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A Feedback Control System for Engineering Technology Laboratory Courses

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Abstract

A feedback control system for incorporation into an Engineering Technology controls laboratory has been developed. The purpose of developing a working control system is to reinforce controls theory taught in the classroom. A control system for positioning an air cylinder driven load platform was selected for the design. By controlling the height of the column of air in the air cylinder and regulating the pressure applied to the air cylinder piston, the platform could be positioned anywhere within the range of the air cylinder travel. To demonstrate the control capability of the system, random load disturbances were generated by adding and removing laboratory weights to the platform, with the desired setpoint position maintained. The instructional benefit of selecting a position control system was that the reaction of the control system could be readily observed as the platform returned to the set point position.

The PID controller gain constants were found first by using the Ziegler-Nichols Method of controller design. In this method, the system gain is increased to the point of oscillation. The gain for oscillation and frequency of oscillation will be inserted into a table of tuning rules to determine the value of PID controller constants.

Next, the dynamics of individual components of the system were characterized through experimentation and modeling. A Laplace transform transfer function representation of the system was found and then analyzed using simulation software and root-locus analysis. The gain for oscillation and frequency of oscillation were determined from the root-locus plot. The Ziegler-Nichols Tuning rules where then re-applied to find the value of the PID controller constants.

A comparison of system performance using experimentally determined PID constants and theoretically determined PID constants was presented. Ideally, the constants and resulting system response using both methods would be equal. The time domain closed loop response of the theoretical model was found by computer simulation and then compared to data taken from the actual system.

I. Introduction

A block diagram of the load platform positioning system is shown in Figure 1-1. The data acquisition system consists of a ZENITH® personal computer, BURR-BROWN® computer instrumentation hardware and LABTECH NOTEBOOK® data acquisition and control software. LABTECH NOTEBOOK is a graphical user interface that allows data acquisition and control, using commonly available computer instrumentation boards, without the need for programming. The portions of the control loop residing in the data acquisition and control system are the setpoint input, setpoint and feedback summing node, and PID controller. The "plant" or system being controlled is a NORGREN® E/P converter(a voltage command to pressure command conversion device) and a vertically oriented Speedaire® air-cylinder driving a load platform. A voltage proportional to the position of the platform is fed back to the PID controller using a MIDORI linear potentiometer.

Background

In the general configuration of a feedback control system, the output signal is fed back and subtracted from the input signal, creating an error signal. The error signal then serves as the input for the system controller which processes the error and generates a control signal to correct the output of the system. One type of controller frequently used in analog industrial closed loop control applications is the PID controller. The control signal of the PID controller is generated by summing scaled amounts of the: error, integral of the error, and the derivative of error signals.

