Self-Aligning Mechanisms, Forgotten Part of ME Curriculum

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Abstract

Mechanical engineering students designing machinery are confronted with the lack of a reliable
method in determining if the machinery will move after assembly, and under what conditions
assembly is possible at all. Gruebler' and Chebyshev's formulas found in the majority of
American textbooks are unreliable. A simple, though almost unknown, loop analysis developed
by Ozol can solve the problem.
The loop analysis allows one not only to check if the mechanism can move, but also provides a
valuable insight into the design of self-aligning mechanisms insensitive to manufacturing and
assembly errors.

Introduction

One of the most important tasks in designing a mechanism is checking if the proposed device
constitutes a mechanism and not a rigid structure. In the language of mechanical engineers, the
procedure is called *checking the mobility of the mechanism*.
The mobility of the mechanism is defined as the number of degrees of freedom that the
mechanism possesses with respect to one arbitrarily chosen link. One can determine the
mobility of the mechanism by "fixing" links one by one, until the mechanism is not able to
move. The number of fixed links that immobilizes whole mechanism is equal to its mobility.
The Gruebler's and Kutzbach's formulas for the mobility of a plane mechanism and found in
majority of textbooks on kinematics, [1], [2], [3], although they are known to produce
misleading results.

The second task of the designer is to formulate geometric conditions (parallel axes, tight
tolerances on some dimensions, etc.) to make assembly possible. The geometric conditions
imposed in this stage on the links of the mechanism are also known as *the redundant
constraints*. An example of a system with redundant constraints is a four-leg table placed on a
warped floor. To avoid shaking of the table one of its legs must have a strictly defined length.

Unfortunately, parts are machined with errors. If errors are too large, the assembly may not be
possible, or would require force to connect its parts together. Reshetov [4], gives examples of
when bad geometry of links caused internal loads on the parts that exceeded many times the
loads for which the machine was designed. To visualize the problem, compare two sets of links
for two four-bar linkages shown in Fig 1. The set shown on the left can be easily assembled

because the axes of the holes in the links are made parallel to each other. The set shown on the right will resist assembly unless links are bent.
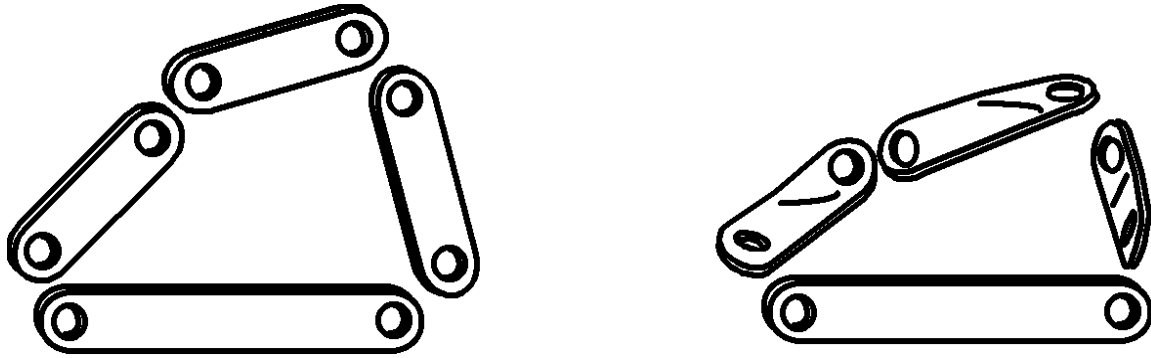

Figure 1 On the left, the links of a four-bar linkage with axes of all holes parallel to each other. On the right, the links with serious geometric errors.

The longer the list of geometric requirements for the mechanism , the more expensive it will be to make (cost of jigs, measuring equipment, etc.). Additionally, such a mechanism will be sensitive to thermal effects and deformations caused by the load. Binding is a frequent phenomenon in mechanisms that have redundant constraints.

It is possible to design mechanisms so that they are insensitive to geometric errors. Such mechanisms have no redundant constraints and are called self-aligning. A self-aligning four-bar linkage can be easily assembled even if its links are severely deformed (even as badly as those shown in Fig. 1b).

A simple procedure was developed by Ozol [4] to determine mobility and the number of the redundant constraints of any mechanism (two and three-dimensional) . This procedure can be used as a tool to design self-aligning mechanisms.

Kinematic Pairs (joints)

Mechanisms are composed of links connected together in a way that makes their relative motion possible. In the analysis that follows, the links will be considered as perfectly rigid bodies with perfect geometry (zero clearances in pin joints and so on). Because of this assumption, their geometry will not change even under extremely large loads.

A connection of two links that permits their relative motion is called a *kinematic pair* or a *joint*. There are many different types, or classes, of kinematic pairs (joints) used in machinery which ensure specific types of contact between the mating surfaces (point contact, line contact, etc.).

In the part that follows, a brief classification of kinematic pairs (joints) is presented. This knowledge is a prerequisite to understanding the procedures used in determining the mobility of a mechanism and the existence of the redundant constraints.

