2006-374: A COGNITIVE STUDY OF MODELING DURING PROBLEM-SOLVING

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A Cognitive Study of Modeling during Problem-solving: An integrated problem solving model

Introduction

A fundamental issue in engineering education is the question of how to improve students' analytical skills.[1] Analysis skills are central to engineering students' abilities to interpret and solve problems and the question of how to improve these skills is a fundamental issue in engineering education research. In recent years, there have been several programs developed to improve students' analysis skills. Amongst these are programs to improve problem solving skills[2], software modules that teach concepts[3], and concept inventories to assess misconceptions[4]. Each of these programs is grounded in a set of beliefs about the underlying causes of students' disappointing analytical skills. If one believes, for example, that students use ineffective problem solving strategies, one is more likely to develop and adopt a program that teaches these strategies. On the other hand, if one's belief is that the quality of underlying knowledge is the critical factor in determining analysis skills, one will turn toward efforts to improve students' knowledge.

Our belief is that there are multiple causes for students' troubles with analysis skills. In the literature, we have found three approaches to understanding analysis that we find to be the most promising. These three are: problem solving processes,[5] translations between symbol systems,[6] and domain knowledge.[7] We are attracted to these approaches because each has compelling empirical and theoretical support. In this article, we report on a cognitive model of the analysis process that was developed by melding these three approaches. This model, the Integrated Problem Solving (IPS) model, treats each of the three approaches as a separate dimension of problem solving and proposes that high quality problem solving processes requires their integration. These three dimensions are organized into three phases with each phase corresponding to a different step of analysis. Because the IPS model incorporates all three dimensions of problem solving, symbol system transformations, and domain knowledge, it is more comprehensive than previous models.

In this research, we collected verbal protocols from engineering students as they constructed free-body diagrams (FBD). We compared the cognitive processes reported in protocols to our model. Verbal protocols were analyzed through a thematic analysis of processing to better understand students' difficulties with analysis skills. The results are used to uncover the source of students' difficulties and to generate suggestions about effective instructional interventions. In the sections that follow, each of the three dimensions contained in the IPS model are discussed independently. This discussion is followed by a presentation of the model itself.

Problem Solving Process

Since Polya's seminal work in mathematics,[8] the utility of learning and using a sequence of steps during problem solving has been widely accepted. Although several specific models exist, a generic 4-step model captures most: (1) Represent the Problem, (2) Goal Setting and Planning, (3) Execute the Plan, and (4) Evaluate the Solution. In the first step, problem representation, the student must read the problem statement and discern the objective. Correct execution of this step is heavily dependent upon the student's ability to determine the deep structure of the problem and recognize the principles that must be applied to reach a solution. During the second step, goal setting and planning, the student must develop a path to reach the solution. A student who plans, determines not only the path to the final solution but also any necessary subgoals that must be achieved along the way. Execution is the third step. This is the step in which students carry out the procedures necessary to complete the problem solving process. Throughout this step, a student must not only apply procedures but must also monitor, or check, the accuracy of progress. Monitoring is necessary to determine if the plan is viable and working. The final step is evaluation. At this point, the student checks the quality of the final solution. During evaluation, the student verifies that the final solution actually and accurately answers the given problem.

Although these steps can be described independently, the reality is that good problem solving is recursive. Students do not proceed in a linear fashion because any one step may lead the problem solver back to earlier steps. For instance, a student who is constructing an FBD may establish the subgoal of identifying the force brought by a distributed load. As she proceeds to this point of diagram construction, she realizes the need to calculate the exact weight of the load before she can locate it on the diagram. At this point, the new calculation subgoal is set and executed. Another example of the recursive nature of the solution process is when monitoring during the execution phase sends the problem solver back to earlier steps. In this case, the student may realize that an earlier portion of the problem was solved incorrectly with this realization sending the student back to re-execute the faulty processes.

Although all steps in the problem solving process are important, the need to accurately represent the problem may be particularly critical.[9] The research in this area has shown that individual differences in how students represent problems are tied to differences in problem solving performance. Sherin, for example, demonstrated that college students who struggle to understand a physical situation also have difficulty expressing that idea as a mathematical equation.[10] Savelsbergh *et al.* divided physics students into groups of strong and weak novices. He found that these two groups could be differentiated by their ability to represent problems.[11]

There are instructional interventions for engineering education that are grounded in this theoretical model of problem solving. For example, Gray *et al*.[12] developed a systematic approach to solving Statics and Dynamics problems. In this intervention, it is recommended that students be taught the sequence of: Road Map (Planning), Modeling (Representation), Governing Equations (Representation), Computation (Execution), and Discussion and Verification (Evaluation). Gray *et al.* explain that this instructional approach was developed in response to concerns that problem solving is taught as a “hodgepodge of tricks” that are problem specific rather than generalizable. By providing students with a set of general problem solving processes, they can approach problems more strategically.

Don Woods completed some of the most thorough work that has been done in this area while developing the McMaster Problem Solving program.[13] In his most recent work,[14] Woods has focused on the processes of problem solving and has developed a model to describe ideal problem solving. In this model, he summarizes the cognitive, meta-cognitive, and attitudinal skills that relate to each stage. Included in the cognitive skills that Woods identifies as required for problem solving are generic problem solving skills and stored solutions to previous problems.

In sum, the available evidence implicates students' problem solving processes as an important contributor to the quality of problem solutions. Although the specific work examining these processes during engineering analysis is not extensive, we believe that the importance of these processes in other domains[15] warrants their inclusion in a model of analysis skills.

Domain Knowledge

Without a doubt, the quantity of prior domain knowledge affects problem solving.[16] It is also widely accepted that qualitative aspects of knowledge matter. Prior knowledge is believed to act as an important scaffold for problem solving. The structure provided by the knowledge base can, for example, act as a constraint during analogical reasoning,[17] support strategic processing during reading,[18] and contribute to positive motivational states during problem solving.[19] In short, the effects of prior knowledge are wide-reaching and powerful.

It is not only the quantity of domain knowledge but also the quality of knowledge that matters during problem solving. Several studies have demonstrated that the organizational properties of students' knowledge are related to problem solving performances.[20] de Jong and Ferguson-Hessler[21], for example (1986), measured the organization of students' physics knowledge and found that these organizational properties correlated with problem solving performance. This study employed a card sorting technique to measure knowledge organization. Participants used the cards to identify meaningful categories of principles related to electricity and magnetism. Results of the card sort showed that students who organized the cards in a manner that more closely resembled those of experts performed better on the related problem solving assessment. Those who performed poorly on the problem solving assessment, the weak novices, were more likely to have sorted cards based upon surface features. These findings are consistent with research showing that qualitative aspects of knowledge are related to problem solving processes in nursing,[22] sound engineering,[23] and medicine[24].

Also captured within the qualitative aspects of knowledge are students' misconceptions. When learners' prior knowledge contains inaccurate conceptions, these underlying errors are passed on to the mental model that is constructed to solve a problem. When the faulty knowledge plays a central role in the solution for that problem, the error in the underlying structure is passed on as an error in the outcome performance.[26] Paul Steif closely examined the role of misconceptions in Statics[25] and developed a concept inventory[26] to determine the effect of these misconceptions on problem solving. The Statics Concept Inventory,[26] a multiple-choice measure, assesses students' knowledge according to the conceptual knowledge required to solve problems. On the inventory, distractor choices were crafted so that the specific choice made reveals the students' misconceptions. The results of a psychometric evaluation showed that the quality of students' knowledge correlated with course grade. Additionally, within conceptual item clusters, error patterns in the choice of incorrect distractors revealed predicted, consistent misconceptions.

In sum, there is ample evidence to suggest that students' prior knowledge plays an important role in problem solving. Any model intended to account for engineering students' problem solving processes will have to take this variable into account. Specifically, this model will not only have to deal with what the student knows but also if this knowledge is available to be accurately applied during problem solving.

Symbol System Transformations

A final approach to understanding problem solving in engineering focuses on the symbol system translations inherent in the analysis process. By symbol system, we refer to the semiotic system used to understand and express elements and their relations. Mathematical expressions are an example of a semiotic system in which numbers and operators act as elements. How these elements are configured in relation to one another communicates the full meaning of the expression. Translations are required when problem solvers move between symbol systems.

