AC 2011-1108: A DIMENSIONAL ANALYSIS EXPERIMENT FOR THE FLUID MECHANICS CLASSROOM

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A Dimensional Analysis Experiment for the Fluid Mechanics Classroom

Overview

Dimensional analysis is a technique used in many fields of engineering to facilitate correlation and interpretation of experimental data. It provides a means of combining the many parameters of an experiment into a lesser number of dimensionless groups. This technique greatly reduces the amount of experimental work needed to determine the effect of parameter variation on the dependent parameter of the experiment.

As an example, consider the flow of a fluid through a smooth tube. As the fluid moves downstream, its pressure decreases due to frictional effects. The amount of pressure decrease depends upon 5 parameters - the diameter of the tube, the distance traveled down the tube by the fluid, the fluid velocity, the density of the fluid and the viscosity of the fluid. The problem therefore involves 6 parameters - the pressure drop, which is the dependent parameter, and the other 5 parameters which are the independent parameters. To fully investigate the pressure drop phenomenon, it would appear that one would have to vary all 5 independent parameters separately to see how a change in a given parameter would affect the value of the pressure drop. This would indeed be a formidable undertaking, and voluminous amounts of data would be generated that would be hard to correlate and interpret. Through the technique of dimensional analysis, the problem can be reduced from one involving six parameters to one involving three dimensionless groups, one group of which contains the dependent parameter. By reducing the number of variables in the problem from six to three, the necessary experimental work is greatly reduced, and correlation of the experimental data is greatly facilitated.

In the mechanical engineering curriculum, dimensional analysis is typically taught in the fluid mechanics lecture course, where students apply the Buckingham-Pi theorem[1, 2] and determine the appropriate dimensionless groups for a given problem. The procedure used for determining the dimensionless groups is generally straight-forward but tedious. It is believed that the use of laboratory demonstrations in the lecture class would increase the students' interest in the subject and would significantly enhance the students' comprehension of the usefulness of dimensional analysis in the planning of experimental programs and the interpretation of the experimental data. The intent is to have students actively participate in the performance of the demonstrations and the gathering of data. They would also correlate the data during the class period using software such as Excel, Matlab, or MathCad. The class session would be lively, with much increased student participation and active learning.

Accordingly, we have developed a fluid mechanics experiment dealing with the flow of fluids (i. e., water and air) through nozzles of different sizes. The pressure drops across the nozzles are measured for a variety of flow rates. Using dimensional analysis, the students determine the appropriate dimensionless groups for the experiment and correlate the experimental data. The students thereby observe first-hand the usefulness of dimensional analysis in the correlation and interpretation of experimental data.

The paper describes the experimental apparatus in detail. Although the apparatus is currently located in our mechanical engineering labs, the intent is to make minor modifications to the apparatus to enable its mounting on a cart for easy transport to and from the lecture classrooms. The paper includes a parts list, a discussion of construction aspects, and includes actual data obtained from the experiment by the author. When the experiment is ultimately performed by the students in the lecture class, they will be asked to complete a survey regarding the usefulness of the experiment in improving their understanding of dimensional analysis. The effectiveness of the experiment will also be assessed by pre- and post-experiment exam problems.

In summary, this paper describes a fluid mechanics experiment which can be used to enhance lectures on dimensional analysis. Through active student participation, the experiment should greatly increase the students' interest in the subject and their comprehension of the use of dimensional analysis in the planning of experiments and the correlation of experimental data.

I. Objectives

The main objective was to design and construct a fluid mechanics experiment to illustrate the concept and usefulness of dimensional analysis. An accompanying benefit of the effort was the addition of an experiment to the mechanical engineering laboratories.

II. Experimental Apparatus

The experimental apparatus is shown in Figure 1. It has been designed to be low cost, relatively easy to construct by lab technicians, and readily transportable from storage location to place of use.

Figure 1 - The Experimental Apparatus

The copper tubing (1/2" and 3/4" Type L), including valves and fittings, is mounted on a ½" plywood board. Air and water supplies enter the copper tubing at the left end of the tubing. Full-port ball valves are used for selection of the fluid and for control of the flow rate. The specimen is mounted at the right end of the copper tubing. Water leaving the specimen (i. e., nozzle) is caught by a 1-1/2 inch diameter drain line which leads to a 12 gallon plastic rectangular tank. The tank has a submersible pump which recirculates the water through the experiment. (It is also easy to run the experiment with water supply from the sink faucets. If this is done, the submersible pump empties the water periodically to the sink so that the tank does not overflow.)

Going from left to right in the figure: The air compressor is on the floor. On the tabletop is the rotameter for the air flow rate and the rotameters for the water flow rate. The blue digital meter measures the fluid pressure just upstream of the specimen. The drainline support and the air flow meter are clamped to the tables for easy removal for storage.