In a three-dimensional space, any link (rigid body) has six degrees of freedom: three translations and three rotations about the axes of a fixed system of coordinates. The motion of the link in space can be a simple translation or a simple rotation, or any simultaneous combination of these motions. When one link is connected to another link, it looses some of its degrees of freedom. To see the effect the connection has on the degrees of freedom, it is convenient to fix one link, and to study the motion of the other link. By making a connection between two links, we

impose geometric constraints on both links. To explain it better, let us consider the situation shown in Fig. 2.
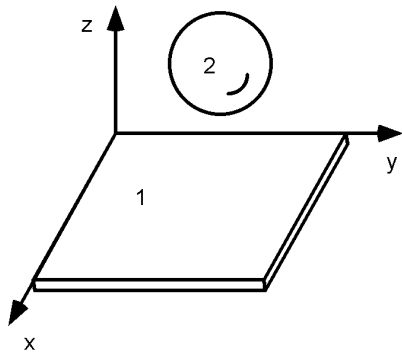

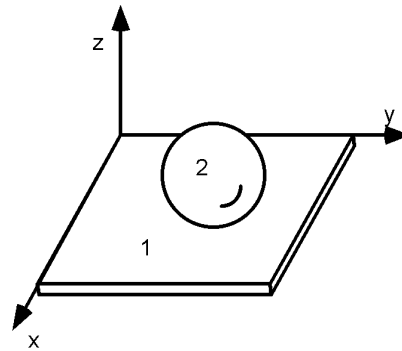**Figure 2** Free body (2) in space.

Figure 3 Kinematic pair of I class.

Link 1 has a perfectly flat surface and is considered as stationary. Link 2, having the shape of an ideal sphere, "floats" above. In this state of levitation, the sphere has all six degrees of freedom. Now, let's impose a constraint on the sphere so that it must touch link 1 (a ball resting on a plane in Fig.3). In this connection link 2 has now five degrees of freedom with respect to link 1. One degree of freedom – the translation in the direction of the normal to the plane-- is lost (constrained). Because one degree of freedom for the sphere was removed, the connection will be classified as a kinematic pair of the first class (class I). In the convention which will be adopted, the class of a kinematic pair is equal to the number of independent degrees of freedom removed from the movable link. The reader should be warned here that a different convention is used in works of other authors. In that convention the class of a kinematic pair corresponds to the number of the retained degrees of freedom.
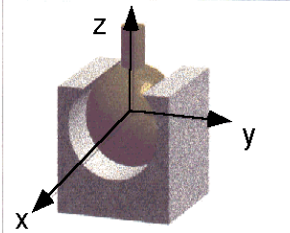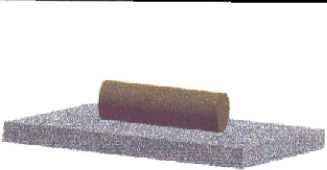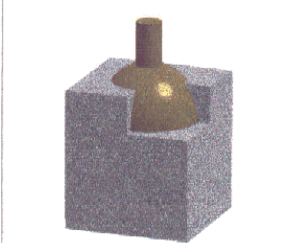According to the convention used in this paper, all possible joints create five classes only.
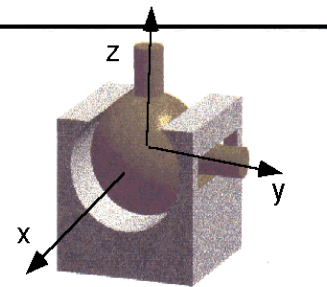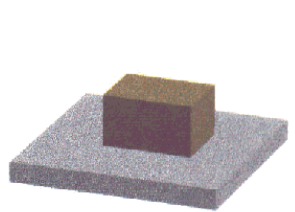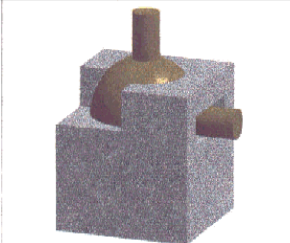
Classification of Kinematic Pairs

A *class I* joint was explained in the previous section, and is not restricted to a sphere and a plane. Any two bodies making contact at one point create a class I kinematic joint.
In a kinematic pair of class II, if one link is fixed, than the movable link must have two degrees of freedom removed. The possible combinations are two translations, two rotations, or a translation and a rotation. The second combination cannot be realized because any kinematic pair requires at least one point of contact between parts. The possible forms of the class II are shown in Table 1 : a sphere inside of a tight fitting round tube, and a cylinder resting on a plane. In the latter case some rotations of the cylinder are forbidden because they change the contact between the links from line to point.
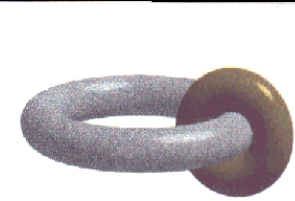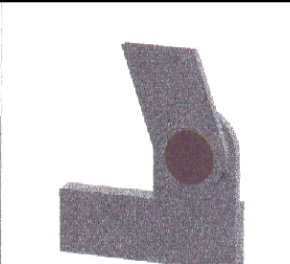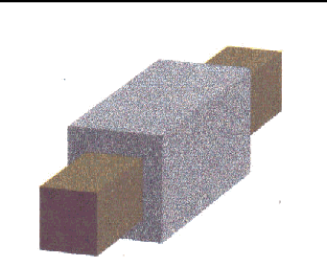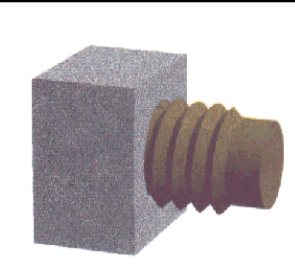
The class III joints are shown in the third row of Table 1. The possible combinations of constrained degrees of freedom are three translations (joint shown in the first column), two translations and one rotation (the joint shown in the second column), or one translation and two rotations (the joint shown in the third column). The examples of the joints satisfying the criteria for class III are shown in the table. The first form, which constraints all three translations, is the commonly used ball-and-socket connection. In the model representing the second form, the ball is constrained by a pin inserted through the slot machined in the fixed link.