McCracken and Newstetter[27] developed the TDS model to capture the transformations that take place during analysis. This model includes verbal (Text), visual (Diagram), and mathematical (Symbol) semiotic systems through which the student must pass to complete an analysis task, with each phase corresponding to a different symbol system. Accurate analysis relies on the ability to move through the phases by transforming a representation of the problem from one symbol system to another. In order, the problem solver must move from the verbal representation, through the diagram, and on to a mathematical expression. The TDS model carries hypotheses about the cognitive processes underlying the transformations between symbol systems. In the first phase, as the problem solver holds a verbal representation, she engages in data gathering. The diagram is the semiotic system used in the second phase. Here, the learner generates hypotheses and makes assumptions about the problem and draws an external diagram to depict these understandings. Finally, the phase of problem synthesis occurs when the student translates the diagram into a set of mathematical equations. McCracken and Newstetter reason that, although these representational systems are tightly bound in the expert, novices must work harder to pass from one phase to the next.

This contention is supported by a body of Cognitive Science research. Haverty et al. [28], for example, demonstrated that college students struggle to translate a data table into a mathematical expression. de Jong and Ferguson-Hessler's[29] research provides a clue about the potential cause of these translation difficulties. Recall from the description in the previous section that, in this study, de Jong and Ferguson-Hessler assessed the organization of students' physics knowledge by having them sort cards related to principles of electricity and magnetism. In addition to verbal representations of principles, a subset of cards depicted these principles as mathematical formulas. Weak novices sorted verbal and mathematical representations into separate piles and tended to match formulas by the variables contained in each. In short, the weak novices in this research did not recognize principled redundancy when these principles were represented by different semiotic systems.

These findings are consistent with a broader literature that addresses the degree to which students integrate, or map between, different representational systems during learning and problem solving. On balance, this research shows that college students often struggle with the task of integration.[30] Consequently, there exists both theoretical and empirical work to show that a comprehensive model of analysis in engineering must pay attention to the processes of symbol system transformation. In keeping with McCracken and Newstetter, we believe that this model must consider the cognitive processes of analysis alongside these transformations.

The Integrated Problem Solving (IPS) Model

The literature reviewed thus far makes a strong case that any comprehensive effort to understand analysis will have to consider the triad of students' problem solving processes, prior knowledge, and symbol system transformations. Fortunately, the tenets held in these approaches are not in conflict and thus, they can be assimilated to construct a more comprehensive model. The IPS model, depicted in Figure 1, is a result of this assimilation.

The structure of this model draws heavily upon McCracken and Newstetter's TDS model. Specifically, we retain the three phase structure, which corresponds to the main semiotic systems involved. We have mapped theoretical problem solving and knowledge-driven processes within each phase to complete the dimensional nature of IPS model. The result is a hypothetical model of ideal analysis skills in which prior knowledge, problem solving strategies, and symbol system transformations operate together to complete an analysis task. Although the IPS model contains Problem Synthesis, in this study, we have only collected data corresponding to the first two phases. Accordingly, this third phase is not fully discussed in the article.

As can be seen in Figure 1, the first phase is the *problem representation* phase. Consistent with the TDS model, the main semiotic system in use during this phase is verbal. During problem representation, the problem solver reads the problem and constructs a mental representation, or mental model[31], of the situation. Held within the problem solving processes of this phase are the cognitive operations involved in understanding the problem and setting goals. Understanding the problem requires that the problem solver recognize the deep structure of the problem. The givens are recorded onto this structure, which allows the problem solver to determine what he must solve for. Goal formation involves setting subgoals and planning how to achieve each one.

Prior knowledge is embedded in the cognitive processes of this phase. Knowledge-based pattern recognition is critical to understanding the problem, which occurs when the problem solver applies accurate knowledge to determine involved principles. These principles contain the deep structure that forms the knowledge representation. Knowledge is similarly applied to derive subgoals and set plans.

The second phase is *problem framing* and, in this phase, the symbol system is primarily diagrammatic. It is during this phase that the problem solver begins to draw a physical diagram to depict the problem representation. The problem solving processes at this stage include the execution of plans and engagement in corrective actions. Examples of the execution processes found within problem solving models include the mapping of givens, activation of prior knowledge, and self-monitoring. When problem solvers detect errors through monitoring processes, corrective actions are taken.

Knowledge-driven processes are again embedded within problem solving processes. As the problem solver executes his plans, he must retrieve and apply appropriate principles and rules to guide behavior. For example, to execute processes relative to the subgoal of diagramming the load force on a frame, the problem solver must retrieve and apply the rules for representing this force.

As the problem solver completes the problem framing phase, she must evaluate the quality of the constructed diagram. In this respect, we view the construction of the diagram as a distinct part of analysis. Accordingly, evaluation occurs here as it does at the end of a problem solving cycle. Once the problem solver has finished the diagram, she must check to ensure that the new representation contains all of the critical components that were given in the problem statement, the engineering principles inherent in the diagram are accurately represented, and that the values necessary for calculating a solution are determined.

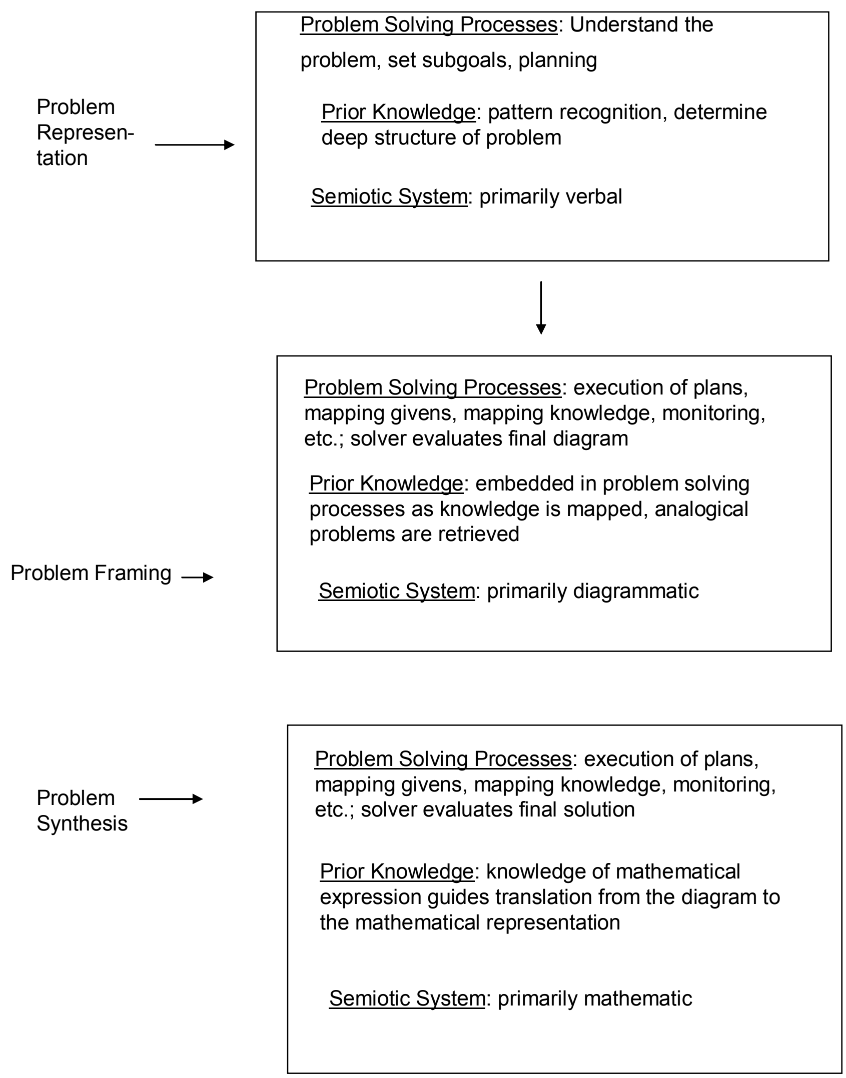

Figure 1: the Integrated Problem Solving Model proposed by this study.

Although each phase of the model is tied to a single, primary symbol system, other symbol systems are involved as well. As the ideal problem solver works to understand the problem in the first phase, for example, it is likely that he immediately begins to retrieve stored diagrammatic representations for parts of the problem even as he is just reading the statement. In our model, we propose that most of the interaction between symbol systems occurs during the problem framing phase. During execution, for example, the problem solver will move between the diagram under construction and the problem statement in order to identify givens and retrieve subgoals. Furthermore, when monitoring processes detect errors in the analysis process, a corrective action that the problem solver employs may send him back to recheck something in the verbal representation.

