# AC 2008-2556: A COMPREHENSIVE LABORATORY CURRICULUM IN SINGLE DEGREE OF FREEDOM (S-D-F) VIBRATIONS; PHASE I – WORKING MODEL EXPERIMENTS

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# A Comprehensive Laboratory Curriculum in Single Degree of Freedom (S-D-F) Vibrations
## *Phase I – Working Model Experiments*


**ABSTRACT**
A package of experiments for examination of the Single-Degree-of-Freedom Vibration Systems is proposed for national adoption. The comprehensive and robust package will examine the Dynamic Characteristics of Free and Harmonically Forced Systems with and without Damping. The modes of vibration are linear with Mass, Spring and Damper in a vertical orientation. The capabilities incorporated in the design of the associated apparatuses allow for adjustments of the values of: a) mass, b) damping coefficient, c) spring constant, d) the setting of the Initial Conditions, e) control of the frequency, and f) the amplitude of the driving forces. The apparatus will be interfaced with Matlab and National Instrument's LabVIEW/Vernier's LabQuest to measure system response and display the results both numerically and graphically. Students in a Vibration, Control Systems, or a measurement related exercise/course may be challenged to generate the mathematical models of the proposed modes of experimentation.

In Phase-I of the project; and to insure the high quality and reliability of the potential designs (for the robust and affordable apparatus), Working Model – 2D software has been employed for the preliminary modeling, simulation and testing of a set of practical systems. The successful implementation of this phase of the project has encouraged the authors to share their practical and cost-effective results with the educational community. This effort should prove valuable to those colleagues who are limited in time (for set up and conducting such valuable experiments and exercises) and also those who struggle with resources (for obtaining the necessary hardware/space). Phase-II of the project will concentrate on the design, fabrication, interfacing, and testing of all the intended modes and will be presented in a future work.


**I - INTRODUCTION**
Experimentation is one of the most effective means for a student to gain a thorough understanding of the materials taught in class. It provides a means to verify the theories through data collection and interpretation. A single degree of freedom vibrations apparatus can be used for either demonstration or experimentation in such classes as Dynamics, Vibrations, Controls, Differential Equations, and of course, an Engineering Laboratory.

Despite the many advantages of using a commercially available single degree of freedom apparatus, they are prohibitively expensive for many institutions, which can cost upwards of $20,000. Commercially available vibration apparatuses are not only expensive but they also tend to be limited. The other apparatuses researched by this group did not provide all the desired modes of operation and modularity of experimentation in Single Degrees of Freedom Modes.

It was therefore desirable to design an apparatus that would be able to be replicated by other educational institutions, with a budget of $3,000 in materials and components and about 75 hours of machining. The unit created would incorporate design features that would allow for a reduced cost and increased quantity of features. The design would incorporate a wide variety of modes and experimentation in order to provide the students with a comprehensive understanding of (S-D-F) vibrations.

*To develop a better appreciation of the proposed simulated experiment (in Working Model), it is essential to have a preliminary discussion of the features of the physical apparatus. This is important in that all the physical parameters of the intended unit are all incorporated in the simulation process and may prove to be a critical link in visualizing the actual environment and the problem at hand. The authors hope to generate sufficient interest in potential use of such easily simulated experiments considering the degree of difficulty involved in the selection, design, and fabrication of the major components and subsystems of the overall design.*

## II - OBJECTIVES OF THE PROJECT

The following were the predetermined parameters that needed to be incorporated:

1. To create a set of experiments in Single Degree of Freedom (SDF) Vibrations.
2. To design and fabricate the associated apparatus such that it would be safe, robust, modular, and relatively inexpensive.
3. To design the apparatus so that it would be reproducible by other educational institutions.
4. To utilize Working Model for simulation, testing, and obtaining the practical range of parameters that are both suitable for a laboratory environment and are preferably available stock items
5. To provide a means to collect and interpret data recorded through experimentation.
6. To share the schematics and parts required to fabricate additional units via a website accessible through The College of New Jersey's website.
7. To develop appropriate instruction for experimentation to optimize the function of the apparatus.

## III - BACKGROUND

A comprehensive review of literature (1-6) was conducted on Single Degree of Freedom (S-D-F) Vibrations as well as the existing pertinent laboratory apparatuses. Appendix (B) reflects on a collection of the relevant equations for the study of S-D-F Vibrations.

After examining what was commercially available, and determining what features were both desirable and undesirable, certain design considerations were made. The final design would be available for other educational institutions to manufacture. As a result, the cost was not to exceed $3,000 and the parts used had to be as commercially available stock parts as possible. An important consideration was that the combined mass of the frame of the apparatus and the added dead mass (at the time of operation) had to be large enough in order to suppress the effects of external vibrations and thereby provide more accurate results.

One-page laboratory experiments would be developed with the intention of trying to keep the time to complete different phases of the experiments between 30 to 60 minutes. To review the results of experiments, a data acquisition system would need to be integrated. This would require data acquisition hardware as well as programmed software to record and analyze the measured data. It was decided that either an accelerometer or a linear encoder would be used to measure the motion and LabVIEW / LabQuest would be used to acquire the data and export it to an Excel file. Once exported it would be inputted into Matlab for analysis and comparison with theoretical results.

The unforced system would incorporate adjustable mass, damping, and spring constants.
It would also incorporate the ability to provide an initial velocity or initial displacement.
The unforced system would need to be designed to allow the operator to study an overdamped, underdamped, and critically damped system (see Figure 1).