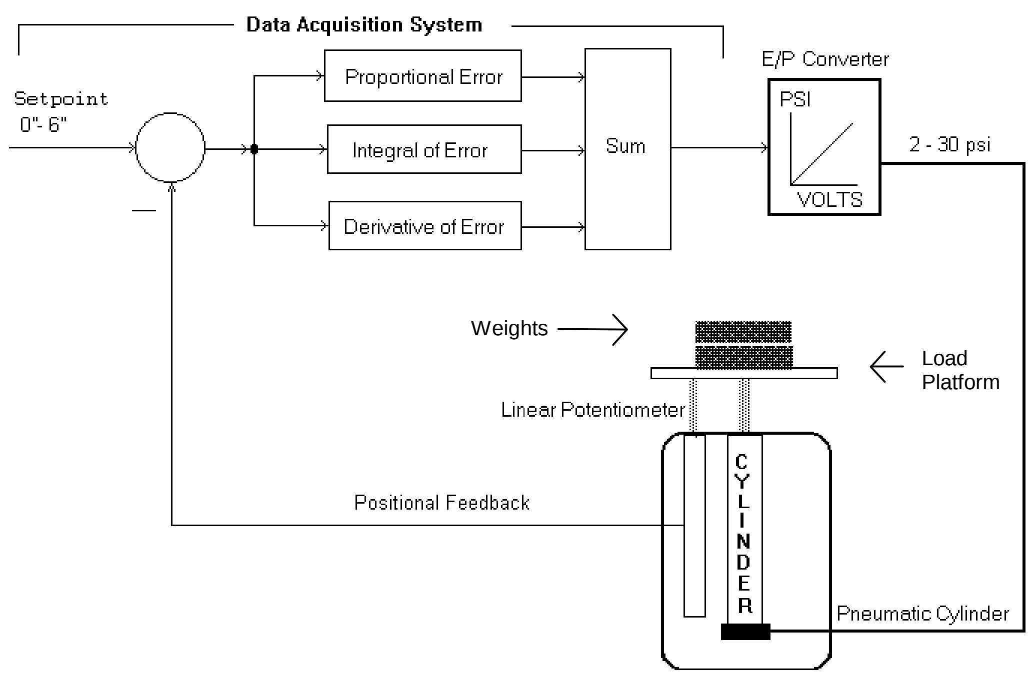


Figure 1-1: Load Platform Position Control System

Proportional control is the most basic control mode of the PID controller. The control signal generated for corrective action of the system is proportional to the error. Using only proportional

control in a system can produce large steady-state errors in its operating point when disturbances are large because the net correction offered by the error signal and controller gain frequently is not capable of bringing the system to the desired set point. Increasing the controller proportional gain will decrease error but there is a limit to this adjustment because too much gain has the potential of sending the system into oscillation. Using PI (proportional plus integral) control solves the offset problem of the proportional-only control system by adding extra output effort only when an error exists. The action of the integral term is to sum the error signal over time until a large enough correction signal is generated to offset the large disturbance, and force the system error to zero. The effect of the proportional control working in conjunction with the integral control is to improve the steady state response.
In systems where an improvement in frequency response is needed, derivative control action can be added to the proportional plus integral control. This speeds up the response of the controller because it responds to the rate of change of error, and can make corrections before the error reaches a large value. Derivative control has the net effect of adding damping to the system. If the gain of the derivative term is higher than necessary, noise spikes that frequently occur on the error signal can be amplified and cause saturation or overloading of the controller.

The response characteristics of the system can be optimized by the choice of PID controller constants. Determining these constants is also called controller tuning. The classic method of controller tuning, known as the, “Ziegler-Nichols Method of Automatic Controller Tuning”, will be applied to the pneumatic cylinder positioning system. In this method, the value of the proportional gain that causes closed-loop oscillation, and the frequency of that oscillation, are experimentally determined. These values are then used to calculate the PID constants by the tuning rules set forth by Ziegler and Nichols.

The Ziegler-Nichols Method may also be used if the transfer function of the system is already known prior to the tuning process. Component characteristics can be measured and used to develop models of the system components. From these component models, a complete system model is found and then analyzed using linear systems analysis. The root locus method can be used to find the closed-loop gain that would cause oscillation, together with the frequency of oscillation of the closed loop model. Then, as in the experimental method, the gain and frequency can be entered into the Ziegler-Nichols tuning rules to determine the appropriate PID controller constants.

Assumptions

It is assumed that the positioning system can be represented as a linear system. This permits use of the Ziegler-Nichols Method of Controller tuning and Laplace transfer function analysis. However, due to the compressibility of air and the stiction (Coulomb friction) of the piston seal, non-linearities do exist in the pneumatic system. The assumption was made that these non-linearities are negligible, thus tuning rules and linear analysis could be used. As stated by Ogata, “Although all physical systems demonstrate non-linear characteristics over a range, if the range of the deviation of system variables is small enough then the system can be treated as linear” [Ref 1].

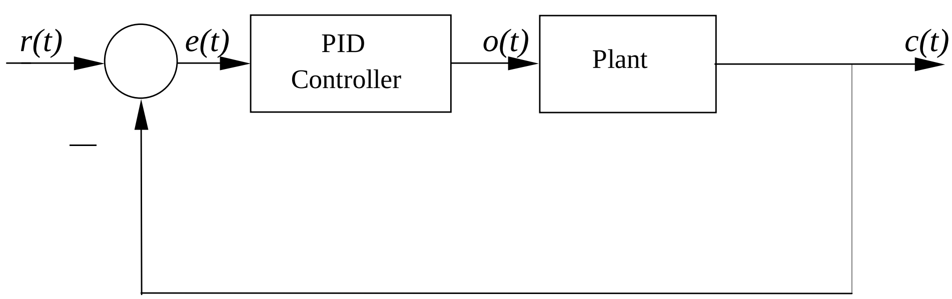
Additionally, the PID controller is implemented using a digital computer. In strict analysis, the sampling process of the digital computer causes the system to be classified as a sampled-data system. However, when the sample period of the digital controller is very short compared to the time constant of the system, the system can be analyzed as if it were continuous. In the case of the load positioning system, the computer can sample much faster than the mechanical components can react, thus continuous methods are applied.