It is likely that movements between these symbolic systems decreases as the diagram begins to take shape and the problem solver focuses on its construction. Once the diagram is complete, however, we believe that movements between these symbol systems take place again. Essentially, upon completing the diagram, the ideal problem solver will check its accuracy and completeness by comparing it to the contents of the problem statement.

In summary, the model shown in Figure 1 describes analysis as occurring in three phases. Within each phase, the model specifies the problem solving and knowledge-driven processes involved as well as the required symbolic representations. Although this model is grounded in previous work, because it is more comprehensive than past models, we believe this framework stands a better chance of uncovering the specific aspects of analysis that cause engineering students such difficulty. Although the model combines processes from each of three distinct approaches, the test of the model reported in this article does not pit one approach against the other. We are not asking, "Which approach is best?", but rather, "How do these approaches interactively account for the processes of analysis?" and "Which specific aspects of analysis are a major source of difficulty?"

The research reported in this article begins to answer those questions. We have undertaken the study of college engineering students as they complete FBDs. The problems were pulled from content covered in a Statics course and required only that the diagram be constructed. As stated above, we did not include the final problem synthesis phase in order to focus this study on a closer examination of students' processes. The research questions that guided this research were:

1. Does the proposed model adequately capture the processes of analysis?
2. Are students' problem solving processes a major source of difficulty during analysis?
3. Are translations between symbol systems a major source of difficulty during analysis?
4. Are students' prior knowledge-driven processes a major source of difficulty during analysis?

Research Methods

Four students majoring in engineering at a large university participated in this study. There were three females and one male. Students were recruited from their classes and consented to participate in the study. Students were targeted for recruitment to represent a range of experience with Statics. One participant was currently enrolled in a Statics course (EMech11) and the other three had recently completed the course. Three of the students were Mechanical Engineering majors and the fourth was an Aerospace major. The students received an A or A- in their Statics course indicating that they were successful in the course. In order to protect the identity of participants, hereafter, we will refer to all participants as female.

All students completed three problems, the first of which was a practice problem. All problems included a verbal problem statement and an illustration of the problem. Students constructed an FBD but were not required to derive the mathematical formulas. The students did not have access to any reference materials while constructing the FBD. The first problem required students to draw an FBD of the man, a task with which they were unfamiliar. This problem required them to make a decision about the presence of friction, to recall the presence of a body force, the normal force, and recognize that the direction of the tension would be away from the man. The second problem involves the concepts of distributed loads and pin joints.

On each problem, students were instructed to draw a fully dimensioned FBD of the target body. Space for these diagrams was provided below each problem statement. These problems are shown in Appendix A.

Students were videotaped, with audio, as they gave verbal protocols concurrent with the analysis tasks. The video camera was trained on the paper with the written problem statements so that verbal statements could be analyzed in light of students' written marks.

All verbal protocols were transcribed verbatim. Protocols were separated into different lines to segment utterances. An utterance was defined a single complete thought. Boundaries between utterances were typically marked by transition words (e.g., because, then, etc.) or shifts in thoughts. Extended silences or atypical physical actions were also marked on separate lines.

The experimenter introduced the study to each participant by explaining that we were interested in studying how students construct free-body diagrams in order to learn about potential instructional interventions. The free-body diagram tasks and verbal protocols were explained and the student gave consent.

To introduce the think aloud task, participants were told that, while they were working on each problem, they should say out loud what they were thinking. Students were told that they did not need to explain what they were doing but that they should report the thoughts in their head. After explaining the verbal protocol procedure, the experimenter gave participants the first problem as practice for thinking aloud. During this practice problem, the experimenter prompted the students' think aloud, if necessary; after the student completed the FBD, the experimented provided feedback on the quality of the think aloud. Following the practice problem, the experimenter gave participants the first experimental problem. The second experimental problem immediately followed the first. No time limit was imposed; the students could spend as much time on each problem as they needed in order to complete the diagram. The average amount of time for the entire procedure was approximately 20-25 minutes.

A thematic analysis was used to explore verbal protocols. This analysis was conducted by considering each research question independently and examining transcripts for evidence related to concepts of each question. Thus, when analyzing the data in light of the second question, transcripts were examined for episodes of verbalizations corresponding to generic problem solving processes. Each episode was noted and collectively, these were examined for meaningful patterns. The patterns that emerged from this analysis form the basis of the results that are reported here. In this respect, the analysis reported here is both exploratory and descriptive.

Results

Does the proposed model adequately capture the processes of analysis? To answer this question, we read verbal protocols and looked for evidence of the processes that are predicted by the IPS model. Descriptions were written, for each student on each problem, to organize these protocols into the framework provided by the IPS model. These descriptions were used to determine how well the IPS model fit the data. As we completed this task, we considered that the categories of the model capture only the processes of analysis and do not evaluate the accuracy of these processes. Thus, the data analysis for this research question addresses students' processes but does not fully consider their accuracy. Accuracy is considered more centrally in response to research questions two through four, which consider the sources of students' analysis difficulties along each dimension.

A summary of verbal protocol descriptions is contained in Table 1. (See Appendix B for the complete set of written descriptions.) In this table, we have summarized the written descriptions. Each column in the table corresponds to a single student and each problem is described in a separate row. Within each data cell, we have included summary information and indicated the presence or absence (i.e., Yes/No) of each process within each phase. Verbal protocols were consulted as necessary when constructing this table.

As can be seen from the table, the length of verbal protocols varied greatly; Problem 1 ranged from 8 to 61 lines, Problem 2 ranged from 37 to 123 lines. Although all 4 students were given credit for correct answers on the second problem and only two correctly answered the first, we do not believe that students' analysis skills improved on the second problem. To the contrary, descriptions of students' analysis skills show that two of the novices did experience extensive difficulty on the second problem. In addition, all students made at least one error of omission or commission on both problems.

This descriptive data presented here supports the conclusion that the IPS model captures students' problem solving processes during an analysis task. There was evidence that students worked to understand the problem and used prior knowledge to recognize patterns during the first phase of analysis. Differences in the quality of the solution processes between instances when students did and did not process to understand the problem suggest that this is a critical step. Students did not, however, form subgoals during the Problem Representation phase. Instead, students dealt with these as they arose. The second phase begins when the student starts to

Table 1a: Summary of Analysis for Novices 1 and 2

Novice 1 (KB)		**Novice 2 (CF)**	
Problem 1		Problem 1	
Errors:	06	Errors:	01
Corrections:	03	Corrections:	00
Omissions:	02	Omissions:	00
Silent periods:	03	Silent periods:	00
Number of Lines:	19	Number of Lines:	24
Correct Answer:	No	Correct Answer:	Yes
Phase 1: Problem Representation		Phase 1: Problem Representation	
Understand the problem:	No	Understand the problem:	Yes
Subgoal Formation:	No	Subgoal Formation:	No
Planning:	No	Planning:	No
Pattern recognition:	No	Pattern recognition:	No
Determine deep structure:	No	Determine deep structure:	Yes
Phase 2: Problem Framing		Phase 2: Problem Framing	
Execution of plans:	Yes	Execution of plans:	Yes
Mapping of givens:	Yes	Mapping of givens:	Yes
Mapping knowledge:	Yes	Mapping knowledge:	Yes
Monitoring:	Yes	Monitoring:	Yes
Evaluates the diagram:	No	Evaluates the diagram:	No
Problem 2		Problem 2	
Errors:	01	Errors:	01
Corrections:	00	Corrections:	00
Omissions:	00	Omissions:	00
Silent periods:	00	Silent periods:	00
Number of Lines:	37	Number of Lines:	37
Correct Answer:	Yes	Correct Answer:	Yes
Phase 1: Problem Representation		Phase 1: Problem Representation	
Understand the problem:	Yes	Understand the problem:	No
Subgoal Formation:	No	Subgoal Formation:	No
Planning:	No	Planning:	No
Pattern recognition:	No	Pattern recognition:	No
Determine deep structure:	Yes	Determine deep structure:	Yes
Phase 2: Problem Framing		Phase 2: Problem Framing	
Execution of plans:	Yes	Execution of plans:	Yes
Mapping of givens:	Yes	Mapping of givens:	Yes
Mapping knowledge:	Yes	Mapping knowledge:	Yes
Monitoring:	Yes	Monitoring:	Yes
Evaluates the diagram:	No	Evaluates the diagram:	No