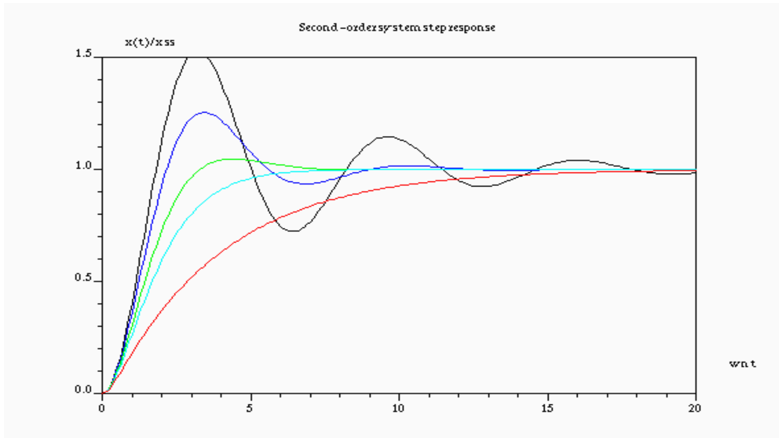


**Figure 1.** Vibration Responses

The forced system would also have adjustable damping and mass, but not spring rate. However, it would allow for adjustable frequency and amplitude of the harmonic oscillator. The initial conditions would not be adjustable and set to zero for both initial velocity & displacement.

## IV - DESIGN OF THE COMPONENTS AND SUBSYSTEMS OF THE APPARATUS

### Section I: Unforced apparatus

General design considerations were that the apparatus would be approximately 24” in width, 18” in height, and 6” in depth. The housing containing the mass would be supported by linear bearings riding on two symmetrically hardened rods. Maximum initial displacement would be about two inches giving a total maximum oscillation of approximately four inches. Figure 2 (below) displays the preliminary design of the system.

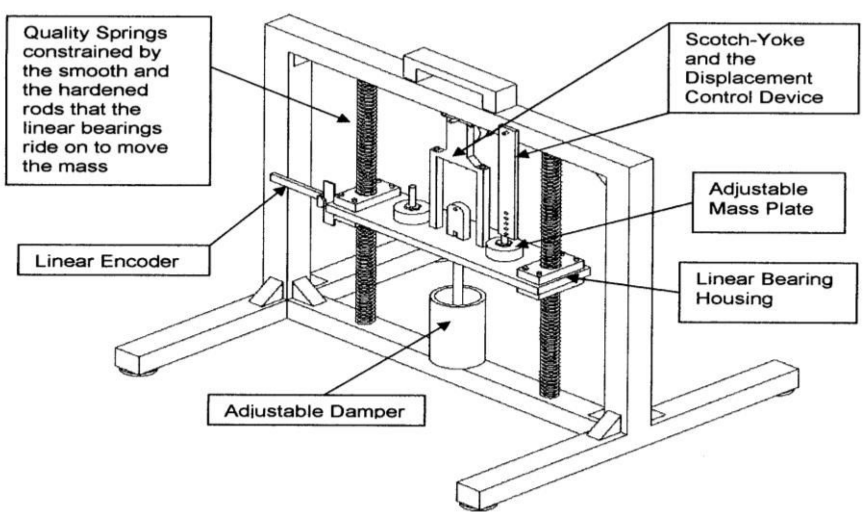


**Figure 2.** Preliminary Design of the Proposed System

As shown, the damping and the loads are all applied symmetrically. This was important in order to reduce friction in the bearings and increase the accuracy of the results. Considering the thrust and the mass loads that would be applied, it was decided to utilize closed fixed-alignment linear ball bearings with a half inch inner diameter.

The primary consideration in designing the unforced system was for it to be able to reach a critically damped state. A critically damped system occurs as a function of the mass, spring rate, and damping. The mass used for the system was assumed to have a range of 2-6 lbs which would provide substantial range and diversity of data.

Taking a predetermined mass and critical damping, the appropriate spring constant per spring was calculated. The desired critical damping value was set at 20 lbs*in/s because this was in the middle of the range of the air damper. This allows the user to operate the system as underdamped for half of the damping range and overdamped for the other half. The only concern is that oscillation for an underdamped system would not occur until a very low damping value. However, this can easily be addressed by ordering springs with stiffer constants. This resulted in a desired spring constant of 6 lbs/in per spring. A simulation of this system was developed using Working Model software (see Figure 3).

