A CONSTRAINT CLASSIFICATION SCHEME FOR TEACHING KINEMATICS

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Abstract

This paper presents a classification scheme of constraints for modeling and simulation of mechanisms and mechanical systems. Current undergraduate kinematic texts deal primarily with pin, slider, cylindrical, ball and a few other types of constraints. Usually other types of constraints and, especially composite constraints are not covered in traditional texts. Only specialized literature deal with composite constraints, and they too are limited to certain types. This paper presents a systematic scheme that is easier to teach, and understand. This scheme is represented by a table that encompasses all types of constraints, and facilitates understanding and developing the constraint equations easily for computer simulation of mechanical systems. In this classification scheme, six constraints (axes, parallel, perpendicular, point, line and plane) have been identified as basic constraint types, and these are then used as building blocks for deriving other constraints. This approach is very effective in modeling mechanical system that can not be modeled just using existing joint types. Furthermore, this building block approach makes it easy to identify suitable composite joints and calculate the degrees of freedom of any joint just by adding the row and the corresponding column number of the classification table. Thus this scheme is very useful in teaching joint types in spatial mechanisms in class rooms. In addition, a mathematical model for each constraint has been constructed using matrix methods making it easier for computer implementation.

Introduction

Constraints or Joints create interconnections between bodies and restrict the relative motion between them in a predetermined fashion. For example, a revolute joint constrains the motion between links to one rotational degree of freedom with respect to a common axis. Currently, the texts by Norton (1992), Erdman and Sandor (1991), Reinholtz and Mabie (1987) and Shigley and Uicker (1981) are widely used for teaching design of machines and mechanisms, and as a reference to practicing engineers. These books provide comprehensive discussion on various joint types and classify joints based on the number of degrees of freedom(dof) permitted/prevented at a joint, the type of contact (point, line, or surface) between the mating elements, or, the type of physical closure (form or force) of the joint. This traditional classification usually covers revolute, prismatic, cylindrical, universal, spherical and a few other joints. Although linkage and several other mechanisms can be modeled using these joints, they do not have the capability to model kinematic constraints necessary to simulate any mechanical system. For example, simulation of the motion of balls in a recirculating ball-screw (or in back-spin game), or, modeling 3-translational motions of a machine tool slide are not directly possible. Thus, in order to model any constraint, composite joints are needed.

Hoeltzel and Chieng (1991) proposed a methodology for combining the degrees-of-freedom of lower kinematic pairs into higher pairs for the creation of kinematic connections using hybrid (symbolic-numerical) optimization process. In their work, Hoeltzel and Chieng provide a table adapted from the work done by a Russian kinematician Reshetov on serially connected kinematic pairs. According to this work, a pair having *i* constraints is classified as Class i pair, and totally four such pairs were defined. These pairs were further subdivided according to type. While this table provides a good classification scheme, it is too difficult to understand and interpret. Furthermore, this approach also does not lend itself to derive the constraint equations for composite joints easily.

Haug (1989) presented the theory behind the commercial software DADS ‘M (available from CAD Software, Inc. Oakdale, Iowa) used for kinematic and dynamic analysis of mechanical systems. In this work, Haug developed the constraint equations for composite joints and introduced the concepts of dot-1 constraint, dot-2 constraint, parallel-1 constraint and parallel-2 constraint. Another commercial program ADAMS ‘M (available from Mechanical Dynamics, Inc., Ann Arbor, MI) introduced the concept of joint primitives that are different from composite joints to eliminate the redundant constraints in a system. These two packages are quite different internally and use different terminology but do not provide any simpler classification scheme.

Sheth and Uicker (1971) introduced a symbolic notation for mechanisms based on matrix methods. They developed a systematic approach that contains the essential parameters for a complete description of the mechanism. Today, their matrix based approach is being widely used for mechanism analysis. In their work, they discussed the representation of kinematic pairs by the functional relationship describing the relative motion of the two contact surfaces. For the symbolism developed in their work, the functional relationship is defined by transformations between two coordinate systems attached to the bodies. They developed such transformation matrices for six lower pairs (revolute, prismatic, screw, spherical. flat) and one higher (gear) pair.