Table 1 Classification of kinematic pairs

CLASS I			
CLASS II			
CLASS III			
CLASS IV			
CLASS V			

The slot constrains the rotation of the ball about the x-axis, but allows rotations about the z and y axes. The length of the slot does not have to be large. In many technical cases, a little amount of rotation or translation is considered to be a sufficient degree of freedom. For example, 0.5 degree rotation of the self-aligning commercial bearing is all that is needed to avoid the excessive stress due to deformation of the shaft caused by the load.

The class IV joints are characterized by four degrees of freedom lost by a movable link. There are three possible forms for this class, and all three are shown in the fourth row of Table 1.

While the first two forms are self-explanatory, the third one requires a comment. In this form, the link --considered as movable (a doughnut shaped ring)-- can spin about its own axis of symmetry, and can rotate about the vertical axis of symmetry of the fixed ring. Only two available degrees of freedom imply that four out of six degrees of freedom were removed, therefore the connection is of class IV. Technical applications of the IV class pairs include two teeth of mating spur gears, or a cam and roller of a follower.

The joints classified as class V connections must allow one degree of freedom in relative motion. Possible forms for this class are shown in the last row of Table 1. The first form is a typical hinge (pin) joint that permits single rotation. The second form, often referred to as prism joint, allows for single translation. The third form represents a screw-and-nut connection. The screw has only one degree of freedom. When it is rotated, it has to move along the rotation axis, but this motion is not independent from the rotation.

Loop Analysis

The method is based on a simple observation that all mechanisms (with the exception of open loop mechanisms like manipulators) are made of loops of links connected by joints. If a mechanism is designed without redundant constraints, then even if the links are deformed, it should be possible in assembly to close each loop without use of force.

The number of independent loops, L, can be calculated with the formula:

$$L = j - (n-1)$$

where j is the total number of joints, and n is the total number of links in the mechanism (base included).

An independent loop corresponds to a chain of links that starts and ends at the same joint. One can physically trace a loop on the mechanism by the following selected links from joint to joint. As an example, let us consider a mechanism shown in Fig. 4.
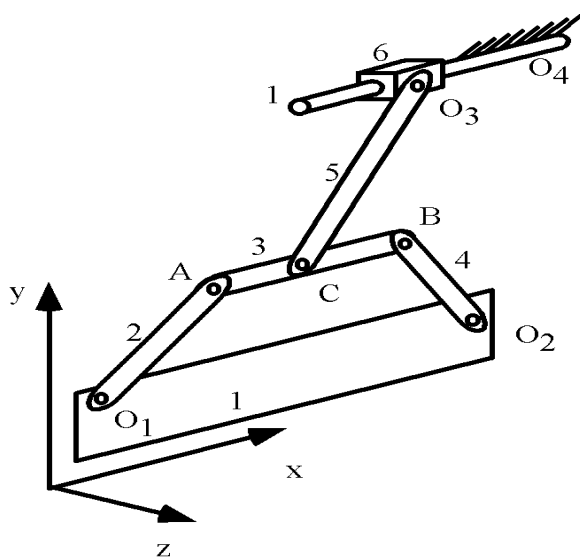
Figure 4 A mechanism with six links.

The mechanism has n=6 links and j=7 joints, and therefore the number of independent loops is L=7-(6-1)=2.

There are different ways to choose loops, as demonstrated in Fig. 5 a, b, and c. Information about a loop can be represented in the form of a code. The code is a sequence of letters corresponding to the joints in the selected loop. For example, the loops shown in Fig. 5 a can be written as O_1-A-C-O_3-O_4-O_1 and O_2-B-C-O_3-O_4-O_2. Point O_4, intentionally located in the vicinity of the slider, is used to show that the loop continues through the slider toward the base. It is obvious that the same joint (same label) may appear in more than one loop code.