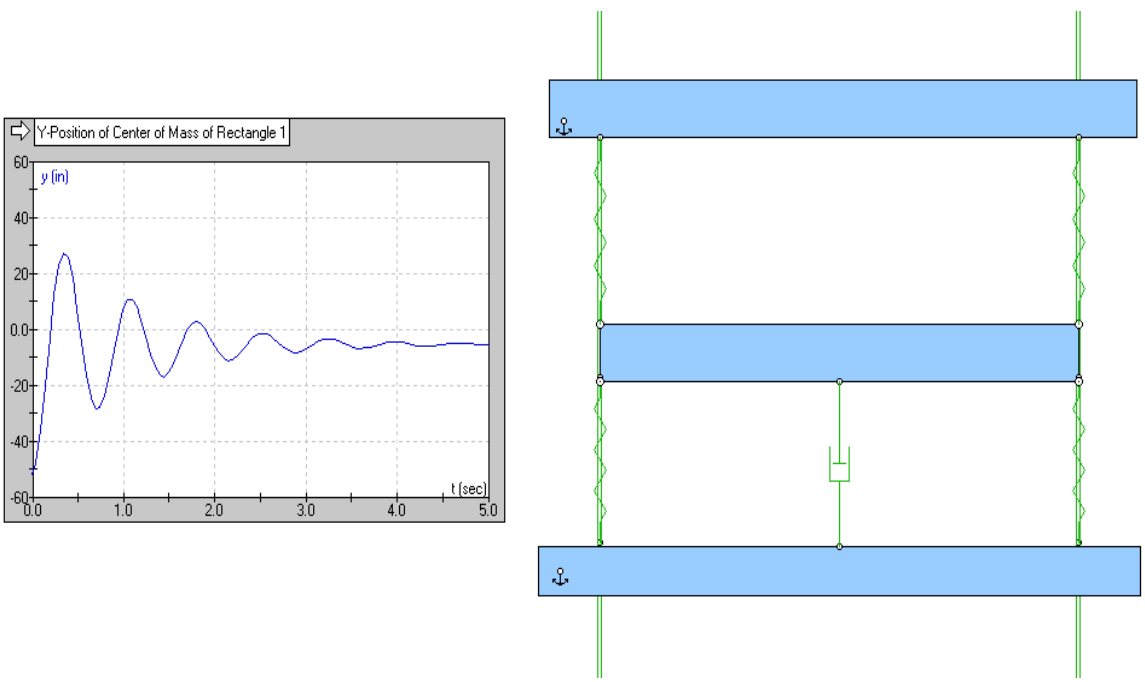


**Figure 3.** Working Model Simulation of the System and the Response

With these parameters the type of springs that would be viable for this apparatus were then considered. Compression springs were decided to be used and due to a desire to have a maximum initial displacement of two inches, the spring itself would have to displace a total of four inches without incurring permanent deformation. Due to lack of availability in commercially available springs that met the proper specifications it was decided to put two springs in series. A Pro/E model of the proposed system was developed and is shown in Figure 4.

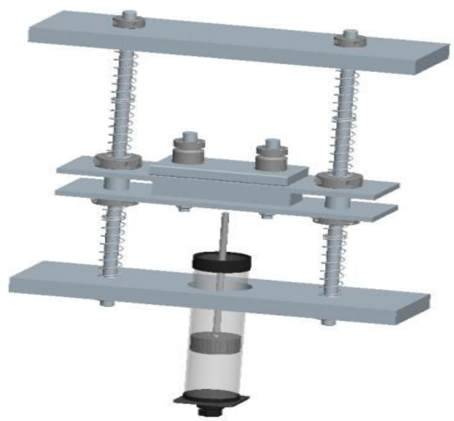

**Figure 4.** Skeleton Representation of the Unforced System - Pro/E Model

**Section II: Forced apparatus**

In order to address the need to explore both unforced and forced vibrations experimentally, a driven system needs to be created which is similar to the undriven system but with the added ability to have a forced harmonic motion applied to the mass.

An important design decision for the driving force was to determine an acceptable frequency for the nominal value of the sinusoid. A speed of 1 Hz was chosen because it allows the system to move at a reasonably fast pace for visualization. The frequency can still be increased or decreased by up to a factor of three and produce results which will not be unrealistic for the driving mechanism to handle the stress components.

The system itself contains the same basic parts as the undriven vibration apparatus. In order to make the design easier to construct, and for experimental results to be more easily compared, the two systems will contain the same damper, range of mass, and spring constants. Although the undriven system will be able to incorporate interchangeable springs, of varying constants, the driven system will not be modular in that respect.

When designing for the motion of the driven system multiple parameters were taken into account. These included: safety, cost, spatial constraints, range of amplitudes and frequencies, life expectancy, control of external vibrations, ease and speed of assembly and disassembly, and aesthetics. But it was also vitally important to determine what excess vibrations any new component might add to the system. To get the highest quality experimental results it was desirable to reduce any and all external vibrations (i.e. those not related to the natural movement of the mass or the sinusoidal driving motion).

To drive the motion of the system, it was deemed desirable to use a motor. To drive the system with the most consistent speed and power, a single-phase A/C motor was selected on. In order to convert the rotary motion of the output shaft of the motor to the sinusoidal motion of the system a number of options were considered, but the scotch-yoke mechanism was eventually decided on (Figure 5). It was the only mechanism that can produce truly sinusoidal output motion. It also had the benefit of being relatively compact and easy to machine.