II. Ziegler-Nichols PID Controller Tuning Rules

The objective of tuning a PID controller is to compensate the control loop so that the control system can behave in an optimal manner, which means that it will exhibit satisfactory transient response and small steady state offset. Tuning means to adjust the controller parameters to produce enough change in the control loop steady-state and dynamic characteristics to compensate for the fixed characteristics of the plant. The controller parameters are defined as the proportional gain P, the integral time T_i the derivative time T_d. The proportional gain is the fraction of the error signal generated by subtracting the output of the system from the operating set point. The integral time is the amount of time it takes the integral of error to reach the same magnitude as the proportional error. The derivative time represents the amount of gain added from the rate change of error, and is given the designation time since it is a simplification of the units in an expression:

(controlled variable) per (controlled variable per time) = time

A block diagram of a closed loop control system using a PID controller is shown in Figure 2-1. Controller parameters can be found by mathematical analysis and modeling of the plant for its transfer function and then calculating the parameters of the PID transfer function. However, in many applications it is not practical to attempt to model a plant mathematically due to the complexity of the system and time required to develop an accurate model. Fortunately, there are numerous experimental methods and auto-tuning devices available for finding the PID constants in an expedient manner without the need for detailed modeling. The objective of this chapter is to illustrate one of these methods.



where:

r(t) = operating set point

e(t) = error (defined as set point - feedback)

$$o(t) = P\left[e(t) + \frac{1}{T_i}\int_{-\infty}^{t} e(t)dt + T_d\frac{de}{dt}\right] \qquad (1)$$

c(t) = system output

Figure 2-1: Control System with PID Controller

An experimental tuning method that was developed during the marketing of the early PID controllers and remains in use today is the Ziegler-Nichols method. This method was developed by John Ziegler and Nathaniel Nichols at Taylor Instruments, who designed the Fulscope, the first controller to include derivative control with proportional and integral control. They published their work in the 1942 Transactions of the American Society of Mechanical Engineers , [13] called "Optimum Settings for Automatic Controllers". The goal of the article was to provide the controls engineer with a quick and efficient method of setting up PID controllers on existing installations, by doing tests on the plant in the field. Two approaches to controller tuning were presented in this article, both with the objective of producing a nominal 25% overshoot of the operating set point when the closed loop system was subjected to a step impulse. Figure 2-2 illustrates the response output $c(t)$ of an optimally tuned closed loop system that has been subjected to a step impulse.

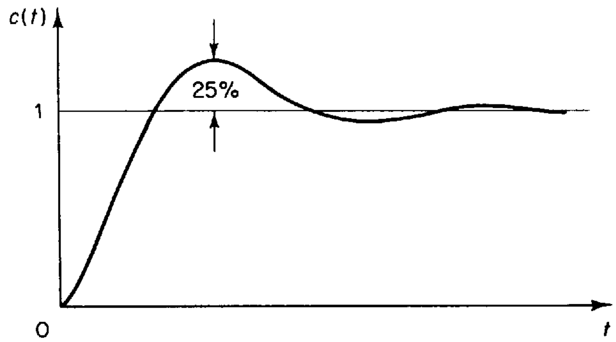


Figure 2-2: Closed Loop Step Response of an Optimally Tuned System

In the first method the integral and derivative effects are disabled by setting the integral time T_i to infinity and the derivative time T_d to 0. The proportional gain P is increased until the closed loop control system just reaches the point of instability and produces continuous oscillation as shown in Figure 2-3.

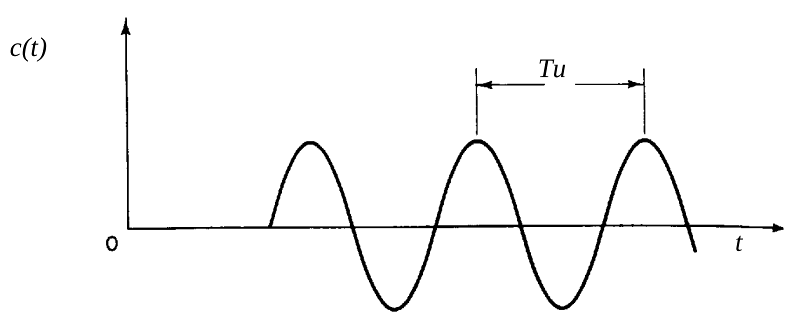


Figure 2-3: Sustained Oscillation with Period *Tu*

The proportional gain P_U and period of oscillation T_U at this point of instability are used to determine the values of the PID constants using the tuning rules set forth by Ziegler and Nichols in Table 2-1.