Figure 2 shows a specimen mounted at the end of the tubing.

Figure 2 - Mounted Nozzle Specimen

Figure 3 is a close-up picture of a specimen. The specimen consists of a 1/2" male pipe-to-tubing adapter with a one-inch long aluminum rod in its end. The rod is 5/8 inch diameter so it snugly fits into the male adapter. Nine (9) specimens were fabricated; each specimen with a different diameter hole drilled through the aluminum rod. The diameters vary from 1/8" to 3/8", at 1/32" increments. We would have preferred using copper rod segments and soldering them to the adapters. However, the cost of copper is much greater than aluminum, so we chose aluminum. To avoid possible problems soldering aluminum to copper, we chose to glue the rod to the adapter. We used Gorilla Glue, and it has worked well.

Figure 3 - One of the Nine Specimens

Parts List

Major equipment and instrumentation items (with approximate current prices) are as follows:

- Air Flow Meter
  Omega Engineering FL4511 (0.5 – 4 scfm) $ 80

- Water Flow Meter
  Figure No. 1 shows three flow meters, covering the range 0.2 to 10 gpm. The first flow meter was the one used for most measurements since the flow was usually 2 gpm or less.

  Omega Engineering FL4601 (0.2 – 2 gpm) $ 147
  Omega Engineering FL4603 (.5 – 5 gpm) $ 147
  Omega Engineering FL4604 (1 – 10 gpm) $ 147

- Digital Pressure Gauge
  The gauge used in the experiment was Cole-Parmer 1200-030. This is discontinued. If buying now, we would get

  Omega Engineering DPG 5600B-100G $ 319

- Submersible Pump
  Flotec (1/3 HP) $ 84

- Air Compressor
  Husky 1.5 gallon Air Scout Compressor $ 100

III. Experimental Runs

Construction of the apparatus is complete, and experimental runs have been performed with water and compressed air. Thus far, most runs have been done with water. The following discussion outlines possible experimental runs to be done by students in the laboratory or in the lecture class:

First, let's discuss the dimensional analysis for the fluid flowing through the specimen. It is desired to determine how the pressure drop across the nozzle varies with the nozzle dimensions and the flow rate. Following an example in the textbook by John and Haberman[3], we will assume that the pressure drop ΔP depends on the the flow area A_1 at the entrance of the nozzle, the flow area A_2 of the nozzle exit, the density ρ of the fluid, and the volumetric flow rate Q of the fluid. (Why did we choose these parameters? Well, hindsight is great. We actually performed the experiment and found that this choice of parameters was consistent with the data.) Hence there are five parameters in the problem.

The dimensional analysis technique [1, 2] will enable us to reduce the problem to one of only two dimensionless groups. These groups are determined via the Buckingham Pi method to be:

$$\frac{A_1}{A_2} \quad \text{and} \quad \frac{\Delta P \ {A_1}^2}{\rho \ Q^2}$$

This means that the dimensionless group $\frac{\Delta P \ {A_1}^2}{\rho \ Q^2}$ is a function of $\frac{A_1}{A_2}$.

The following describes the determination of these pertinent dimensionless groups for this problem:

We wish to determine how the pressure drop across the nozzle varies with the nozzle dimensions and the flow rate of the fluid. We first make an educated guess that the pressure drop ΔP depends on the the flow area A_1 at the entrance of the nozzle, the flow area A_2 of the nozzle exit, the density ρ of the fluid, and the volumetric flow rate Q of the fluid. We then express the five parameters of the problem in terms of either the primary dimensions mass (M), length (L), and time (t) or Force (F), length (L), and time (t). We will choose M, L, and t. In terms of the primary dimensions, the parameters of the problem are

$$\Delta P = M\ L^{-1}\ t^{-2}$$
$$A_1 = L^2$$
$$A_2 = L^2$$
$$\rho = M\ L^{-3}$$
$$Q = L^3\ t^{-1}$$

We have 5 parameters in the problem, and they can be expressed in terms of 3 primary dimensions. The Buckingham Pi theorem states that the parameters can be arranged into $(5 - 3) = 2$ dimensionless groups. To determine these groups (historically called π groups), we proceed as follows:

Of the five parameters, we select three (i. e., the number of primary dimensions) which can possibly appear in both groups. These are called "repeating" parameters. (Note: Since ΔP is the dependent parameter, we want it to be only in one of the groups. Therefore we do not choose it as a repeating parameter.) We will choose A_1, ρ, and Q to be repeating. This leaves A_2 and ΔP to be non-repeating; i. e., they each appear in only one of the two groups.