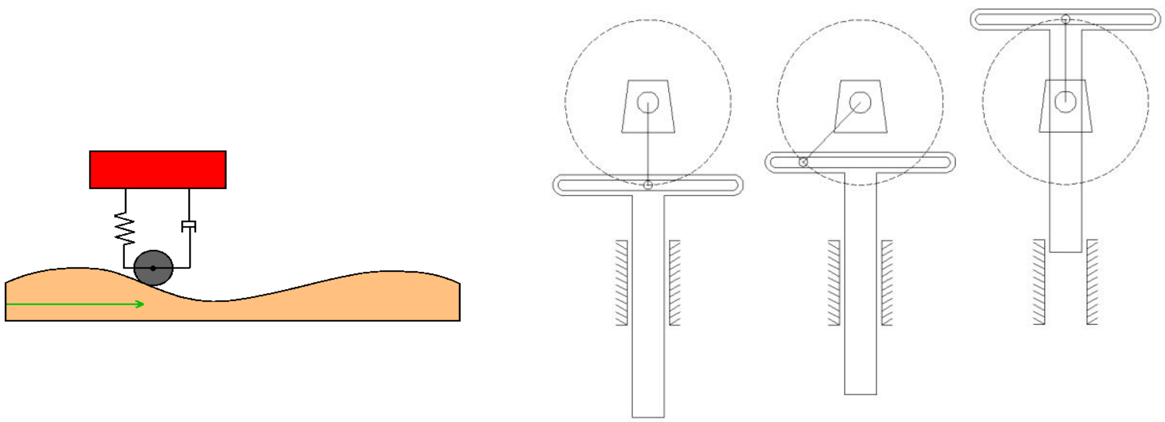

**Figure 5.** Harmonic Oscillator and the Scotch-Yoke Mechanism

To test the feasibility of the motor speed, system constants (spring rate, damping coefficient, et cetera), and drive mechanism geometry, a Working Model simulation was created. Shown in Figure 6, the model demonstrates that the scotch yoke mechanism will drive the system as believed and also shows the spatial constraints of attaching both the damper and the scotch yoke. Realistically, they should both be applied as close to the center of the mass as possible to allow the bearings to operate smoothly. In the simulation, the slots that the mass rides on are perfectly frictionless. The model also helped to visually confirm how the various motor speeds will operate.

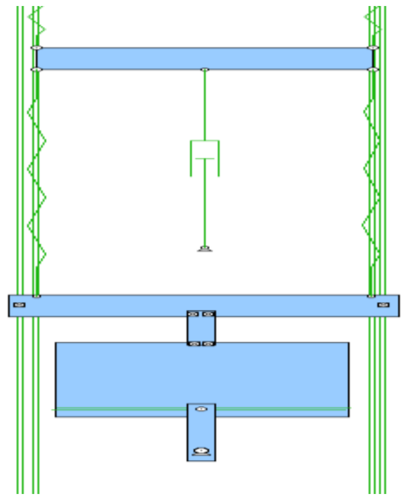

**Figure 6.** Working Model Simulation of the Drive System

Once the system is fully constructed the motion will be analyzed with either a low- or high-G accelerometer. Experimental results will determine which is more appropriate. The information from the accelerometer sensor will be integrated within Matlab to find a function that mathematically describes the actual motion of the mass plate. It will be compared to a theoretically solved system that will take all the measured system parameters (mass, damping, spring rate, driving force) to create an associated curve. The two can then be compared.

**Section III: Frame design**

Spatially, the apparatus needs to be able to allow for both mechanisms to be attached and to work without interaction with each other. Also, since the device needs to be functional in an undergraduate laboratory environment, it needs to be set up such that both mechanisms are accessible simultaneously.

A preliminary frame design was created and is shown below in Figure 7. The two mechanisms will be placed on the extreme ends of the frame. The frame will be constructed of the aluminum framing from MiniTec.  Placing the two mechanisms at the extreme ends also allows both systems to be accessed simultaneously (by two different groups of students).

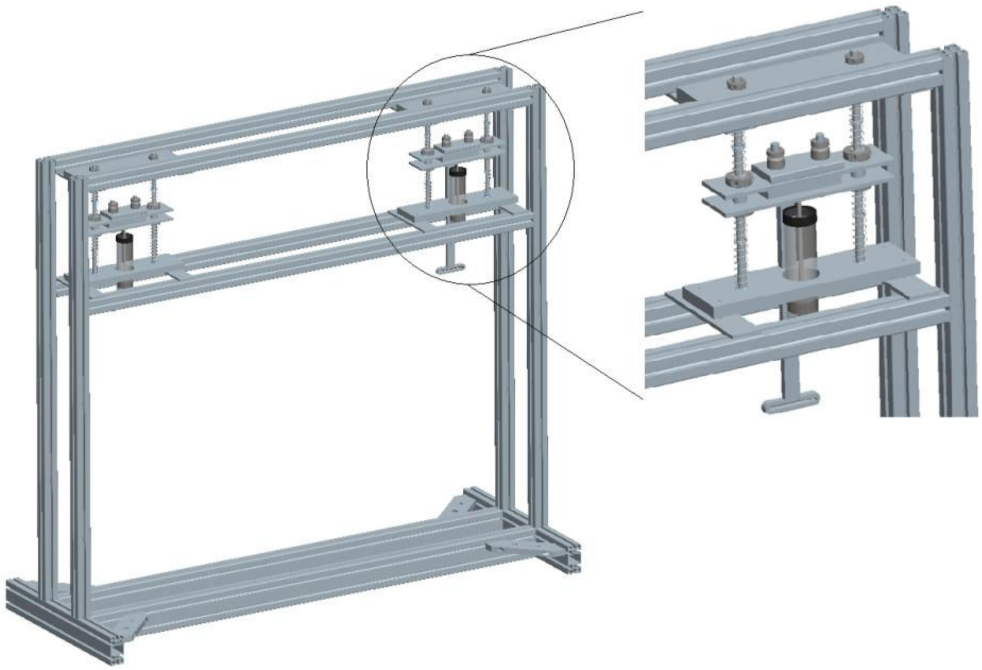

**Figure 7.**  MiniTec Frame Supporting the Two Systems at its Extreme Ends

An important function of the frame is to help minimize external vibrations (outside of the mass-plate system). The geometry of the frame itself will help reduce vibrations. In between the frame and the ground will sit a flat rubber mat, which will absorb extraneous vibrations.  A 10:1 weight ratio was taken as the minimum necessary to reduce the effect of the vibrations of the apparatus on the frame. To increase the weight of the frame, mass will be added externally by filling PVC tubes with sand and using end plugs to close them. The PVC can be attached to the frame with U-bolts.

## V - WORKING MODEL

Working Model is a computer-based motion simulation software that is the result of a 15-year collaborative effort between professional engineers and software specialists. The reliability of the mathematical models incorporated and the results provided (by the software) have gained the trust of many professionals. Different versions of the software are available for companies and educational institutions (7). For the purposes of this project, a 2-D version of the software addresses all the requirements. The company provides tutorials to start the undergraduate students.