In this paper, a classification scheme of kinematic connections is presented. This scheme is based on the definition of six basic constraints that are used as building blocks to create other constraints. This approach is very different from the one where lower pairs are used to create higher pairs because the six basic constraints defined in this work include both lower and higher pairs. The salient features of this scheme are that it is easy to teach, understand, and useful for defining composite joints.

Classification Table

In this scheme, a table (Table 1) with four rows and four columns has been created. Rotational dof is represented columnwise in increasing order from left to right. The first column prevents all rotational dof and the last column permits all three rotational dof. Rows are assigned to translational dof in increasing order from top to bottom. Each element in the table represents a constraint that can be uniquely identified by an element number. For example, element (0,0) represents a joint that has neither rotational and translational dof, meaning the body is fixed (ground link). Similarly, element (3,3) has all six degrees of freedom representing a free body. The degrees of freedom permitted by any constraint can be found by adding the corresponding row and column number. For example, the joint (T’, R’) has one degree of translational and one degree of rotational freedom yielding a total of two degrees of freedom as shown in Table 1. This table includes all joint types that are available and can also represent any composite joint that can be obtained by combining basic constraints.

Table 1: Constraint Classification Table

Translational DOF	Rotational DOF			
	0	1	2	3
0	**Ground Link** (T^0R^0)	T^0,R^1	T^0,R^2	T^0,R^3
1	T^1,R^0	T^1,R^1	T^1,R^2	T^1,R^3
2	T^2,R^0	T^2,R^1	T^2,R^2	T^2,R^3
3	T^3,R^0	T^3,R^1	T^3,R^2	**Free Body** (T^3,R^3)

It must be noted here that some of the joint types in the table may not be physical] y available nevertheless this classification includes all possible types that are theoretically available and useful in specific situations. For example, in order to represent the motion of a ball on a flat surface, one must define a composite constraint that allows two translations and three rotations. Such a constraint and its mathematical model are readily available (element 2,3) in this classification.

Basic Constraints

The classification system is based on the definition of six basic constraints that occupy the last row and the last column of the table. These are axes, parallel and perpendicular, point, line and plane constraints. These six constraints serve as basic building blocks in defining other constraints.

Point Constraint

A point constraint allows 3 rotational dof and it is defined when one point on a coordinate system attached to a body can not translate with respect to another coordinate system of another body. This condition will be true only if a point on the first body always coincides with a point on the second body. Fig. la shows the relative position of two bodies i and j and their local coordinate systems, Xiyixi and Xjyjzj. In order for the above condition to be true, points, 0, and O_j are always coincident as shown in Fig. lb. Such a constraint eliminates all three translational dof and allows three rotational dof.

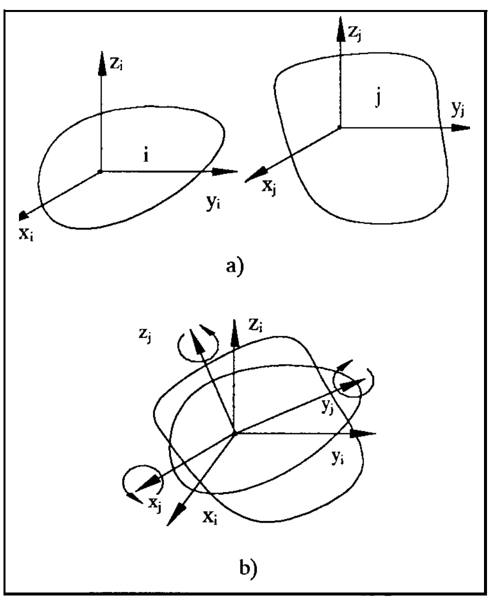

Fig. 1: Point Constraint

Using the concept of elementary transformation matrices and the matrix notations (Uicker, Denavit, and Hartenberg, 1964; Sheth and Uicker, 1971; Sheth, 1972), a joint with three rotational dof can be represented as follows:

$$J' = T(\theta_1)T(\theta_2)T(\theta_3) \qquad (1)$$

where P represents Point constraint and $T(\theta)$ represents elementary (4x4) transformation for rotations. Thus, this joint allows rotational freedom around all three axes of body relative to another. Typically, these rotations are described by Euler angles. The final matrix that describes this joint after Eulerian transformation can be shown to be:

$$J^P = \begin{bmatrix} (c\theta_1 c\theta_3 - s\theta_1 c\theta_2 s\theta_3) & (-c\theta_1 s\theta_3 - s\theta_1 c\theta_2 c\theta_3) & (s\theta_1 s\theta_2) & 0 \\ (s\theta_1 c\theta_3 + c\theta_1 c\theta_2 s\theta_3) & (-s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) & (-c\theta_1 s\theta_2) & 0 \\ (s\theta_2 s\theta_3) & (s\theta_2 c\theta_3) & (c\theta_2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

In Equation 2, there are three independent variables and hence, the point constraint has three degrees of freedom. Physically, point constraint represents a ball and socket or what is known as spherical joint. For these and other joints described in this paper, the mathematical formulation is based on the well-known concept of elementary transformation matrices and therefore, the details are omitted for brevity.

Line and Plane Constraints

A line constraint (Fig. 2) is when one point of a coordinate system attached to a body can only move along a line. A point in space has six dof. When this point can move only along a line, it will have four dof (T^1+R^3). Similarly, a plane constraint (Fig. 3) restricts the motion of this point to a particular plane. Therefore, the plane constraint has five dof (T^2+R^3).

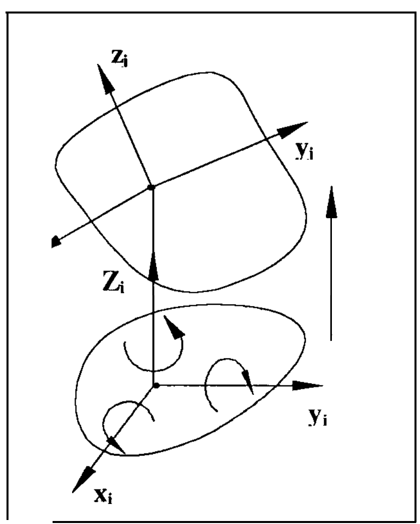

Fig. 2: Line Constraint

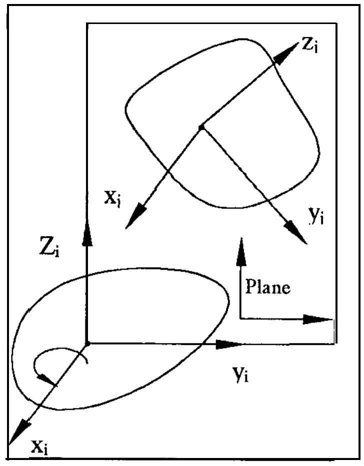

Fig. 3: Plane Constraint

Axes Constraint

Axes constraint does not permit any change in the angular orientation between two bodies. This is possible only when one coordinate system can not rotate at all with respect to the other, thus allowing only translations without changing mutual orientation (Fig 4). The mathematical model for this constraint can be represented as follows:

$$J^A = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

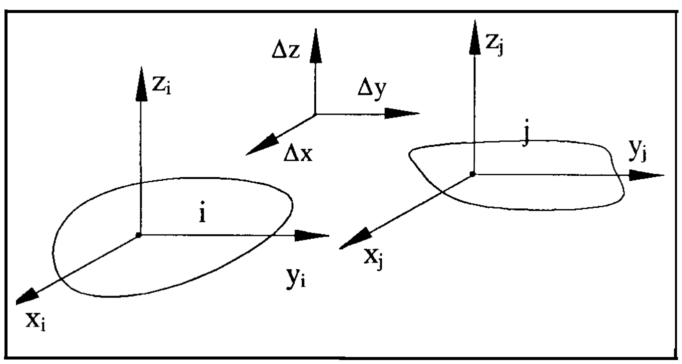

Fig. 4: Axes Constraint

Parallel and Perpendicular Constraints

A parallel constraint (Fig. 5) is described when one axis of a body is forced to remain parallel to another axis of the second body. Thus, this constraint will allow three translations as axes constraint but will allow also allow one rotation (T^3+R^1). A perpendicular constraint (Fig. 6) requires one axis of a coordinate system to remain perpendicular all the time to another. This constraint eliminates only one rotation and has five dof (T^3+ R').

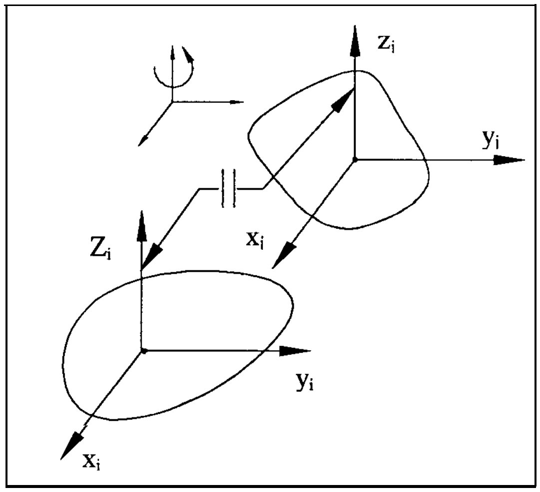

Fig 5: Parallel Constraint

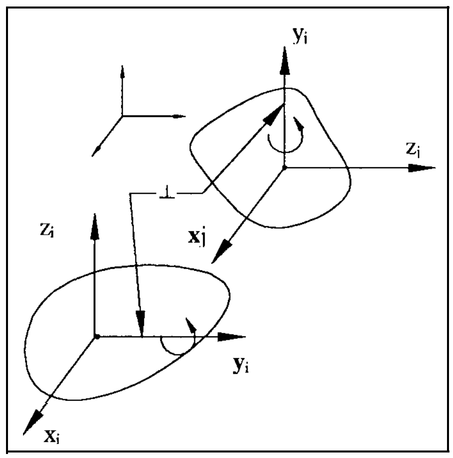

Fig. 6: Perpendicular Constraint

Derived Constraints

Using the definition of these six basic constraints, all other composite constraints can be defined by selecting the-appropriate basic constraints from corresponding rows and columns as shown in Table 2. If one of the axes of j^{th} body is to remain parallel to another axis of i^{th} body all the time while satisfying the point constraint, then only one rotation is possible with respect to this axis. Thus the composite Point-Parallel constraint has one rotational degree of freedom. Physical example of this constraint is the revolute joint. Point-Perpendicular constraint is when one of the axes of j' body remains perpendicular to another axis of the i^{th} body while a point constraint is imposed. This constraint permits two relative rotations. Example of this constraint is the Hook joint. These constraints can be represented as:

Revolute Joint: J'-'"'= T(6) and Universal Joint: J^{P-Per} = T A T In a similar fashion, all other constraints are defined as shown in Table 2.