For the purpose of the procedure, which has yet to be described, information about the degrees of freedom of a particular joint will be entered in the calculations of the mobility of the mechanism. To avoid counting the same degrees of freedom twice, the restriction is made that a joint can be taken into consideration only once. To show which joints are excluded from a loop (to avoid a double count), a letter corresponding to the relevant joint is enclosed by parentheses, like in this example: O1-A-C-O3-O4-(O1) and O2-B-(C)-(O3)-(O4)-(O2). It is arbitrary which joint will be counted in which loop. In the considered case, we could as well exclude joint C from the first loop and make it available for counting in the second. The only requirement is that a loop should have at least one joint that will be counted.
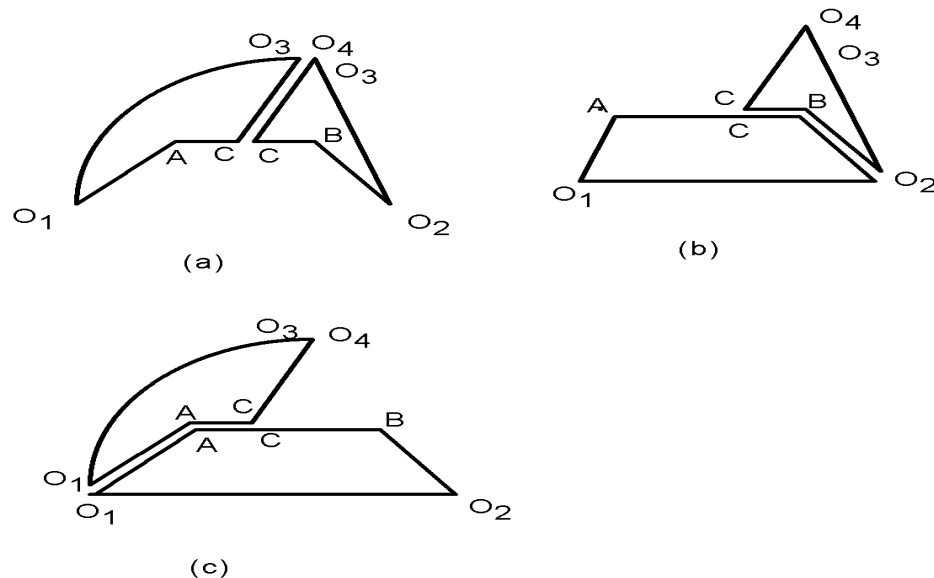
Figure 5 Possible loops for mechanism shown in Fig.4.

Rationale of the Method
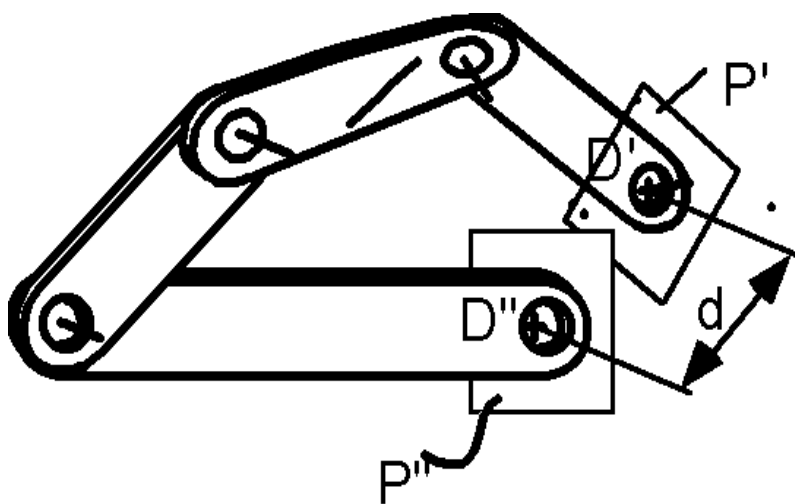
Figure 6 A loop, made of links with geometric errors, before closing.

Let's now consider a partly assembled loop, as shown in Fig. 6. The loop needs to be closed at joint D to make a four-bar linkage. Because link BC is twisted, the centers of the holes D' and D" cannot be aligned. To align the parts, and make closing of the loop possible, the joints of the loop must provide translations and rotations. Merely overlapping points D' and D" is not enough, rotations in a partly assembled loop are needed to make plane P' coincide with plane P". Any three-dimensional open

loop of links needs six degrees of freedom in order to be closed:

$$t_X = 1 \quad r_X = 1$$
$$t_y = 1 \quad r_y = 1$$
$$t_Z = 1 \quad r_Z = 1$$

where t_X, t_y, t_Z, and r_X, r_y, r_Z represent the number of required degrees of freedom in translation (letter t) or rotation (letter r) with regard to arbitrarily located and fixed x, y, and z axes. These translations and rotations come from the degrees of freedom available in the joints of the links making the loop.

Lack of any of the six degrees of freedom <u>is an indication of the existence of redundant constraints.</u>

For example, in the following situation for a loop:

$$t_X = 0 \quad r_X = 1$$
$$t_y = 1 \quad r_y = 0$$
$$t_Z = 1 \quad r_Z = 1$$

the presence of two zeros indicates the existence of two redundant constraints.

In order to have mobility of the loop after closing to create a four-bar linkage that can move, the joints must provide at least one additional degree of freedom.

For example, an open loop with the following degrees of freedom:

$$t_X = 1 \quad r_X = 1$$
$$t_y = 1 \quad r_y = 2$$
$$t_Z = 1 \quad r_Z = 1$$

will have mobility equal to one after closing. The mobility comes from the "surplus" degree of freedom in rotation about y-axis ($r_y = 2$).

What is interesting is that the surplus rotational degrees of freedom can compensate for the missing degrees of freedom in translation in the loop. This property can be explained with the help of Fig.6. The chain of links that needs to be closed has three rotational degrees of freedom about the y-axis --each pin joint contributes one degree of freedom. In this case point D' can be moved in, say, x direction by properly rotating links in joints A, B, and C. A better study of this phenomenon leads to the principle that <u>a missing translational degree of freedom can be compensated for by a surplus rotational degree of freedom about the axis that is perpendicular to the direction of translation.</u> This means, for example, that an additional rotational degree of freedom, r_Z, can be used in lieu of one of the translational degrees of freedom, t_X or t_y, should one of them be missing. To make things clear, here is another example.