## VI - WORKING MODEL SIMULATIONS AND EXPERIMENTS

As mentioned earlier, the simulations of the desired modes of experimentation [using Working Model (WM)] provided the design team with a higher degree of confidence to proceed to the next steps. The process enabled the team to test the feasibility of the targeted ranges for: a) the mass values, b) damping coefficients, c) spring constants, d) the setting of the initial velocities and displacements, e) frequencies, and f) the amplitude of the driving forces. The program played an instrumental role for obtaining a combination of the desired ranges based on the system's parameters.

It was in this process that the team made the observation that the software has a significant range for examination of a large array of different scenarios. Further, the issues of control of friction were easily addressed by simply calling for a frictionless guide/joint – a condition that may prove challenging to establish in the actual physical environment. The values of the mass, spring constant, damping coefficient, frequency, amplitude are all easily entered into the simulated system. It is critical to note that such values would be significantly more difficult to obtain from an off-the-shelf unit/component.

Six distinct examples of Single Degree of Freedom (S-D-F) Vibrations with the use of Working Model – 2D are enclosed in Appendix (A). Appendices (C) and (D) provide two sample handouts for the proposed experiments. *The interested instructors may choose different sets of parameters for each (sets) of groups and such parameters may be changed for every class or academic year. It is important to note that the spatial constraints and the costs involved prohibit the availability and use of several actual/physical units by several different groups of students simultaneously. Therefore, another added advantage is the unlimited number of groups that may simulate the experiment(s) at any given time with different sets of reasonable values for the system's parameters.*

## VII - CONCLUSIONS

The first stages of research, design, and material collection for design and fabrication of a cost-effective and reproducible Single Degree of Freedom (S-D-F) Vibrations apparatus are completed. The use of Working Model – 2D enabled the team to test the feasibility of the targeted ranges for: a) the mass values, b) damping coefficients, c) spring constants, d) the setting of the Initial velocities and displacements, e) frequencies, and f) the amplitude of the driving forces. The capabilities of the WM-2D are robust and reliable to simulate potential experiments in S-D-F Vibrations. Although simulations should be accompanied by actual experimentations, for situations where and when space and budget are limited, this approach may be justifiable if not the only practical solution. Another added advantage is a higher number of existing (simulated) units in contrast to limited number of physical apparatus.

## ACKNOWLEGMENTS

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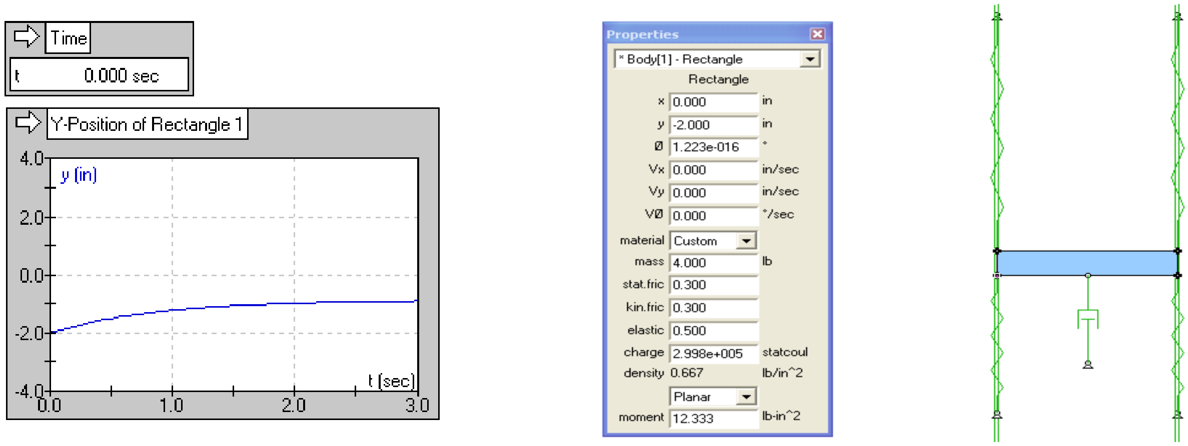
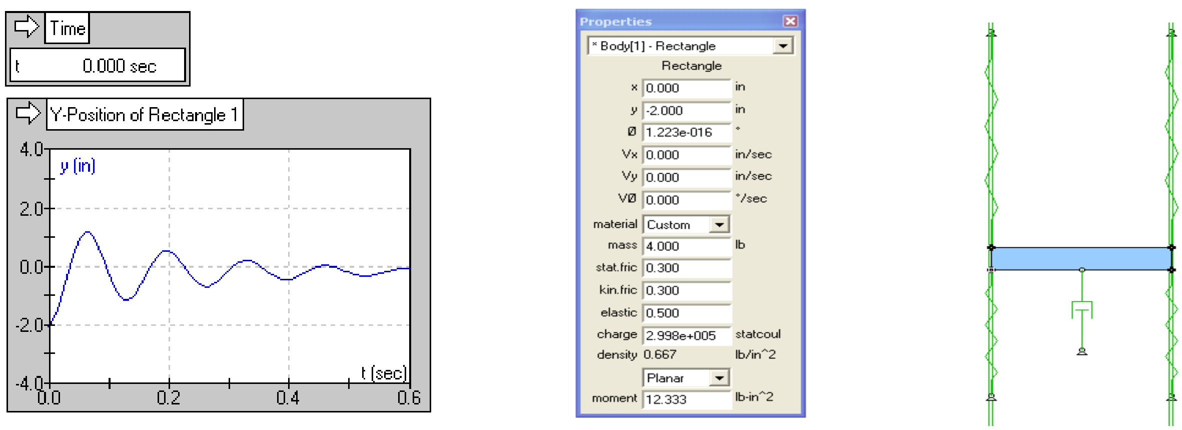
## Appendix A: Six different Modes of S-D-F Vibrations using Working Model