Table 1b: Summary of Analysis for Novices 3 and 4

Novice3: (JM)		**Novice 4(DG)**	
Problem 1		Problem 1	
Errors:	01	Errors:	00
Corrections:	01	Corrections:	00
Omissions:	00	Omissions:	01
Silent periods:	00	Silent periods:	00
Number of Lines:	61	Number of Lines:	08
Correct Answer:	Yes	Correct Answer:	No
Phase 1: Problem Representation		Phase 1: Problem Representation	
Understand the problem:	(Phase 2) No	Understand the problem:	Yes
Subgoal Formation:	(Phase 2) No	Subgoal Formation:	No
Planning:	No	Planning:	No
Pattern recognition:	No	Pattern recognition:	No
Determine deep structure:	(Phase 2) No	Determine deep structure:	No
Phase 2: Problem Framing		Phase 2: Problem Framing	
Execution of plans:	Yes	Execution of plans:	Yes
Mapping of givens:	Yes	Mapping of givens:	Yes
Mapping knowledge:	Yes	Mapping knowledge:	Yes
Monitoring:	Yes	Monitoring:	Yes
Evaluates the diagram:	Yes	Evaluates the diagram:	(incorrectly) Yes
Problem 2		Problem 2	
Errors:	03	Errors:	01
Corrections:	03	Corrections:	02
Omissions:	02	Omissions:	01
Silent periods:	03	Silent periods:	01
Number of Lines:	123	Number of Lines:	40
Correct Answer:	Yes	Correct Answer:	Yes
Phase 1: Problem Representation		Phase 1: Problem Representation	
Understand the problem:	Yes	Understand the problem:	Yes
Subgoal Formation:	No	Subgoal Formation:	No
Planning:	No	Planning:	No
Pattern recognition:	No	Pattern recognition:	No
Determine deep structure:	No	Determine deep structure:	No
Phase 2: Problem Framing		Phase 2: Problem Framing	
Execution of plans:	Yes	Execution of plans:	Yes
Mapping of givens:	Yes	Mapping of givens:	Yes
Mapping knowledge:	No	Mapping knowledge:	Yes
Monitoring:	Yes	Monitoring:	Yes
Evaluates the diagram:	No	Evaluates the diagram:	No

diagram the problem. Written descriptions contained evidence to support the processes proposed during this phase of the IPS model. As students worked to construct the diagram, they executed processes including the mapping of givens and prior knowledge. In addition, students monitored their progress and evaluated the quality of the final solution. As shown in Table 1, however, students evaluated in only two of the eight opportunities to do so.

In summary, we conclude that the IPS model does capture the processes of analysis. We found evidence for not only the phase structure of this model but also the problem solving and domain knowledge dimensions contained within phases. Furthermore, a close reading of the written descriptions shows that there were qualitative differences in the solution processes produced when students did, and did not, match the phase description of the IPS model.

Sources of Students' Difficulties

The purpose of Research Questions 2 through 4 was to examine the verbal protocols in a search for the major causes of students' difficulties during analysis tasks. Each research question focused attention within one of the three specific dimensions of the IPS model. We used this focus to evaluate not only the role of each dimension during analysis but also to determine if we could attribute students' difficulties to any one specific dimension. To answer these questions, we used the structure of the IPS model to guide data interpretation and identified the themes that emerged with respect to each question. Appendix C contains excerpts from verbal protocols that illustrate some of these themes. These excerpts are numbered and correspond to the numbers that appear in parenthesis as themes are discussed.

Are students' problem solving processes a major source of difficulty during analysis? During the Problem Representation phase of analysis, students' problem solving processes include understanding the problem, setting subgoals, and planning. Problem solving processes in the Problem Framing phase capture the execution of the solution process including mapping of givens and prior knowledge, monitoring, and evaluation. To suggest that these processes are a source of students' difficulties, we would need to find that students not only struggled with some problem solving process but also that associated errors were unrecoverable. Although we did find evidence of students' problem solving errors, the second condition was not met: Students were generally able to overcome problems caused by a problem solving process error. In the section below, we explain this conclusion within the structures of the first two phases of the model.

<u>Representing the problem</u>. During the Problem Representation phase, for example, there was evidence that, although understanding the problem is a correlate of analysis quality, students were generally able to execute this process easily. In most instances, students began the task by reading the problem statement and then explicitly stating a representation of the problem. [1, 2]

Two cases, however, provide evidence that students can make errors when understanding the problem. On the first problem, both KB and JM shifted focus to the givens without verbalizing a representation of the to-be-diagrammed body. [3] Not surprisingly in both cases, the failure to begin by understanding the problem leads to problems. Importantly, however, both students were able to overcome this error and correct their problem representation. The first several lines of KB's transcript, for example, were marked by silence and corrections until she states that she needs to represent the forces on the man. [4] Similarly, JM did not form a correct understanding until later in the transcript. In this case, she has identified the drum as the body that she needs to dimension and it is not until she is diagramming the forces due to gravity that she realizes the error. [5]

The Problem Representation phase also includes the processes of subgoal formation and planning. The IPS model holds that, during ideal analysis processes, students will follow up understanding the problem by formulating subgoals and planning a course of action. In this study, we did not find any evidence of either goal formation or planning. Instead, once the problem was understood, students immediately began to construct the diagram and dealt with subgoals as they arose. Contrary to model predictions, however, there was no evidence that the lack of explicit subgoal or planning statements harmed students' analyses. A typical sequence was for a student to achieve one subgoal and move directly to the next with subgoals addressed as they came up. [6, 7]

In summary, students' problem solving processes during the Problem Representation phase may be less than ideal according to the IPS model but these shortcomings did not appear to be a major source of students' difficulties. In cases when students did make representational errors, these were overcome. In addition, there was no evidence that the failure to state subgoals at the outset of the task harmed analysis quality in any way.

Framing the problem. In the Problem Framing phase, problem solving processes include the steps taken to execute plans and complete analysis, monitoring and evaluation of solution quality mapping of both givens and prior knowledge. The evidence available in these transcripts shows that students are capable of coordinating these generic problem solving processes. Transcripts included several examples of successful analysis processes in which students moved fluently between these problem solving processes. Within these episodes, students shifted smoothly across processes such as mapping prior knowledge, executing mathematical procedures, and mapping givens. [8, 9]

Additionally, this data also suggests that students' monitoring and evaluation processes are not a major source of analysis difficulties. Transcripts held several examples of students' detection and correction of errors. On the first problem, for instance, both KB and JM detected the error they made when representing the problem. On the same problem, KB also realizes that she has incorrectly mapped the man's weight and later corrects this error. On the second problem, JM's evaluation of the final product resulted in the correction of an error. As she finished, JM realized that she had not fully dimensioned the diagram, and she added the information she had initially omitted.

In the segments described above, once students detected the error, they were able to correct it. This was not always the case, however. On Problem 2, for example, both JM and DG were aware of the errors they were making with respect to distributed load but this awareness did not help them to correct the error. [10, 11]

Although these excerpts demonstrate that awareness alone is not sufficient for error correction, they also demonstrate that neither monitoring nor evaluation can be considered major sources of students' difficulties during analysis tasks. That is, the most troubling instances are marked by an inability to resolve detected errors rather than a failure to detect those errors.

The problem solving process that did cause significant problems for students was the mapping of prior knowledge. Mapping of prior knowledge, which occurs in the Problem Framing phase, requires that students recall and apply prior knowledge as needed. Because these processes are more fully discussed under the fourth research question, we will leave the details of this analysis for that section of the paper. For the time being, however, we can state that, with respect to the dimension of problem solving processes, only the process of mapping of prior knowledge can be cited as a major source of difficulties. On the other hand, it is also worth noting that we found no evidence to suggest students' generic problem solving processes provided any help when students were unsure of how to proceed. Thus, although problem solving problems may not cause students' difficulties, they do not act to resolve these either.

Are translations between symbol systems a major source of difficulty during analysis? In this study, we examined only the translation from the verbal to the diagrammatic representation, a process that takes place as the students move from the Problem Representation to the Problem Framing phase of analysis. We did not find any evidence to support the hypothesis that these translation processes are a major source of students' difficulties during analysis tasks. Students in this study were consistently able to represent and connect elements of the verbal problem statement in diagram form. Furthermore, we were unable to find any instances in which students had difficulty constructing the diagram or were unsure of how to represent a set of elements from the verbal statement in the diagram. Rather, in all cases, students were able to begin constructing the diagram immediately after reading the problem statement and no student ever expressed uncertainty of how to represent an element in diagrammatic form. In interpreting these findings, however, we must consider that illustrations were given for each problem statement. The implications of this methods point is addressed further in the discussion section of this article.