The loop described below is equivalent to the first loop in this section in the sense of degrees of freedom, and therefore can be closed:

$$t_X = 0 \quad r_X = 1$$
$$t_y = 1 \quad r_y = 1$$
$$t_Z = 1 \quad r_Z = 2$$

One of the two rotational degrees of freedom, r_Z, can be "transferred" to the missing translational degree of freedom, t_X. An important observation made by Ozol was that the surplus rotational degrees of freedom available in one loop can be used to compensate the missing translational degrees of freedom in other loops. These concepts can be used to develop a procedure to determining the mobility of the mechanism and the number of redundant constraints. The procedure will be explained in an example.

Let's consider a structure (we do not know if it is a mechanism) shown in Fig. 4. The task is to determine the mobility and the number of redundant constraints.

The solution starts from identification of the joints. There are six joints of the V-th class created by pairs of links: 1-2, 2-3, 3-4, 3-5, 5-6, and 1-4. Connection 1-6 belongs to the IV-th class. All V-th class joints provide rotations about the z-axis, the IV-th class joint provides both rotation about the x-axis and translation along the same x-axis.

The mechanism is made of two independent loops, as determined before. Let's choose the loops as shown in Fig. 5a. We start the procedure by drawing a box for each loop, as shown in Fig. 7, step 1. The code for each loop is written in a corresponding box, and the number of degrees of freedom --translational and rotational-- is determined for each loop. The degrees of freedom are counted only for the joints that are included in the loop. Because loop O_1-A-C-O3-O4-(O1) represents a flat configuration of the links, each joint O1, A, C, and O3 contributes one rotational degree about the z-axis, so $r_Z = 4$. There is another rotational degree of freedom, $r_X=1$, contributed by the joint made by links 1 and 6. The only translational degree of freedom, $t_X = 1$, comes from joint O4, created by the slider and the base. There are no other degrees of freedom, so zeros are entered next to proper symbols.

The box for loop O2-B-(C)-(O3)-(O4)-(O2) is filled out in a similar manner. Because only two joints are included in the loop, $r_Z=2$. There are no other rotational or translational degrees of freedom.

In step 2, shown in Fig. 7, the surplus rotational degrees of freedom are converted to translational degrees of freedom. Out of four degrees of freedom, $r_Z = 4$, two are transferred: one to translation along the y-axis for the first loop, and the second to translation in the x direction for the second loop. This procedure changes values of t_X and t_y from zero to one. Out of two remaining rotational degrees of freedom $r_Z=2$ in the second loop, one degree is converted to translation along the y-axis in the same loop. All of the described conversions are marked in step 2 with arrows. After these reallocations, the distribution of the degrees of freedom is shown in Fig.7 step 3.

The surplus rotational degree of freedom from the first loop (everything above value of one) corresponds to the mobility of the mechanism, and is symbolically transferred to the "mobility counter" located above the boxes (see the broken arrow). All zero values for the degrees of freedom correspond to the redundant constraints and are symbolically transferred (see broken arrows) to the "counter of the redundant constraints " located below the boxes.

Step 4 in Fig.7 shows the final distribution of the degrees of freedom for the closed loops, the mobility of the mechanism, and the number of the redundant constraints. The mobility of the mechanism is m=1, and the number of redundant constraints is r=5.

Because r is not equal to zero, the mechanism is not self-aligning. It will operate if the crank is rotated, but assembly and operation is possible only if 5 geometric constraints are imposed:

1. The axis of pin joint A must be parallel to the axis of joint O_1,
2. The axis of pin joint C must be parallel to the axis of joint A,
3. The axis of pin joint B must be parallel to the axis of joint A,
4. The axis of pin joint O_3 must be parallel to the axis of joint C,
5. The axis of the slider's guide must be perpendicular to one arbitrarily chosen pin joint axis.n the

mobility

STEP 1

loop O_1-A-C-O_3-O_4-(O_1)	$t_x = 1$	$r_x = 1$
	$t_y = 0$	$r_y = 0$
	$t_z = 0$	$r_z = 4$

loop O_2-B-(C)-(O_3)-(O_4)-(O_2)	$t_x = 0$	$r_x = 0$
	$t_y = 0$	$r_y = 0$
	$t_z = 0$	$r_z = 2$

redundant constraints

mobility

STEP 2

loop O_1-A-C-O_3-O_4-(O_1)	$t_x = 1$	$r_x = 1$
	$t_y = 0$	$r_y = 0$
	$t_z = 0$	$r_z = 4$

loop O_2-B-(C)-(O_3)-(O_4)-(O_2)	$t_x = 0$	$r_x = 0$
	$t_y = 0$	$r_y = 0$
	$t_z = 0$	$r_z = 2$

redundant constraints

Figure 7 Step by step loop analysis of mechanism shown in Fig.4 (continued on the next page).