We set-up the two groups as follows:

The first group is

$$\pi_1 = \Delta P\ A_1^a\ \rho^b\ Q^c = [M\,L^{-1}\,t^{-2}]\ [L^2]^a\ [M\,L^{-3}]^b\ [L^3\,t^{-1}]^c = M^0\ L^0\ t^0$$

The second group is

$$\pi_2 = A_2\ A_1^a\ \rho^b\ Q^c = [M\,L^{-1}\,t^{-2}]\ [L^2]^a\ [M\,L^{-3}]^b\ [L^3\,t^{-1}]^c = M^0\ L^0\ t^0$$

Explaining these two lines: After the first equal sign is the parameter grouping. We have to determine exponents a, b, and c which make the group dimensionless. After the second equal sign are the parameters expressed in terms of their primary dimensions. Finally, at the end of the line, we show that, in order to be dimensionless groups, the exponents for M, L, and t must be zero.

For the first group, equating exponents for dimensions M, L, and t, we have:

$$M:\ 1 + b = 0$$
$$L:\ -1 + 2a - 3b + 3c = 0$$
$$t:\ -2 - c = 0$$

Solving these equations for a, b, and c, we get $a = 2$, $b = -1$, $c = -2$

Therefore, the first dimensionless group is $\frac{\Delta P \ {A_1}^2}{\rho \ Q^2}$

For the second group, equating exponents for dimensions M, L, and t, we have:

M: b = 0
L: 2 + 2a -3b + 3c = 0
t: c = 0

Solving these equations for a, b, and c, we get a = -1, b = 0, c = 0

Therefore, the second dimensionless group is $\frac{A_2}{A_1}$

We will actually take our second group as the reciprocal of this, since, in our experiment, A_1 is greater than A_2 . We prefer to work with values greater than one rather than values less than one.

In summary, the Buckingham Pi theorem gives us the dimensionless groups to correlate our experimental data. We should be able to have our data show that

the dimensionless group $\frac{\Delta P \ {A_1}^2}{\rho \ Q^2}$ is a function of $\frac{A_1}{A_2}$.

We performed experimental runs and used the data to determine the actual functional relationship between the two groups. Once we have the functional relationship, we can use it to predict pressure drops for situations for which we have not made an experimental run.

As mentioned above, there are nine (9) nozzle specimens with holes ranging from 1/8" to 3/8" diameter at 1/32" increments. The specimens are numbered from 1 to 9, with 1 being the 1/8" hole and 9 being the 3/8" hole. Let's use data from Specimens 3, 5 and 7 to determine the functional relationship between the two dimensionless groups. We will then use this relationship to predict pressure drops for Specimens 4 and 6. Finally, we will compare these predictions with actual data from Specimens 4 and 6.

The diameter of nozzle entrance, for all specimens, is 5/8". Therefore A_1 is the area of a circle of diameter 5/8", or 0.00019793 m^2. A_2 varies with the nozzle. Specimen 3 has a hole of size 3/16" with an area of 1.7814×10^{-5} m^2. Specimen 5 has a hole of size 1/4" with an area of 3.1669×10^{-5} m^2. Specimen 7 has a hole of size 5/16" with an area of 4.9489×10^{-5} m^2.

Hence, for Specimen 3, $A_1 / A_2 = 11.11$. For Specimen 5, $A_1 / A_2 = 6.25$. For Specimen 7, $A_1 / A_2 = 4$.

Experimental runs were made for Specimens 3, 5, and 7 to determine the functional relationship between the two dimensionless groups. Pressure drops across the nozzles were measured for a variety of flow rates through the nozzles. For all three specimens it was found that the variation of the $\frac{\Delta P \; {A_1}^2}{\rho \; Q^2}$ group was small for the different runs of a given specimen. For Specimen 3, the average value of the group was 92. For Specimen 5, the average value was 29. For Specimen 7 it was 15.

Using Microsoft Excel, a second-degree polynomial was fit to the experimental data, and the functional relationship was determined to be

$$\frac{\Delta P \; {A_1}^2}{\rho \; Q^2} = 0.7938 \; (A_1 / A_2)^2 - 1.9828 \; (A_1 / A_2) + 10.481$$

The curve fit was very good. The fit used 21 data points and R^2 was 0.9959

Having determined the functional relationship between the two dimensionless groups, we then picked some random data points from experimental runs done for Specimens 4 and 6. We used the functional relationship to predict the pressure drop for the runs and then compared this predicted pressure drop with the actual pressure drop determined through experimentation. Specifically, we considered flows of 0.91, 1.35, and 1.72 gpm for Specimen 4 and flows of 1.26, 1.49, and 2.1 gpm for Specimen 6. The results of the comparisons are in Table 1. It is seen that the differences between the predicted and measured pressure drops varied from 7 to 25 percent.