Consistent with what we found regarding students' problem solving processes, when students did experience problems with diagram construction, the source of the problem was not their ability to translate verbal elements into diagrammatic representations but rather the conceptual prior knowledge underlying these marks. In the second problem, for example, when JM struggles with the depiction of the distributed load, this struggle has more to do with her conceptual knowledge of distributed loads than with the diagrammatic symbol system. Although she continues to be uncertain, once she resolves the conceptual dilemma, she proceeds effortlessly to represent this solution on the diagram. [12]

One finding in this data that we found particularly surprising was that movements between symbol systems were not as recursive as we expected. Rather than shifting between verbal and visual symbol systems, students typically read the problem and used some form (e.g., underline, write notes, etc.) to make notes of the givens in the problem statement, which they then used to map givens onto the diagram. Once having made these notes, they did not return to the verbal problem statement but maintained their attention on the visual, or diagram, representation. The notable exceptions, discussed above under monitoring and evaluation processes, were the two students, KB and JM, who misrepresented the problem in Problem 1. In both cases, these students were forced to return to the verbal problem statement to correct their representation. In short, it seems that, because the diagram is fully redundant with the verbal problem statement, students do not need to move between the two.

Although the task required students only to complete the diagram, we expected some evidence that students were constructing a representation based on the equilibrium equations. We based this expectation on the belief that, if students have integrated across symbol systems, the mathematical representation of a given set of diagrammed elements would be activated when the diagrammatic representation was activated. In short, if internal representations are tightly bound, by principle and across semiotic systems, mathematical representations should be readily called forth as the student worked to construct the diagram.

Not surprisingly, this was true when dimensioning the diagram called for simple mathematical calculations to derive values from the givens. In Problem 2, for example, all students easily calculated the values for the distributed loads and the application of these mathematical procedures was fluent.

There was less evidence that students applied more principled mathematical representations during these phases of analysis. There were, however, two rather telling instances in which the strength of connections to mathematical representations appeared to play a role. In the first case discussed here, we see that a students' failure to consider mathematical properties provided a missed opportunity to correct a problem she was experiencing; in the second, we find evidence that a students' mathematical knowledge can be activated during the Problem Framing phase. The first example comes from student JM, who was struggling with the second problem. Throughout Problem 2, JM had a difficult time recalling how to handle the distributed load and her analysis process was marked by uncertainty. She also made several comments indicating if a given force from the load would be in a negative or positive direction. [13]

These statements by JM stand in contrast to the other students who dismissed the importance of positive or negative values. These differences suggest that JM's struggle with distributed loads was not limited to the conceptual but extended through to the equilibrium equations. Or said another way, if she had a more complete and integrated understanding of these equations, JM may have been able to use the mathematical representation to resolve the problem of the distributed load.

Support for this assertion comes from the contrasting way in which CF used her mathematical representations while solving the second problem. In short, CF's transcript is peppered with specific words that suggest she was thinking of the problem mathematically. For instance, she identifies that A and C *"…could turn out to be zero but, there is going to be an Ay and a Cy…"* Later, she shows that her knowledge of how to handle distributed loads extends beyond memorized application of a rule. [14]

In summary, students' abilities to translate from a verbal to a diagrammatic representation did not pose any major obstacles. This data does suggest that, when the derived diagrammatic representation is fully redundant with the given verbal representation, there are fewer recursive movements between the two than initially predicted. Finally, there was little evidence that students used knowledge of mathematical representations while constructing the diagram.

Are students' prior knowledge-driven processes a major source of difficulty during analysis? The answer to this question is a clear "Yes". Student's verbal protocols reveal that the application of prior knowledge was the leading cause of students' problems on these analysis tasks. As noted in the discussion surrounding the second research question, when students struggled, it was most likely to be attributable to the mapping of prior knowledge onto the diagram representation. These struggles with recall and application of prior knowledge were apparent in both problems, although not for all students, and during both the Problem Representation and Problem Framing phases of analysis.

In the Problem Representation phase, prior knowledge drives the recognition of patterns given in the problem statement by allowing students to map corresponding patterns from prior knowledge. In the data collected here, students' abilities to recognize patterns in the problem statement was a correlate of analysis quality. On the second problem for example, both KB and CF immediately recognized the pin reactions and moved easily to identify the forces to include. [15, 16] In addition, for both KB and CF, subgoals were similarly influenced by pattern recognition. [17]

Although the excerpts above show how prior knowledge can support Problem Representation, students' transcripts also revealed how failure to successfully match the patterns in the problem can harm the rest of the analysis. As we reported earlier, for example, on the first problem, KB did not verbalize an accurate representation of the problem before she began the analysis. Following from this, KB does not recognize the force-related pattern implication in the problem statement, and she is unable to use this knowledge to drive subsequent analysis processes. Instead, she is initially distracted by the angle and does not know how to proceed. In summary, students' abilities to recognize the patterns in problem statements, and map these against prior knowledge, affected subsequent analysis processes.

As noted earlier, students' knowledge-driven processes exerted a powerful influence on analysis quality. Students were most fluent when they first identified a principle and followed with more surface level processes such as mapping givens or applying routine formulas. By contrast, when superficial processes came before, or in the absence of, prior knowledge principles, errors were made and analysis suffered. In the second problem, for example, when DG cannot recall the principles associated with distributed loads, she is reduced to an effort at matching the givens to prior knowledge. [18]

Furthermore, the application of prior knowledge seemed to be the one aspect of the analysis from which errors were unrecoverable. As noted earlier, students recovered from errors made as they represented the problem, left out a dimensioning step, or translated givens onto the diagram. They did not fully recover, however, when they could not recall some specific aspect of knowledge and this was true regardless of whether the knowledge weakness occurred during Problem Representation or Problem Framing phases of analysis.

For instance, on the first problem, after KB fails to recognize the required force principles, she never recovers. Although she completes a diagram, two forces are omitted. Of the three dimensions of problem solving included in the IPS model, knowledge-driven processes exert the strongest influence on analysis quality. Prior knowledge affected students' abilities to recognize patterns during the Problem Representation phase and to engage in knowledge-driven processes during the Problem Framing phase. It is likely that failures of prior knowledge were unrecoverable because, unlike other aspects of the analysis process, there are few knowledge supports available in the problem. That is, neither the problem statement itself nor the students' processes during the analysis task are likely to uncover anything that will tell the student something she does not already know. Students, it seems, either knew what needed to be done, or they did not.

Summary of Results

On the whole, the data collected here supports the conclusion that the IPS model provides an accurate and comprehensive description of students' processing while completing analysis tasks. Students' verbal protocols included statements indicating that processes related to all three dimensions included in the model are active during analysis. Students employed generic problem solving processes, activated prior knowledge, and translated between semiotic systems. The data also suggested two major revisions to the model. First, although the depiction of the model shown in Figure 1 indicates that each phase is dominated by a specific semiotic system, our expectation was that students' movements between these systems would be recursive. Based on the data collected here, however, students did not move between semiotic systems. Instead, once students noted the givens from a particular verbal statement, there was little movement toward reinspection of the verbal statement.

A second revision pertains to the role of goal and subgoal formation in the first phase of the model. In this study, students did not generate subgoals at the outset but rather handled subgoals as each arose during the construction of the diagram. Thus, this data suggests that subgoals are more closely tied to execution than to understanding and representing the problem. Given the data collected here, it is unclear if this method of handling subgoals impacts analysis quality. Perhaps these students' abilities to construct FBDs would have been improved if they had developed subgoals in a more forward-reaching manner than they did here.

This study supports the major contention underlying the model. Specifically, by including all three dimensions in the model, and showing how these dimensions are related to one another, we were able to capture a more comprehensive view of students' analysis behaviors than has been captured by previous models. As demonstrated here, all three dimensions operate to support analysis and thus, any effort to understand or improve students' analysis processes must also adopt a similarly comprehensive view.