mobility

STEP 3

loop O_1-A-C-O_3-O_4-(O_1) $t_x = 1$ $r_x = 1$
                                        $t_y = 1$ $r_y = 0$
                                        $t_z = 0$ $r_z = 2$

loop O_2-B-(C)-(O_3)-(O_4)-(O_2) $t_x = 1$ $r_x = 0$
                                              $t_y = 1$ $r_y = 0$
                                              $t_z = 0$ $r_z = 1$

redundant constraints

mobility   **1**

STEP 4

loop O_1-A-C-O_3-O_4-(O_1) $t_x = 1$ $r_x = 1$
                                        $t_y = 1$ $r_y = 0$
                                        $t_z = 0$ $r_z = 1$

loop O_2-B-(C)-(O_3)-(O_4)-(O_2) $t_x = 1$ $r_x = 0$
                                              $t_y = 1$ $r_y = 0$
                                              $t_z = 0$ $r_z = 1$
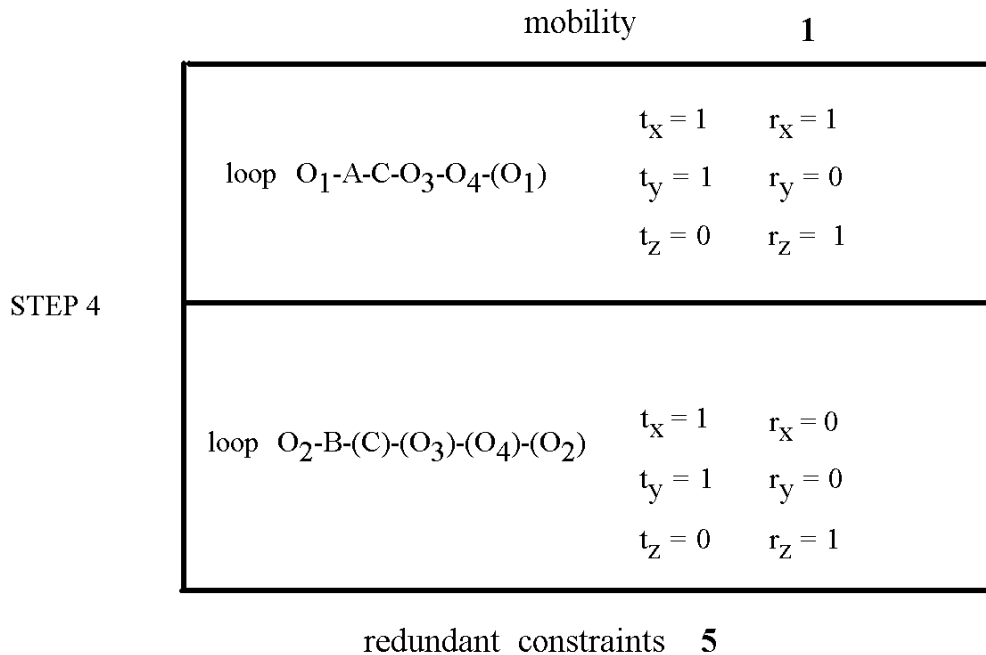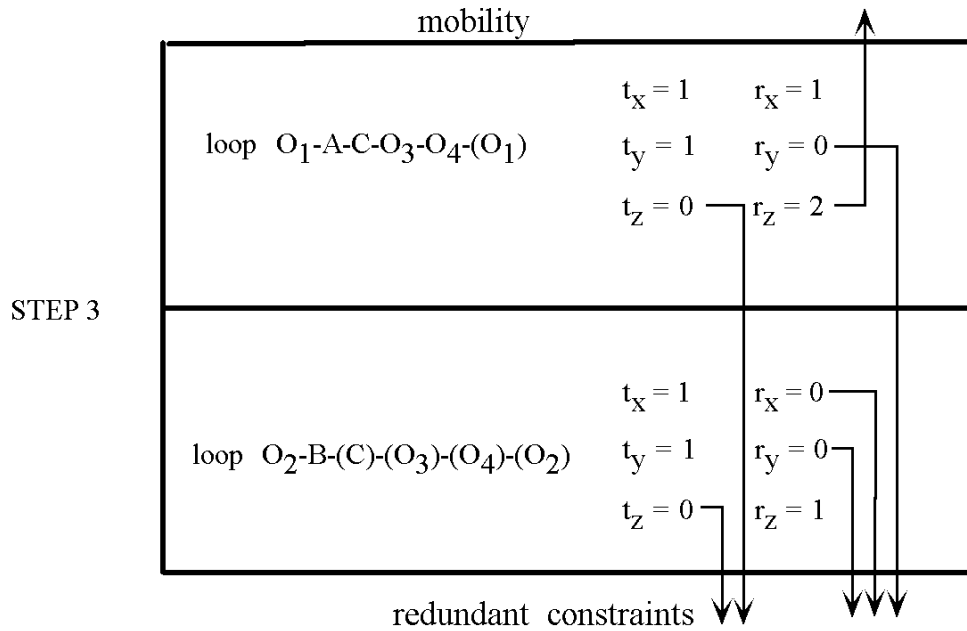
redundant constraints   **5**

Figure 7 (Continuation from previous page) Step by step loop analysis of mechanism shown in Fig. 4.

