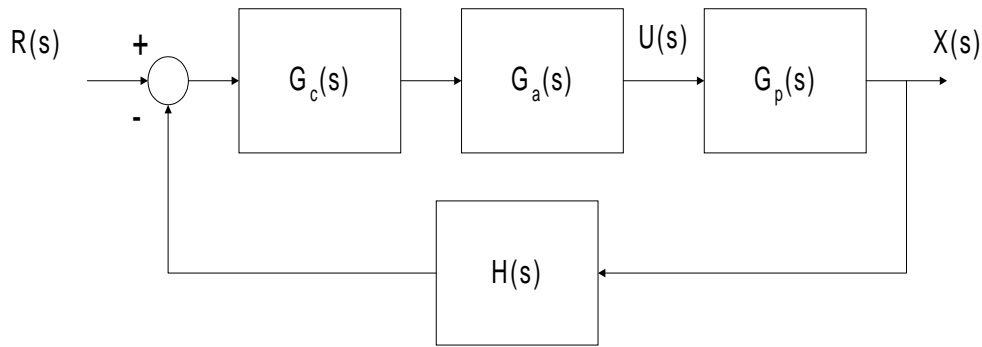


Figure 6: Reconfigured closed loop system with accelerometer feedback

The accelerometer is modeled as a mass-spring-damper system and provides another example of a 2<sup>nd</sup> order system. The closed loop system is simulated for several sets of accelerometer dynamics. These simulations demonstrate that a fast accelerometer provides the best results because it most closely matches the ideal case. In this simplified settling, these simulation may lead the students to believe that the accelerometer response can be made arbitrarily fast. It is important to discuss the limitations on the accelerometer parameters such as size, cost, and availability as well as the propagation of measurement noise into the steering and lateral position response. The later situation can cause discomfort to the passengers and excite unmodeled dynamics.

### Design Project # 3

This design project builds upon the results on the first two design projects. The design of an automated steering controller is considered for the scenario presented in Figure 3. In this project, the analysis and design tools accumulated throughout the semester are applied to the problem. Specifically, the root locus techniques are used to determine the desired location of the dominant closed loop poles. The specification on maximum steering angle is converted into a more useful specification on time-to-peak using MATLAB simulations to determine a least squares parabolic fit.

The students are asked to sketch the root locus by hand and to draw some preliminary conclusions from this sketch. This exercise demonstrates that useful information can be obtained from the root locus sketch without resorting to computer simulation. Specifically, the root locus sketch shows that the dominant poles move towards the imaginary axis and, eventually, into the right-half plane as  $K$  increases. As a result, the specifications on the transient response and steady-state error impose conflicting requirements on the value of the control gain  $K$ . In particular, the designer must trade higher overshoot and larger maximum steering angle for reduced steady-state error.

## IV. Conclusions

In the three design projects, the complex problem of designing an automated steering control system has been investigated. By examining the design in several stages, the students can gain

insight into the various aspects of the problem without being overwhelmed. Originally, a simplified version of the problem was considered in order to determine the relationship between the control gain and the closed loop response. Next, the effect of an accelerometer on the closed loop response was considered. Finally, using the results of these design projects and root locus techniques, it was possible to design a control system for the more complex problem including accelerometer feedback, actuator dynamics, and a wind gust disturbance.

The projects serve to reinforce classroom material on 2<sup>nd</sup> order systems, feedback control, proportional control design, and root locus techniques.

#### **RICHARD T. O'BRIEN, JR.**

Rich O'Brien is an Assistant Professor in the Systems Engineering Department of The United States Naval Academy. He received his Bachelor of Science in Engineering from Brown University in 1991 and his Masters of Science in Engineering (1994) and Ph.D. in Electrical and Computer Engineering from The Johns Hopkins University (1998). At the Naval Academy, Dr. O'Brien teaches courses in modeling, simulation, analysis, and design linear control systems. He is currently pursuing research into automated steering control for passenger vehicles and tractor-trailers.