Major Sources of Students' Difficulties

The results of this study shed light on those aspects of the analysis task that posed the greatest challenge for students. Students' abilities to recall and apply prior knowledge were the single biggest obstacle to analysis quality. A student who was able to recognize the patterns embedded in the problem statement, recall requisite principles and rules, and apply these to the FBD had a distinct advantage over those who could not similarly use prior knowledge. Although we did not measure either the accuracy or organization of students' prior knowledge in this study, it is likely that these qualitative aspects did impact performance. This assertion is based on previous work[32,33], which has demonstrated that not only the quantity, but also the quality, of prior knowledge impacts problem solving performance. In this study, it is likely that the students who were able to recognize the principles embedded in a particular problem had higher quality knowledge structures representing those principles than students who were less able to recognize these same principles.

Recognition alone was not enough, however. It was particularly evident on the second problem that students' analysis processes were also dependent upon their ability to recall and apply specific aspects of prior knowledge. On the second problem, for example, two of the four students could not recall how to handle a distributed load and, in both cases, this caused serious disruptions to their processing. These processes were most evident during the Problem Framing phase of the model. Higher quality analysis processes in this phase can be best described as knowledge-driven. When students were able to apply their prior knowledge, this knowledge led the way as students constructed the diagram, mapped givens, monitored progress, and applied mathematical procedures. In short, what one knows and how one uses that knowledge is a strong predictor of analysis quality.

Although students' prior knowledge exerted the strongest influence on analysis quality, students' generic problem solving processes were also lacking. With respect to the Problem Representation phase of the model, all students demonstrated that they were capable of understanding the problem but students did not always carry out this step. In two cases, for example, students failed to adequately understand the problem and their analysis suffered as a result. In both instances, the students ultimately had to return to the Problem Representation phase and explicitly engage in efforts to understand the problem. Similarly, students did not explicitly establish subgoals at the outset of the analysis task. Thus, although students in this study were able to understand the problem and identify subgoals, they did not always do so in the most beneficial ways.

On the other hand, the students in this study did not evidence any difficulties with respect to generic problem solving processes during the Problem Framing phase. Once students represented the problem, they were able to construct the diagram, map givens, complete mathematical calculations, monitor progress, and evaluate solution quality. When these processes followed from an accurate and complete representation of the problem, and the recognition of relevant subgoals, students' executions were smooth and fluent. Disruptions to execution were more closely tied to a failures of prior knowledge than to difficulties with the execution processes themselves.

Accordingly, we conclude that there is a significant difference between the difficulties students face when they struggle with generic problem solving processes and when they struggle with prior knowledge processes. In short, students were able to recover from errors that stemmed from their use of generic problem solving processes. They did not, however, recover when the error was initiated by an inability to activate and use relevant prior knowledge. Unlike other dimensions of analysis, prior knowledge appears to operate in an all-or-none fashion: a student either knew the principle or she did not. Importantly, however, these prior knowledge failures are likely exacerbated by weak generic problem solving processes at the Problem Representation phase. Specifically, students' abilities to recall relevant prior knowledge would likely be enhanced by more dedicated efforts at understanding the problem and identifying subgoals. Students in this study may have benefited, for instance, if they had spent more time thinking through what knowledge was required by each problem and how that knowledge should be organized and used in the context of that problem.

Students' abilities to translate between symbol systems also did not appear to be a major source of difficulties during the analysis tasks. Contrary to McCracken and Newstetter's[34] hypothesis, students in this study moved quickly and accurately to the task of diagramming the elements presented in the verbal problem statement and this was true for both problems. Thus, although translations sit as a necessary aspect of analysis, these processes did not contribute to any difficulties students experienced during construction of the FBD. Before dismissing the importance of these translations, however, some differences between the context of the current study and other potential analysis tasks should be identified. Namely, for each problem in this study, students were provided an illustration that depicted the elements given in the verbal problem statement. It is possible that students in this study did not need to move between the verbal statement and the constructed diagram because the provided illustration mediated the translation. Additionally, because we did not require students to derive a mathematical representation, we are not able to draw conclusions about the ease or difficulty of that translation process. In short, although this study does not support the hypothesis that symbol system transformations are a major source of students' difficulties, it is quite possible that a different methodology would have resulted in a different conclusion.

Conclusions and Implications

There are three main conclusions that stem from the results of this study. The first of these is the demonstration that the IPS model did adequately and comprehensively capture students' analysis processes. For each problem and for every student, we found evidence that problem solving processes, prior knowledge, and symbol systems play a role in the construction of the FBD. As noted earlier, however, the data collected here suggests some revisions to this model and follow up research questions. Specifically, it is unclear based on the current data, if goal formation should be moved to the Problem Framing phase or the degree to which recursive movements between phases should be expected.

A second major conclusion concerns the importance of students' prior knowledge. The data set obtained in this study suggests that the interventions that would most likely improve students' analysis processes are interventions that target the quality of students' prior knowledge. To be effective, these interventions would need to work toward assisting students in recognizing the patterns embedded within problems and improving the overall quality of students' knowledge. The current study demonstrates that students would benefit from a better understanding of how to recognize principles when these are embedded in problems and how to map the structure of the problem onto the structure of principled prior knowledge.

The final conclusion is a cautionary point that highlights the role of both problem solving processes and semiotic transformations. Our finding that processes tied to these two dimensions did not act as a major source of students' difficulties on analysis tasks should not be understood to mean either that these dimensions are unimportant or that no instruction is necessary. We did find, for instance, that students' problem solving processes were a correlate of analysis quality. Furthermore, although students did not have "difficulty" constructing the diagram, they did seem to use this process as a means of prompting subgoal formation. In short, processes tied to both dimensions were found to be important during analysis. Regarding questions of instruction, we should not overlook the possibility that neither dimension acted as a major source of difficulty precisely because of the instruction students had received. It is possible that, without instruction on the conventions of diagrams or some instructional reference to generic problem solving processes, either or both of these dimensions would have acted as a major source of difficulty for students.

In closing, we believe that the IPS model represents an important step forward in understanding students' analysis processes and the interventions that might effectively improve these processes. By integrating across disparate approaches to understanding analysis, the IPS model is more comprehensive than existing models. Although the small number of participants included in the current study precludes an analysis of individual differences, we believe a potential consequence of this more comprehensive approach will be our ability to identify and define different profiles of students who are struggling with analysis tasks.

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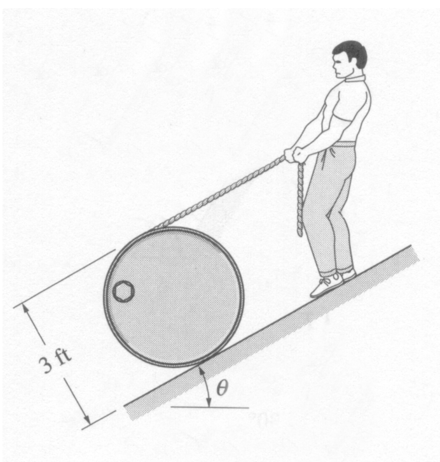
Appendix A

This appendix contains the two experimental problems.

Problem 1

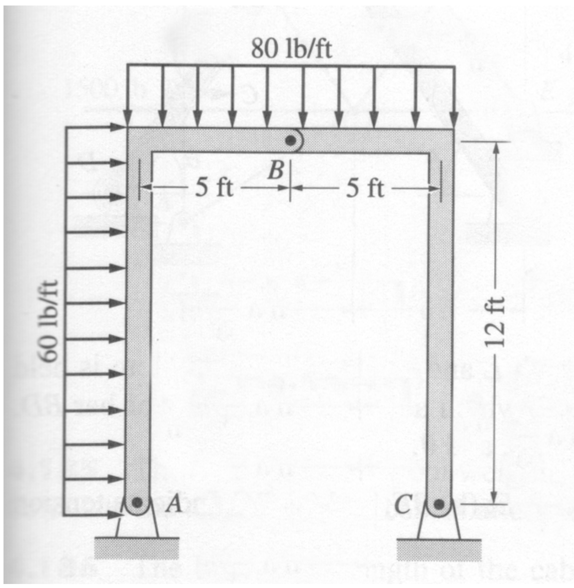
A man, weighing 170 lb, keeps an 800 lb drum from rolling down an inclined surface by pulling on a rope wrapped around the drum. The surface is inclined at an angle, θ, equal to 20°. Draw a fully dimensioned free-body diagram for the man.



Problem 2

Part a) Draw a fully dimensioned free-body diagram for the complete frame A-B-C.

Part b) Draw a fully-dimensioned free-body diagram for the member A-B.



Appendix B

This appendix contains the written descriptions of students' analysis processes that were derived from verbal protocols. A single description was written for each student on each problem.