Making a Self-Aligning Mechanism

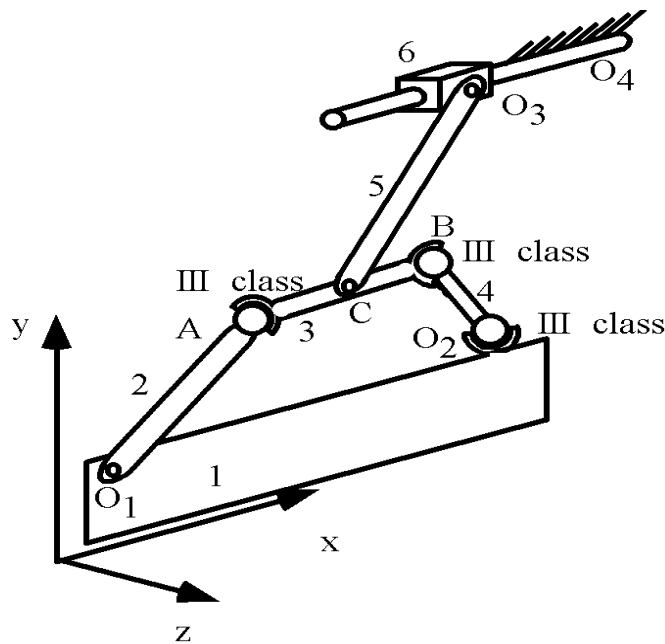To make the mechanism a self-aligning one, more rotational degrees of freedom must be available at the joints. The simplest way to do this is to replace some of the joints by joints of the lower class, for example III-rd class (ball-and-socket type). These self-aligning bearings are available from many bearing manufacturers. The replacement can be gradual, one joint at a time with loop analysis performed after each modification to check if r=0.

In the considered case, three of the pin joints have to be replaced by self-aligning joints of the III-rd class to make the mechanism self-aligning. The modified mechanism is shown in Fig.8, and the corresponding loop analysis shown in Fig.9.

Figure 8 A self-aligning mechanism obtained by modification of the mechanism shown in Fig. 4.

The analysis shows mobility m=2. The original mechanism had mobility m=1. After modification, link 4 gained additional mobility. Because it has ball-and-socket joints at both ends, it can be rotated about its own axis. This additional mobility does not affect the ability of the mechanism to move as before. The mobility of a link that does not affect the general mobility of the mechanism is called *local mobility*. Because the number of redundant constraints in the modified mechanism is r=0, the links of the mechanism can be fabricated without any special precautions with regard to parallelity of the joints axes. Also, the axis of the slider's guide does not have to be perpendicular to any axis. No matter how badly the links are machined, the mechanism will be easy to assemble and operate because it became self-aligning.
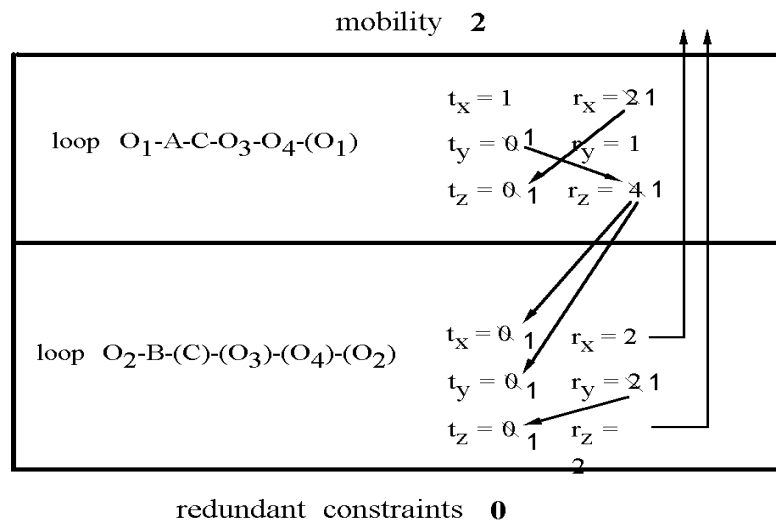

Figure 9 Loop analysis for the mechanism shown in Fig. 8.

Place for the Method in the ME Curriculum

ME students taking capstone design courses design complicated mechanisms. Checking if mechanisms will move as anticipated is sometimes the most difficult task. The author has a collection of preliminary drawings conceived by the students of devices which ended-up not moving. One can find in this collection ideas for three dimensional mechanisms of folded wheel chairs, car jacks, hoists, powered car seats for handicapped drivers, etc. In each case, students were certain that the device was correct. One can only imagine the amount of frustration and material loses if these projects were to be built. Simply, a three-dimensional imagination of the students at this level does not work. The loop method provides an easy solution. The results of the analysis also gives an insight into how sensitive the device is to geometric errors. The larger the number of redundant constraints, the more problems the device with cause.

The author of this paper taught the loop method to students taking MECH 446 Design I, MECH 445/845 Special Concepts of Mechanical Design (design elective course), and in MECH 442 Intermediate Kinematics (technical elective course).
The material presented in this paper requires about three 50-minute periods to be sufficiently explained. A 20 page handout on the subject, with examples how real engineers design self-aligning mechanisms, accompanies lectures.
The main emphasis in assignments is put on training the students to determine classes of real joints used in machinery and counting degrees of freedom in the loops. Photographs and technical drawings of real mechanisms and joints are used in class and home assignments.

