Teaching by Analogy: The Use of Effort and Flow Variables

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Engineering principles taught well to students become the foundation for lifelong understanding of physical processes. No set of engineering principles is more useful or pervasive than the concepts of effort variables and flow variables. By analogy, these can be applied to almost any situation involving transfer of something from one location or situation to another.

Effort variables cause action to occur. They can express the tendency for literal or figurative movement, and are sometimes thought to be potential, or field variables. Flow variables are the things that move because of the presence of an effort variable.

Transport processes such as fluid flow, heat transfer, and mass transfer conform very well to the effort and flow variable concept. Temperature and heat are the effort and flow variables for heat transfer, whereas pressure and fluid flow are the effort and flow variables for flow systems.

Students seem to realize that flow can only occur between points where the effort variable is higher at one and lower at another. They have had sufficient experience with electricity, heat, and fluids to know that electric current must flow from higher voltage to lower, that heat must flow from higher temperature to lower, and that water must flow downhill. If it is explained to them that higher to lower potentials needed for flow to occur actually expresses the second law of thermodynamics in another, more general, way, then a very abstruse concept can be made more real.

The analogy can be extended to other physical systems including mechanics (force, velocity) electricity (voltage, electric current), magnetics (magnetomotive force, magnetic flux), and others not normally taught to undergraduate engineering students as transport processes.

The effort and flow variable analog can extend much farther into such disparate areas as the spread of disease, traffic flow, technology transfer, psychological motivation and attainment, politics, economics, and even responses to perfume. All of these have some cause and an accompanying flow.

What limits the rate of flow in the presence of a certain amount of effort? The answer is resistance, defined as effort divided by flow. Thus, one way to contain the spread of disease is to erect barriers (resistance) to its movement, perhaps through vaccination of the susceptible population.

What happens to the flow once it reaches its destination? It can be stored on capacity, defined as...
the integral of flow divided by effort. Thus, when money crosses the borders of a country, it accumulates and adds to the wealth of a nation.

How fast can changes take place? This is limited by inertia, defined as effort divided by the time derivative of flow. Thus, despite being highly motivated to begin taking courses (high effort), and with sufficient cash in hand to pay for them (low resistance), a student may still take an extra semester to begin that education because of the need to change habits (inertia).

Relationships between resistance, capacity, and inertia lead to time constants and natural frequencies that can show how flow variations can be dampened or magnified. Thus, the more wealth a nation accumulates, the longer is its economic time constant, and the less sensitive it is to external fluctuations in commodity prices.

Several examples are in order. First we consider the very simple ecological case of a large herd of caribou migrating through a constrictive narrow valley. The effort variable is migratory pressure and the flow variable is the movement of caribou. The valley can be represented by a resistance that regulates the flow of animals. On the downstream side of the valley, the caribou are presumed to migrate freely without significant impediment. The downstream side can thus be considered to be connected to a point of nearly zero potential. Representation of the upstream side of the valley could be one of three possibilities:

1) A capacity element. Using this element would signify that the number of migrating animals is finite, whatever could be stored in the capacity element. Flow of animals from the element would decrease as the migratory pressure, and the number of animals upstream, decreases.

2) A pressure source. Using this element would signify that the migratory pressure of animals attempting to move through the valley does not decrease with the passage of animals. However, the constriction of the valley still has an effect on the flow of animals through it.

3) A flow source. Use of this element would mean that flow of animals would be constant, and not affected by the resistance represented by the valley.

Of these three, the flow source is clearly wrong. We know that a more restrictive valley would slow the flow of animals. There are some properties of the capacity element and the pressure source that make each of these at least partially correct. The capacity element represents the fact that the number of animals is finite; the pressure source, however, indicates that the flow of migrating animals is not likely to decrease over time (except, perhaps, for the very last animals). The best choice for a representation of the upstream side of the valley is thus the pressure source.

Another engineering example is the power required for an artificial heart. The vascular system of the body can be represented as a compliance (a fluid capacity element) of about $1.44 \times 10^{-9}$ m$^5$/N. From this model, flow rate ($\dot{V}$) can be found to be:

$$\dot{V} = \frac{p_s}{R} + C \frac{dp_s}{dt}$$

where $p_s$= aortic pressure, $R$= resistance, $C$= compliance, and $t$= time. If aortic pressure is
assumed to vary in any particular time-varying way, as a sinusoid for instance, then flow rate can be found. Required power is pressure times flow rate. Details about these and other applications can be found in *Biological Process Engineering*, by A. T. Johnson (John Wiley, 1999).

Teaching students about the principles underlying transport processes, and illuminating the possibilities of analogous application can enable engineering students to conceptualize in a way that will be forever valuable to them, whether they remain in engineering careers or take other future career pathways. To be agents of change, they will realize very simply that they must increase motivation (effort), reduce resistance, overcome inertia, and maintain sufficient capacity storage that will not be depleted.

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