KB:Novice 1, Problem 1

On the first problem, KB made 6 errors. On the final FBD, both reaction and friction forces were omitted. Her verbal protocol contained 19 lines of coded text.

Phase 1: Problem Representation. KB begins by reading the verbal problem statement and very quickly asks for clarification about friction. She states that she needs to start with the givens but makes an error identifying the man's weight. KB does not stop to identify the body to be diagrammed but moves to the next phase before forming a problem representation or setting subgoals.

Phase 2: Problem Framing. Consistent with the model, in this phase, KB both executes and corrects her analysis performance. She draws the man and then erases it and redraws. The first part of the FBD that she focuses on is the angle theta but, this brings her to a roadblock. After identifying theta, there is extensive silence (12 seconds) with no diagramming. At this point, KB appears to be stuck. Although she has clearly monitored, she seems unable to determine a helpful corrective action.

Following the silence, she identifies and draws the man. She does not state a goal but it seems that her intent is to start with a different component of the diagram. She knows to include the force "being pulled down" and, although she cannot correctly name the force, she adds it to the diagram. KB adds the weight of the man and repeats her earlier error but soon corrects it. She points to the drum and identifies the weight but then sits in silence for 3 seconds with her pencil on the drum. She makes a correction to the FBD and identifies where the weight would be. KB points to the drum, writes down the weight, and then sits in silence for three seconds. During the silence, she continues to point at the drum. She finishes the diagram by erasing a previous mark to correct where the weight would act and labeling it.

Throughout this process, KB's performance is disfluent. Although she follows the framework of processes laid out in our model, she seems to have difficulty identifying the forces involved. She was unable to name included forces and failed to include both reaction and friction forces. She did not evaluate her diagram when she finished. It is possible that KB's difficulty recalling and including all of the necessary forces could be tied to her failure to adequately represent the problem. Because she did not stop to determine the deep structure of the problem, she was unable to recall the force principles needed for a correct diagram. Her attention to the surface details of this problem, the givens in the problem statement, are consistent with this hypothesis.

KB : Novice 1, Problem 2

Performance on the second problem was better. KB only made 1 error and the final FBD was correct. The verbal protocol was 37 lines long.

Phase 1: Problem Representation. After reading the problem statement, KB explicitly identifies the body to be diagrammed. She does not state any subgoals or explicitly identify any givens at this point. Instead, as with the previous problem, she moves quickly to begin diagramming the frame.

Phase 2: Problem Framing. As soon as KB starts the diagram, she recognizes the importance of the pins and the rule of X and Y components. She dismisses B, noting that all reaction forces are internal. She states the principle that a distributed load is replaced by one load at the center. From here, her problem solving proceeds smoothly as she calculates the loads and dimensions the diagram. In each case, she first identifies the principle (e.g., force components) or rule (e.g., distributed loads) involved and then applies the givens from the problem. Her performance on Part B is consistent with Part A. As she begins this diagram, she immediately states that there are two forces and proceeds to add X and Y components and loads. Again, calculations and additions to the diagram follow from statements of principle.

In short, throughout this phase, she moves easily between identifying force principles, mapping in givens, and calculating needed values. Although there is evidence that KB was monitoring her performance, she found the need for little corrective action. Contrary to model predictions, there is little movement between the diagram and the verbal problem statement.

The differences between the solution process for Problems 1 and 3 are striking. In the first problem, KB has difficulty identifying the principles involved, and she reports mapping givens much more frequently than she does in the second problem. Instead, in the second problem, references to the givens follows from her recognition of principles. The attention paid to the givens match the content of her verbalizations which are much more focused on the surface details of the problem.

CF : Novice 2, Problem 1

CF made only one error while completing the first problem and her final FBD was accurate. Her verbal protocol contained 24 lines.

Phase 1: Problem Representation. She begins by reading the problem statement and explicitly identifies the body to be diagrammed. Similar to KB, she does not state any subgoals at this time but moves quickly to begin diagramming.

Phase 2: Problem Framing. The translation from the problem statement to the sketch of the body is easy for CF. Once she completes the diagram of the man and the slope, she begins to map principles onto to the diagram rather fluently. In each case, she identifies the associated force principle and follows that with the specific values related to this problem. CF proceeds through the problem as if guided by a checklist of forces to go through. She quickly identifies the principles related to center of mass, normal force, incline planes, friction, and tension and maps these onto the diagram. When the FBD is complete, she indicates that this is all she knows of. She extends her thinking beyond the given problem and states what would have to be done to also create an FBD of the drum. During this phase, CF does not look back to the problem statement.

Throughout this phase, CF's processes are consistent with those predicted by the model. She executes the solution process and readily maps givens and principles onto the diagram. Her processing seems to be primarily principle driven as she uses her knowledge of relevant forces to guide her actions. There are few instances of monitoring, which is consistent with the few errors that CF makes. Once the diagram is complete, CF does not verbalize any evaluation processes.

CF: Novice 2, Problem 2

On the second problem, CF committed only one error and the final FBD was correct. There were a total of 37 lines in the verbal protocol.

Phase 1, Problem Representation: CF began the problem solving process much like the first problem. She read the problem and immediately began drawing the frame. Although she does not make an explicit statement about the problem representation or any subgoals, as she begins the diagram she states that she needs to "start with the frame".

Phase 2, Problem Framing. CF moves easily from the verbal statement to the start of her diagram. She transitions with the frame drawing statement above and then quickly identifies the importance of the pins at A and C and applies knowledge of the X and Y components. From this point, she recalls how to handle a distributed load and maps this easily onto the diagram. Consistent with the solution process in the first problem, there is a pattern in which she states the principle first and then calculates the correct value. Throughout, she is moving between the cognitive processes of mapping her prior knowledge and pulling in givens.

As she executes through Part B, the problem solving process continues to move smoothly. She identifies B as a pin and determines the X and Y components. At this point, she repeats a behavior from the first problem in which she extends beyond the problem given to state what would happen if you also had to diagram member BC. As she does so, she notes that the forces would be acting in the opposite direction. Once she adds the forces onto the diagram, she easily calculates the load values and finishes dimensioning the diagram. At the end of the solution process, she states that she is finished but does not engage in any evaluation processes. Again, consistent with her processing in Problem 1, there is little movement between the diagram and the verbal representation during this problem framing phase.

What is most salient about CF's processing during both problems is the ease and flexibility with which she applied analysis skills. We see flexibility in her solution processes because she approached and solved the two problems differently. In the first problem, the process appeared to be scaffolded by an internal list of forces she believed she must consider. In the second problem, her reasoning appeared more rule-based as she stated and applied rules pertaining to pins and to distributed loads. In both instances, she quickly represented the problem and moved on to executing the steps to construct the diagram. During the problem framing phase, her verbal protocols show that her analysis processes were driven by her knowledge of the involved principles.

JM: Novice 3, Problem 1:

JM verbalized only one error as she solved the first problem and the final FBD was considered correct. There were 61 lines in her verbal protocol.

Phase 1: Problem representation. Although JM's final FBD was reasonable, she had a difficult time constructing the diagram. She began the task by reading the problem but did explicitly state the body to be diagrammed. She does not set any goals as she reads the problem but instead moves immediately to the diagram.

Phase 2: Problem Framing. JM's failure to represent the problem causes problems as she begins the diagram. She misunderstands the problem and begins by drawing the FBD of the drum rather than the man. Her starting point for the diagram is with the angle of the slope. As with KB, the first novice discussed, this starting point does not lead to a solution path. The beginnings of her diagram is marked by uncertainty of how to proceed. She sets a number of subgoals and makes statements such as, "I'm just looking at what I need to draw." and "…you have to decide what you're going to make as your body…". Following these statements, JM consults the problem statement again and corrects her problem representation but she does not do this until the 20th line of the protocol.

Once JM correctly represents the problem, she begins to make progress. She identifies principles related to normal, reaction, gravitational, and friction forces. What marks this solution process, however, is both disfluency and uncertainty. Even though JM verbalizes knowledge of these forces, her verbal protocols are peppered with "uhm's" and statements of indicating she is unsure. Identification of the forces appears to be driven at least as much by superficial characteristics of the problem. For instance, when talking about tension forces, she begins by labeling this "force due to drum". Thus, although she attempts to use a list of known forces to guide the solution process, her uncertain knowledge of these hampers her efforts. At the end, she does evaluate her diagram. Although this evaluation leads her to state that she is not sure of the it's accuracy she does not follow up with any corrective action.