Table 1 - Comparison of Actual and Predicted Pressure Drops

Specimen No.	Nozzle Diam (in)	Flow (gpm)	Predicted ΔP (psi)	Actual ΔP (psi)	% Difference
4	7/32	0.91	0.58	0.70	17
4	7/32	1.35	1.27	1.61	21
4	7/32	1.72	2.06	2.66	23
6	9/32	1.26	0.47	0.50	6
6	9/32	1.49	0.66	0.71	7
6	9/32	2.1	1.30	1.02	27

One of the reviewers of the draft paper kindly made suggestions for modifications which could possibly decrease the % difference values. In particular, it was suggested that the less-than-perfectly fabricated nozzles might be the source of the differences. We agreed that this might be the case, and, following the suggestion of the reviewer, we purchased four accurately-fabricated nozzles. [4] One of these nozzles is shown below in Figures 4 and 5. The nozzles had hole diameters of 0.07", 0.10", 0.13", and 0.16". We used the first, second, and fourth nozzles in experimental runs to determine the correlation equation and used four runs from the third nozzle to demonstrate the usefulness of the dimensional analysis technique. Unfortunately, the % differences using these accurate nozzles were between 12 and 26%; i. e., no better than the nozzles of Figure 3. We believe that the high % differences are due to developing the correlation equation using data over too large an A_1 / A_2 range. Indeed, when we used the data from only two of the fabricated nozzles (nozzles 2 and 4) to develop the correlation equaion, we found that the % differences for the nozzle 3 runs were much lower, ranging from 4.4 % to 8.3 %.

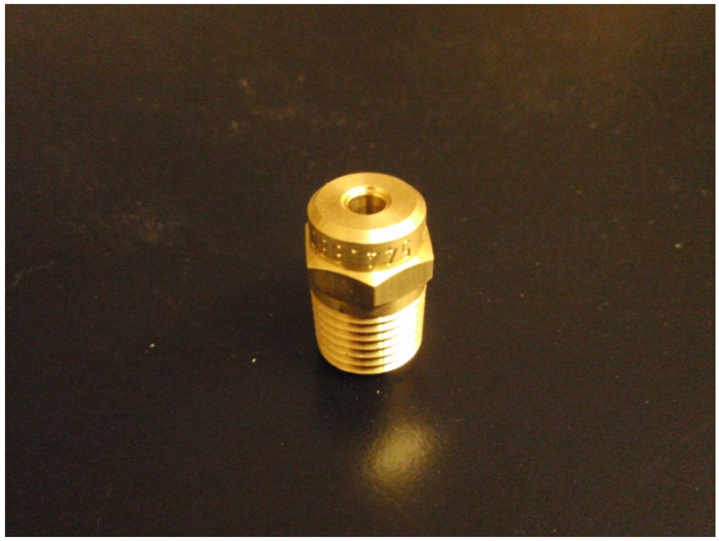

Figure 4 - Manufactured Nozzle

Figure 5 - Manufactured Nozzle Mounted on Experiment

In summary, this section has outlined a possible exercise for students to do in the laboratory or in the lecture class. The students perform experimental runs of pressure drop versus flow rate for some specimens to determine the functional relationship between the two dimensionless groups. They then use this relationship to predict pressure drops for a few runs of the other specimens. Finally, the experimental runs are performed for the other specimens and the predicted pressure drops are compared with the actually measured pressure drops. By doing this, the students will see the advantages of using dimensional analysis to correlate experimental data and to reduce the number of required experimental runs.

IV. Rubric

A rubric similar to the following could be used to assess the effectiveness of the experiment in improving the students' comprehension of the use of dimensional analysis. The rubric could be administered before the performance of the experiment and after the performance.

		Unsatisfactory	Satisfactory	Exemplary
1.	Able to identify the pertinent parameters for the problem			
2.	Able to determine the number of primary dimensions for the problem			
3.	Able to express the parameters in terms of their primary dimensions			
4.	Able to determine the number of dimensionless groups			
5.	Able to determine the repeating parameters for the dimensionless groups			
6.	Able to obtain the equations for determining the exponents of the parameters			
7.	Able to correctly solve the equations and determine the dimensionless groups for the problem			
8.	Able to curve fit the experimental data to determine the correlation equation between the groups			
9.	Using the correlation equation, able to determine the predicted pressure drop across a new nozzle specimen.			

V. Planned Future Work

It is planned to make additional experimental runs for other types of specimens; e. g., specimens made from pipe fittings. Runs will be made with compressed air as well as water. It is also planned to modify the apparatus to enable its easy transport from storage to the lecture class. The equipment will be mounted on a cart and reduced in size as necessary.

VI. Conclusion

A fluid mechanics experiment has been designed and constructed to illustrate the topic of dimensional analysis. The experimental apparatus has performed successfully and has provided results which show the application and usefulness of dimensional analysis in planning experiments and correlating experimental data.

VII. Bibliography

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