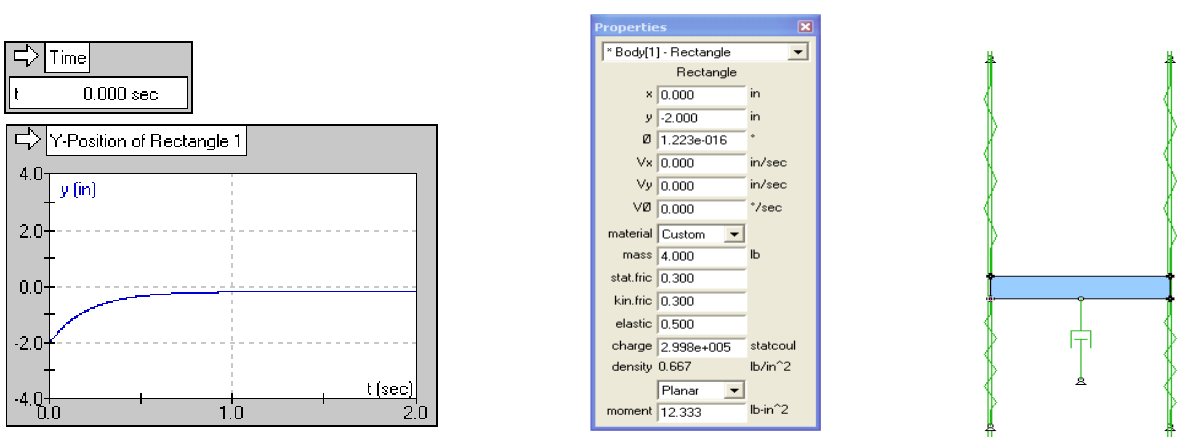
Overdamped (Non-Driven)




Underdamped (Non-Driven)




Critically Damped (Non-Driven)

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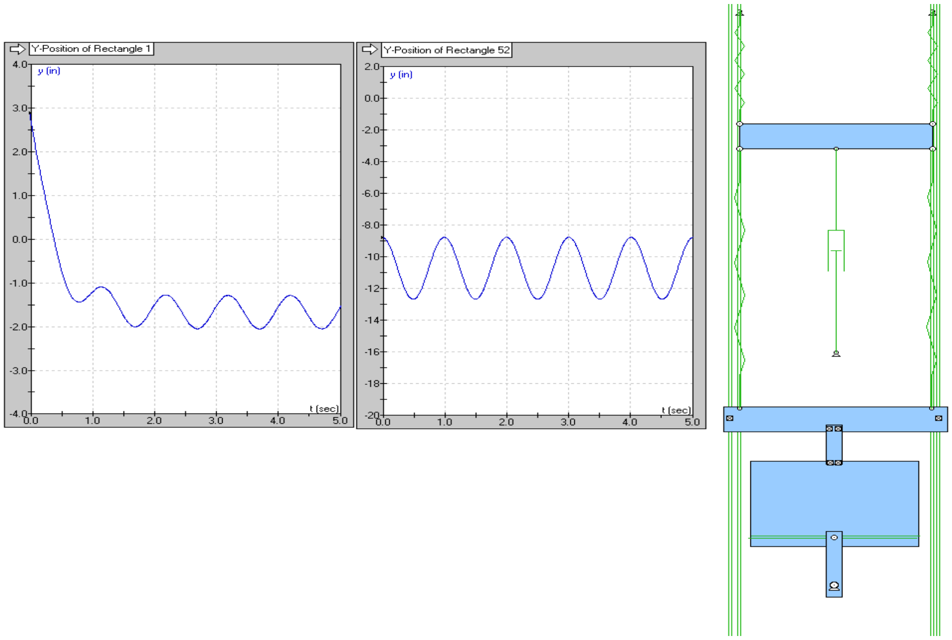
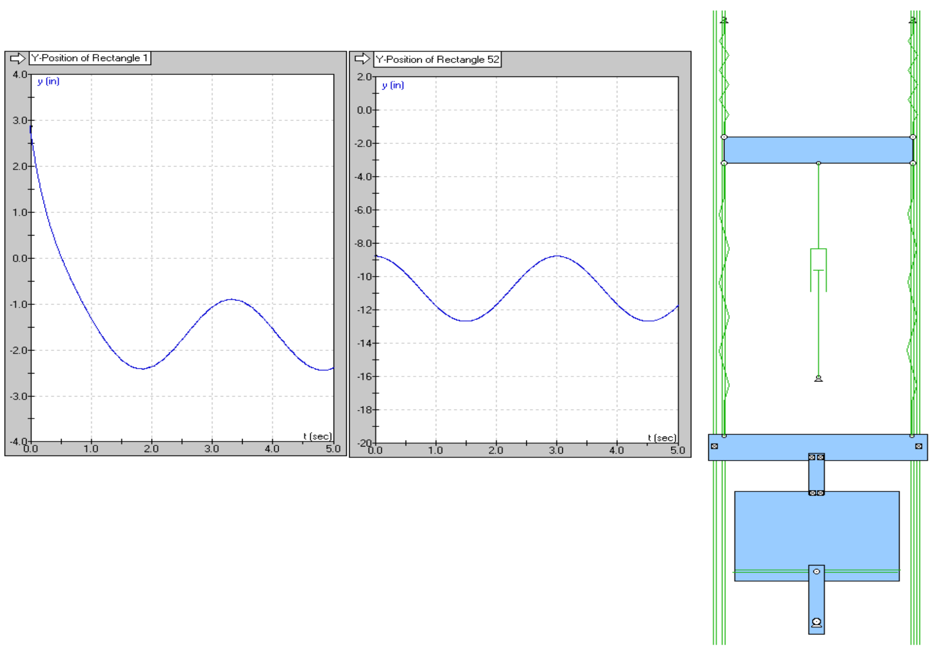
Driven system with:

| Excited Mass = 4 lbs | Spring Rate = 10 lbs/in | Damping Coefficient = 15 lbs*in/s | Amplitude = 2 in [ +/-1 inch ] | Motor Speed = 1/3 RPS |
|---|---|---|---|---|



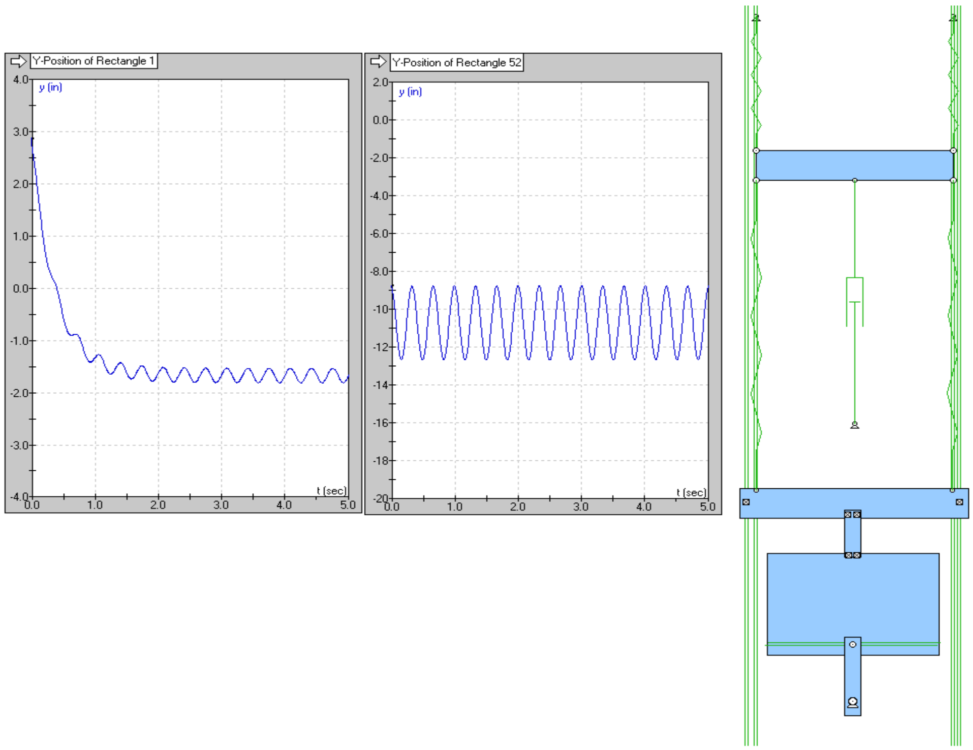

Driven system with:

| Excited Mass = 4 lbs | Spring Rate = 10 lbs/in | Damping Coefficient = 15 lbs*in/s | Amplitude = 2 in [ +/-1 inch ] | Motor Speed = 1 RPS |
|---|---|---|---|---|



Driven system with:

| Excited Mass = 4 lbs | Spring Rate = 10 lbs/in | Damping Coefficient = 15 lbs*in/s | Amplitude = 2 in [ +/-1 inch ] | Motor Speed = 3 RPS |
|---|---|---|---|---|



## Appendix B: Relevant Equations

### General Equations

Spring constant can be determined from the relationship: $\boldsymbol{F = k\delta}$ **(1)**

Where ***F*** is the force, in pounds, placed on top of the top plate and $\boldsymbol{\delta}$ is the deflection, measured in inches, of the plate.

Damping coefficient $\boldsymbol{c = \frac{Fd}{t}}$ **(2)**

Where ***F*** is the force in pounds, ***d*** is the distance traveled by the mass in inches, and t is the time it took to travel that distance in seconds.

Force of Spring: $\boldsymbol{f_k = -kx}$ **(3)**

Where ***k*** is stiffness of the spring, ***x*** is the change in length of the spring from its initial position, and $\boldsymbol{f_k}$ is the force on the spring.

Natural Frequency, $\boldsymbol{\omega}$ (**from**): $\boldsymbol{\omega^2 = \frac{k}{m}}$ **(4)**

Damping Force: $\boldsymbol{f_c = c\dot{x}(t)}$ **(5)**

***c*** is the damping coefficient and $\boldsymbol{f_c}$ is the force provided by the damper.

Critical damping coefficient: $\boldsymbol{c_{cr} = 2\sqrt{km}}$ **(6)**

Damping ratio: $\xi = \frac{c}{c_{cr}} = \frac{c}{2m\omega}$ **(7)**

Pendulum Free Response: $\boldsymbol{\ddot{\theta} + (\frac{g}{l})\theta = 0}$ **(8)**

Where $\boldsymbol{\theta}$ is the angular position, ***g*** is the acceleration due to gravity, and $\boldsymbol{l}$ is the length of the unit.