JM: Novice 3, Problem 2

Phase 1: Problem Representation. Unlike the first problem, JM begins by explicitly identifying the body to be diagrammed. Again, however, she does not set any subgoals but moves to the diagram.

Phase 2: Problem Framing. As she begins her diagram, she immediately states that the A and C are pins and breaks these into the X and Y components. Although she mentions internal forces, she does so while stating that she does not need to worry about those. Once she has labeled the X and Y components, however, she becomes stuck on how to deal with the distributed load. She states that she is confused between content from two different courses and does not know how to proceed. Following this uncertainty, she attempts to diagram some of the problem attempting to use this process to help her determine what to do next. Thus, she appears to use diagramming as a corrective process.

She restates that there will not be any internal cuts and decides to draw the distributed load at a point load. Although the other novices in this set indicated that the positive and negative values did not matter, JM did not dismiss these and instead thought about whether X and Y would go in positive or negative directions. JM does correctly locate the load at the midpoint, but she expresses uncertainty and her calculations, although correct, are disfluent. Once the FBD has been constructed, she briefly repeats some of her reasoning to check the accuracy of her solution.

As she moves to the Part B, a similar pattern repeats. She again notes the importance of the pins but is unsure how to handle the distributed load.

JM's solution to the second problem is marked by her uncertainty of how to deal with a distributed load. Although her resolution is correct, her problem solving process is disfluent. At times, when she cannot recall what to do, she adds to her diagram as if using this as a strategy to help her determine a principle she is uncertain of.

As with the previous two novices, her analysis processes can be captured by the model. The process begins with the verbal representation and moves to the diagram. JM illustrates the importance of representing the problem at this phase. In the first problem, she failed to represent the problem in Phase 1 and had a difficult time during Phase 2 until she returned to the verbal representation. From that point, she was able to move more easily to generate the FBD. On the second problem, her problems occurred in during Phase 2. She was unable to access the prior knowledge of distributed and struggled to determine how to deal with this aspect of the situation. Also consistent with the first two novices, JM made very few moves between symbol systems. The notable exception was when she reread the problem statement when solving the first problem.

DG: Novice 4, Problem 1

DG did not verbalize any errors on the first problem but she did omit normal forces from both her verbal report and the final diagram. Her transcript was 8 lines long.

Phase 1: Problem representation. DG began by questioning the body to be drawn and then identifying the man. The first 3 lines of her transcripts are dedicated to representing the problem. She does not set any subgoals but moves directly to diagramming.

Phase 2: Problem Framing. She begins by drawing the man and then adds gravity, friction, and tension. At two points in the transcript she indicates uncertainty by stating. "I think" but does not follow up on this uncertainty. One of these statements of uncertainty occurred as she finished the diagram and stated that "I think those are the only three forces". Although her evaluation was incorrect, she does not engage in any efforts to check this and instead states that she is finished.

DG: Novice 4, Problem 2

DG does not make any errors of commission but does express the force of the distributed load as a "point" rather than a force. She also has difficulty managing the distributed loads in this problem and, although she correctly resolves, she is never certain of her response. Her transcript is 40 lines in total.

Phase 1: Problem representation. DG begins by asking clarification questions to determine if she is to resolve the forces. Although she does not explicitly state a problem representation, she does follow up her reading of the problem statement by drawing the frame. No subgoals are stated and she moves directly through the diagramming after drawing the frame.

Phase 2: Problem framing. Her first action is to draw the forces associated with the pin reaction. She then encounters a problem as she deals with the distributed load. At this point, her transcript is marked with numerous statements and behavior indicative of her uncertainty. For instance, there are silences broken by statements such as "not sure" and "okay, but…". She states that she should know how to do this because it was covered in class. She then begins to inspect the givens provided in the problem looking for helpful information. She states that she believes she is to find the area but she cannot because the height is not given. She compares the current task with her previous experience and states that this one is different from what she has seen previously. Although she resolves the distributed load, she is uncertain throughout. Additionally, she talks about where the "point" should go and does not identify the associated force. She then moves on and completes the diagram. There is no evidence that she evaluated her diagram of the first part.

As she begins the second part of the problem, she moves more quickly and does not state any thoughts about the distributed load. Instead, she maps the givens on to the diagram and makes few corrections to the diagram. Again, she uses the word "point" rather than force. There is no evidence that she evaluated the quality of her final solution.

Appendix C

This appendix contains excerpts from verbal protocols that illustrate the themes described in response to research questions two through four. Numbers in paranthesis correspond to numbers given in the main body of the results section.

I. Problem Solving Processes

Representing the problem

(1) [Reads the problem quietly] I have to draw it for the drum or the man? Oh…okay, so…

DG Problem 1 Ln 1 – 3

(2) [Reading the problem] The complete frame ABC…[Reading]

JM Problem 2 Lns 1 – 3

(3) Alright, so I'm just reading through and I'll just write the givens out.

JM Problem 1 Lns 1 – 2

(4) I'm just going to draw the diagram here. [Erases line, adds new line] So the total of theta is at 20 degrees from the bottom of the incline…[silence]…[silence for 12 seconds, swiping paper with hand]…I guess they want the forces on the man.

KB Problem 1 Lns 6 – 9

(5) …so I'll have to break that down into components due to gravity…Oh the fully dimensioned diagram for the man!! Okay, that's the problem.

JM Problem 1 Lns 19-21

(6) ...and there's also going to be a Cx and an Ax as well since they are both pins. And then as far as the distributed weights go...

CF Problem 2 Lns 7-9

(7) ...and we'll call it um... force due to drum and then that will be the tension force due to the drum and then the drum will also have the force would be the 800 pounds and that would be straight down so you would have to break that into components...

M Problem 1 Lns 45 – 49

Framing the problem

(8) Um, distributed loads can be replaced by one load um, acting at the center I think I'm going to have to multiply, yeah, so that's 12 feet times 60 that's... alright so 720 for this one acting through the center at this connecting it at point B which is also at the center of

80 times 10 which should be 800 pounds

KB Problem 2 Ln 10 – 18

(9) this is how you can resolve them and since they're squared you can resolve it with a single force acting at the midpoint of the distribution which would be at five feet. And that would be what 800 pounds since it's ten feet long and 8 pounds per foot and then for this load it would be similar frame...

CF Problem 2 Ln 11 – 17

(10) and... I don't know I guess I'm struggling with the distributed load again

JM Problem 2 Ln 76 – 77

(11) And there's some force being pulled down [drawing arrow]. let's call it P [labeling] I guess...

KB Problem 1 Ln 13 – 15

II. Translations Between Symbol Systems

Translations to diagrammatic representations

(12) [pause]... [about 12 seconds]... right now I'm just looking at um the distributed loads. And I'm getting confused now between uh, what we've done what we did in EMEC 11 and what we did in EMEC 13 now I'm not really sure what this problem would be but um... *I think I'm...* [long pause] well I'm not going to make any internal cuts or anything since it's the entire frame ABC so I'm just going to draw this distributed load at a point load and know that it is going to act in the middle.

JM Problem 2 Lns 24 – 39

Translations to mathematical representations

(13) so its going to be the 60 pounds per 12 feet and that will be a force in the middle and the positive Y direction…and just by looking at uh... sum of the forces you know that these are probably going to be in the negative X direction and I don't know...

I guess I'm struggling with the distributed load again this except that little bit right there. But the negative direction...

JM Problem 1 Lns 44 – 45; 52 – 55; 77 – 80

(14) since you know both the weight distribution load and the length that is you can resolve them and since they're squared you can resolve it with a single force acting at the midpoint of the distribution

CF Problem 2, Lns 10 – 13

III. Students' Prior Knowledge-Driven Processes

Representing the problem

(15) And you have two pin reactions which means that it's free to move but it has both and X and Y components. There is no reaction forces at B because they are all internal so you don't have to focus that.

KB Problem 2 Lns 4-9

(16) but there is going to be... an Ay Cy and there's also going to be a Cx and an Ax as well since they are both pins.

CF Problem 2 Lns 6-8

(17) since you know both the weight distribution load and length that is you can resolve them

CF Problem 2 Lns 10-11

Problem framing

(18) but... cause I think you have to find the area of it, but i'm pretty sure you need to know the height to be able to do that and... if this has 80 pounds over the top usually the ones I've seen says that one of these is 80

DG Problem 2 Lns 18-21