In two elective courses, with smaller number of students, in a hands-on exercise, students build self-aligning mechanisms and test their behavior. According to the students, this part of the course is an eye opener.
In a typical exercise, a small team (3 students) is given a task of conceiving and building a three-dimensional mechanism with given specifications. Some of the past assignments were:

*   design a four-bar linkage in which the axis of the crank (input) is perpendicular to the axis of the follower (output),
*   design a mechanism in which a single crank imparts motion to two pistons with skewed axes.
*   design a self-aligning linkage in which one of the links operates with dwells.

At the beginning of the project, the mechanism is sketched on paper and analyzed for the mobility and the redundant constraints. Then a self-aligning version of the same mechanism is designed . The loop method is the only tool available to the students.

The teams build mechanisms from parts purchased at hardware and hobby stores. The most frequently used parts are: brass tubing with slightly different diameters, used in building joints of the IV-th class, small self-aligning joints used by hobbyists to build RC cars, brass flats and miniature screws and nuts (see Fig. 10).

The cost of parts per team is about $8.00, and money comes from the laboratory fees paid by the students. The parts are glued together with a glue gun, the holes are drilled and parts cut with a small Dremel tool. Some safety precautions have to be taken (safety goggles available for drilling and cutting operations).
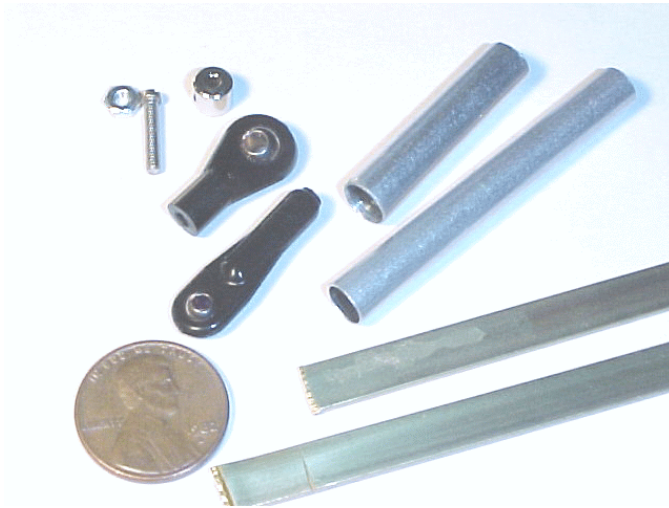Fig. 11 shows an example of a mechanism built by the students in one hour. The advantage of

self-aligning mechanisms is very obvious to the students, because with primitive means of fabrication their mechanisms work amazingly well.

It is important to explain to the students that modification of a mechanism to obtain a self-aligning one, whatever the cost, is not a good engineering practice. The self-aligning mechanisms should be built in case of necessity, and only after cost analysis.
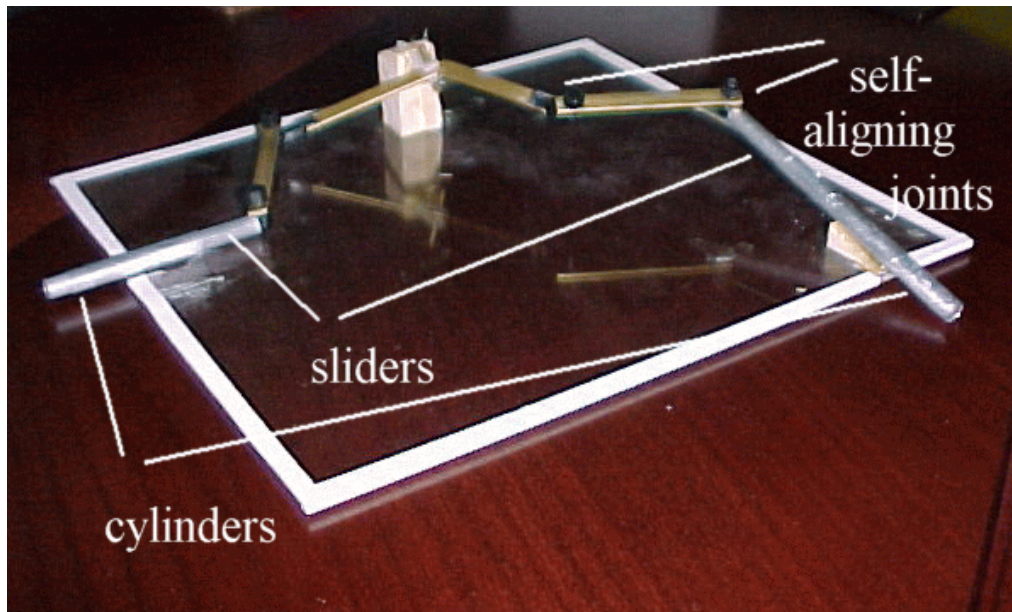
Figure 10 Simple parts for building models of self-aligning mechanisms.

Figure 11 An example of a self-aligning mechanism built by the students.

Conclusions

Presented in this paper, and little known (in the United States), loop method fills a compromising gap in the mechanical engineering curriculum. The information about the redundant constraints provided by the method allows, in case of need, to modify the analyzed mechanism and make it self-aligning. Practical and inexpensive experiments done by the students in the classroom help build intuition by helping the students understand the concept of mobility, redundant constraints, and self-alignment.

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