Table 2: Constraint Classification Scheme

	0	1	2	3
0	**Ground**	(Point, Parallel)	(Point, Perpendicular)	(Point)
1	(Axes,Line)	(Line,Parallel)	(Line, Perpendicular)	(Line)
2	(Axes, Plane)	(Plane,Parallel)	(Plane,Perpendicular)	(Plane)
3	(Axes)	(Parallel)	(Perpendicular)	**Free Body ‘**

Discussion

Table 2 includes all types of joint that can be conceived and Fig. 7 shows a schematic representation of these joint types. While some of the joint types shown Fig. 7 are easily recognizable, some of them may not physically exist. Also, one may ask the reason for defining an axes constraint as a basic constraint when it can be modeled using three prismatic joints. In many applications, the only function of a body is to connect two other bodies using a combination of lower pair joints such as revolute and prismatic. This intermediate body is known as the coupler and need not be treated as a separate body. This simplifies both modeling and computer implementation. This approach is also different from other approaches where lower pairs are usually used to build higher pairs. In this work, all the constraints including the single dof lower pair joints are derived by using a-combination of basic constraints.

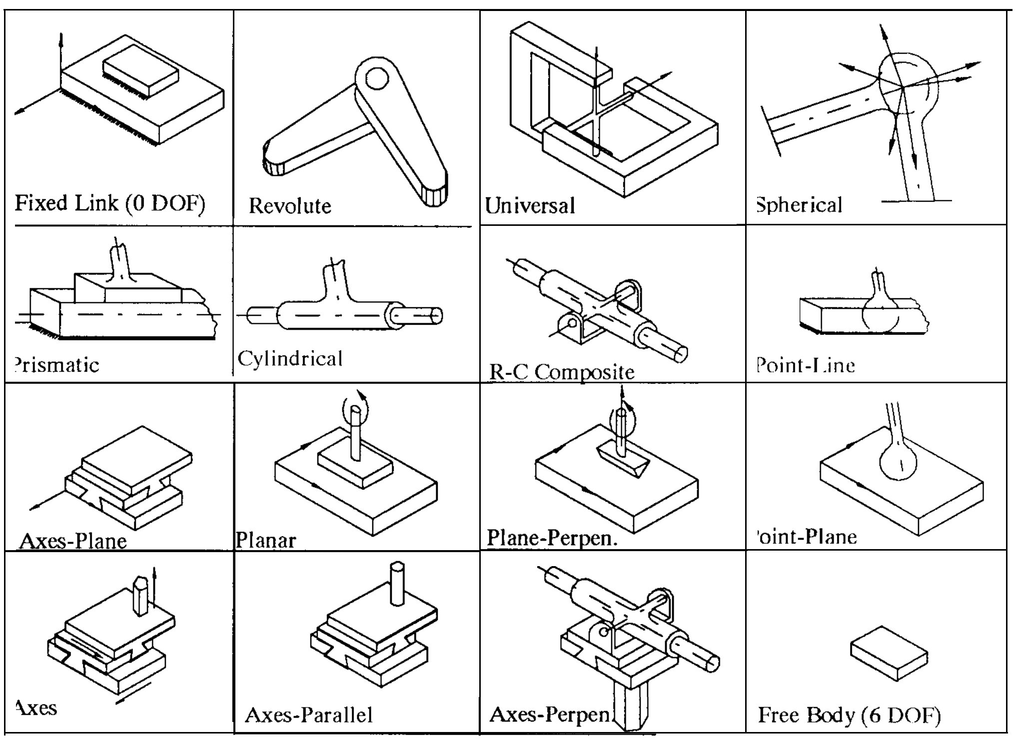

Fig. 7: Constraint Table

Conclusion

A constraint classification scheme is presented. In this scheme six basic constraints are defined. All other possible constraints are derived by combining the basic constraints. The basic constraints are arranged in a simple table. Using this table, composite constraints can be defined very easily and effectively just by selecting appropriate basic constraints from a row and the corresponding column. The mathematical model for each constraint (each entry in the table) is represented by elementary transformation matrices and is automatically generated when a particular constraint is selected. This approach is effective in modeling and simulation of mechanical systems, easy to calculate the dof of any joint by adding the row and column number and hence, very useful in teaching kinematic constraints in class rooms.

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Biography

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