### Unforced System

General solution: $\boldsymbol{m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0}$ **(9)**

Equation of Motion: $\boldsymbol{x(t) = A\,sin(\omega t + \varphi)}$ **(10)**

Amplitude: $\boldsymbol{A = \frac{\sqrt{\omega^2 x_0^2 + v_0^2}}{\omega}}$ **(11)**

Phase shift: $\boldsymbol{\varphi = tan^{-1}\left(\frac{\omega x_0}{v_0}\right)}$ **(12)**

## Underdamped

Unforced equation of motion: $\boldsymbol{x(t) = A\sin(\omega_d t + \varphi)}$ **(13)**

Damped natural frequency: $\boldsymbol{\omega_d = \omega\sqrt{1-\xi^2}}$ **(14)**

Amplitude: $\boldsymbol{A = \sqrt{\frac{\omega_d^2 x_0^2 + (v_0 + \xi\omega x_0)^2}{\omega_d^2}}}$ **(15)**

Phase shift: $\boldsymbol{\varphi = tan^{-1}\left(\frac{\omega_d x_0}{v_0 + \xi\omega x_0}\right)}$ **(16)**

## Overdamped

Equation of motion:

$$\boldsymbol{x(t) = e^{-\xi\omega t}\left(a_1 e^{-\omega t\sqrt{\xi^2-1}} + a_2 e^{\omega t\sqrt{\xi^2-1}}\right)} \quad \textbf{(17)}$$

$$\boldsymbol{a_1 = \frac{-v_0 + \omega x_0\left(-\xi + \sqrt{\xi^2-1}\right)}{2\omega\sqrt{\xi^2-1}}} \quad \textbf{(28)}$$

$$\boldsymbol{a_2 = \frac{v_0 + \omega x_0\left(\xi + \sqrt{\xi^2-1}\right)}{2\omega\sqrt{\xi^2-1}}} \quad \textbf{(19)}$$

## Critically damped

Equation of motion:

$$\boldsymbol{x(t) = e^{-\omega t}(a_1 + a_2 t)} \quad \textbf{(20)}$$

$$\boldsymbol{a_1 = x_0} \quad \textbf{(21)}$$

$$\boldsymbol{a_2 = v_0 + \omega x_0} \quad \textbf{(22)}$$

## Forced System

Driving force: $\boldsymbol{F(t) = F_0\,cos(\omega_{dr} t)}$ **(23)**

$\boldsymbol{F_0}$ is the maximum amplitude of the applied force and $\boldsymbol{\omega_{dr}}$ is the input frequency.

General solution: $\boldsymbol{m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t)}$ **(24)**

Equation of motion: $x_p(t) = \frac{F_0}{\sqrt{(c\omega)^2 + (k - m\omega^2)^2}} cos\left(\omega t - tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right)\right)$ **(35)**

## Appendix C: Free Response of Single Degree of Freedom Vibrations Experiment

OBJECTIVE

The purpose of this experiment is to interpret the results of an unforced response system and the related mathematical models.

EXPERIMENTAL PROCEDURE

1. Open the Working Model filed named "Unforced System."
2. Set the mass of the excited plate by double clicking the block and setting the mass to 1.0 lb.
3. Set the spring rate by double clicking each spring and setting the spring constant to 6 lbs/in.
4. Set the damping coefficient by double clicking on the damper and setting the damping coefficient to 1 lbs- in/ sec.
5. Run the program until several oscillations are recorded.
6. Go to: File, Export, and save a file as meter data (*.dta).
7. Change the mass to 2 lbs, then 4 lbs, then 6 lbs, and repeat steps 5 through 6.
8. Change the damping rate to 5 lbs- in/ sec, 20 lbs- in/ sec, and then 35 lbs- in/ sec, and repeat steps 5 through 7.
9. Change the spring rate to 1 lb/in, 3 lbs/in, 6 lbs/in, and 9 lbs/in, and repeat steps 5 through 8.
10. Close Working Model.
11. Open the meter data files in excel and graph the results.

DATA ANALYSIS

1. For which part of the experiment do you anticipate the percent error of the real world application to be the lowest? The highest? Why?
2. At what value was the system critically damped? How will this change with real world application?
3. Evaluate when the system is overdamped and discuss how the graph changes with the change in the parameters. Do the same for the underdamped system.

## Appendix D: Forced Response of Single Degree of Freedom Vibrations Experiment

OBJECTIVE

The purpose of this experiment is to interpret the results of a forced response system and the related mathematical models.

EXPERIMENTAL PROCEDURE

1. Open the Working Model filed named “Driven System.”
2. Set the mass of the excited plate to 1 lb. by double clicking it and entering 1.0 next to “mass.”
3. Set the motor speed by double clicking the motor and setting the velocity to 1 rev/sec.
4. Run the program until several oscillations are recorded.
5. Go to: File, Export, and save the file as meter data (*.dta).
6. Change the mass to 2 lbs, then 4 lbs, and repeat steps 4 through 5.
7. Change the damping rate to 5 lbs- in/ sec and then 25 lbs- in/ sec, and repeat steps 4 through 6.
8. Change the motor speed to 0.5 rev/sec, 2 rev/sec, 5 rev/sec, and 10 rev/sec, and repeat steps 4 through 7.
9. Close Working Model.
10. Open the meter data files in excel and graph the results.

DATA ANALYSIS

1. Make a graph of input magnitude vs. output magnitude.
2. Make a graph of output phase shift.
3. Construct a Lissajous pattern for each set of mass and damping values.