**Session XXXX**

# A dislocation near a cylindrical hole: A numerical treatment

**Abu B Siddique, Tariq A Khraishi**
Department of Mechanical Engineering
University of New Mexico

## Abstract

Analytical solutions for a dislocation inside an infinite medium of matter need to be adjusted with correction terms to ensure zero traction on internal free surfaces, i.e. voids or holes. The current article proposes a numerical approach to calculate this correction term for a screw dislocation near a cylindrical hole. This method can be implemented in a finite medium also. The numeric solution is verified against an analytical solution.

## Introduction

The solution for a straight screw dislocation[1] in an infinite medium is a classic problem important for the theory of plasticity. In a semi-infinite medium, Eshelby[2] formulated the solution for a screw dislocation eccentrically situated in a thin circular rod. Friedel[3] presented the analytical solution for a screw dislocation near a cylindrical void. This article introduces a numerical method to solve similar problem to [3] but is not limited to only cylindrical void and is based on the collocation point method by Khraishi et al.[4,5,6].

## Theory

A screw dislocation introduces stresses in a medium of matter. If the medium is semi-infinite or finite, the solution needs to satisfy the boundary conditions depending on the problem configurations. For the current problem, traction $\vec{T} = \sigma\vec{\eta} = \vec{0}$ on the surface of the cylindrical void. Since the medium is infinite in all directions, for simplicity, we treat it as a 2D problem shown in Figure 1. The analytic solution in [3] is based on one screw dislocation in the medium along the x-axis. For any other dislocation not on the x-axis, that will require second rank tensor transformation for stress.

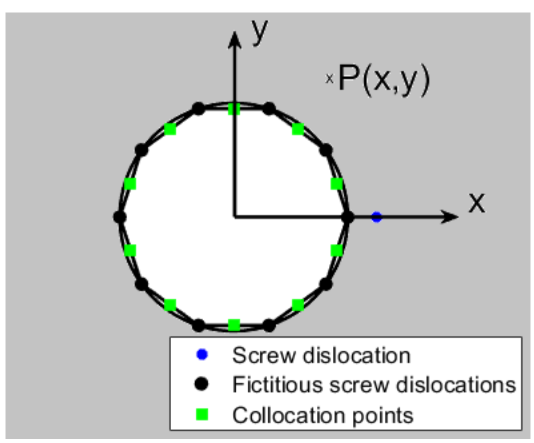



Figure 1: Cylindrical void inside the medium nearby a positive screw dislocation.

## Numerical Framework

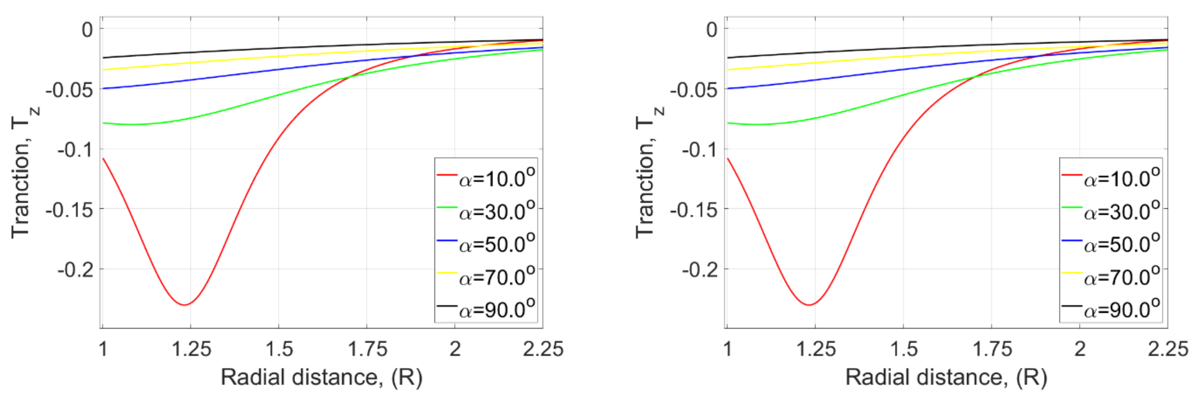
The classic solution of a screw dislocation in an infinite medium[7] does not satisfy the traction free boundary conditions on the void surface. To enforce the zero traction on the surface, N infinite dislocation loops of unknown Burgers vector with collocation points at the centers are padded on the surface of the cylindrical void shown in Figure 1. This method requires the boundary conditions to be satisfied on the collocation points.

For the current configuration of the problem, traction vector on a collocation point is $\{0, 0, \sigma_{xz} \cos\alpha + \sigma_{yz}\ \sin\alpha\}$ where $\alpha$ is the angle created by the collocation point and the $x$ −axis. Now the traction free boundary conditions can be written as follows,

$$\sum_{j}^{N} \sigma_{xz}^{j\to i} \cos\alpha_i + \sigma_{yz}^{j\to i}\ \sin\alpha_i = -\sigma_{xz}^{i} \cos\alpha_i - \sigma_{yz}^{i}\ \sin\alpha_i \qquad (1)$$

Where, $\sigma^{j\to i}$ represents the stress caused by loop $j$ and $\sigma^{i}$ represents the stress caused by the screw dislocation on collocation point $i$, respectively. The right-hand side of eq. (1) is of finite value acting as a forcing vector, and the left-hand side is linear in the Burgers Vectors of the padded loops, which allows solving the unknown Burgers vector of the fictitious loops using any linear equation solver.

## Results




(a) Without considering the surface effect (b) Considering the surface effect

Figure 2: Effect of surface treatment. Lines show the traction component along the radial directions. Unit of Traction $T_x$ is $G/2\pi$.

Figure 2 shows the traction component $T_z$ dies out for the field points near to the surface of the cylindrical void when the surface effect is considered.

## Summary and Conclusions

The authors of this article presented a general numerical framework that can be extended to solving irregular geometries and also works for multiple dislocations present in the medium. Besides, the numerical results show a good agreement with the analytical solutions.

## References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