Ziegler-Nichols Tuning Rule Based on Critical Gain P_U and Critical Period T_U (First Method)			
Type of Controller	Gain P	Integral Time T_i	Derivative Time T_d
P	$0.5P_U$	∞	0
PI	$0.45P_U$	$\frac{1}{1.2}T_U$	0
PID	$0.6P_U$	$0.5T_U$	$0.125T_U$

In the second method, the control loop is opened, the plant is subjected to a unit step and the resulting reaction curve is observed. If the system response to the step input resembles an S-shaped curve, the lag time L and time constant T can be measured, as shown in Figure 2-4. If the reaction curve does not resemble an S-shaped curve, the second method cannot be used to find the controller PID constants.

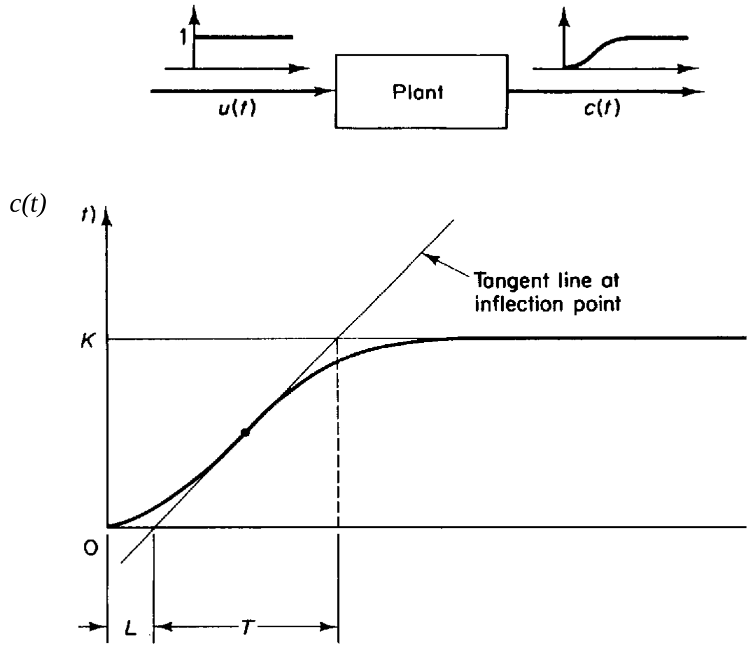


Figure 2-4: Determination of Plant Lag Time and Time Constant

The data is used to determine the appropriate parameter values for the PID controller using the tuning rules for step response testing in Table 2-2 lists.

Ziegler- Nichols Tuning Rules Based on Step Response of Plant (Second Method)			
Type of Controller	Gain P	Integral Time T_i	Derivative Time T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

Table 2-2

Zigler and Nichols determined the tuning rules of Tables 2-1 and 2-2 by testing many systems, and observing that these constants, on the average, produced optimal results. Actual response of a given system could result in a set point overshoot range of 10% to 60%. However, the above controller settings give a starting point at which the control system becomes functional and can be fine tuned if needed.

<u>Tuning the Pneumatic Cylinder Positioning System PID Controller</u>

The pneumatic cylinder positioning system performance specifications were to maintain the operating set point over a range of loading disturbances between 5 and 45 lb. that can occur at random. The PID controller gives the system the capability to handle a range of loads because the integral term integrates the error term until the system output is at the setpoint with no steady state error.

Another benefit of using the PID controller in this design is that it can also respond to the rate of change of error and provide corrective output generated by the derivative term. When the error suddenly increased, with rapid placement of weight of 10 lb. or more on the loading platform, an abrupt change in the output signal was generated as the integral term quickly ramped to correct the system output. These changes have the potential for causing system instability. The derivative output contribution can start to make system corrections before the error reaches the oscillation causing value. The derivative controller effort to counteract rapid movement of the load platform effectively added dampening to the system.

The PID controller parameters were found by the first method of Ziegler and Nichols, where the closed loop system was tested and the amount of proportional gain that caused oscillation was determined. The second method, in which the process reaction curve is found, was not used because the rapid and violent acceleration of the pneumatic cylinder when operated in open loop mode did not produce an S-shaped curve. Using the closed loop method, the value of proportional gain P was changed in the PID controller until the ultimate gain value P_U was reached. The period of oscillation T_U was observed on the data acquisition system monitor.

The PID control algorithm written in the LABTECH NOTEBOOK data acquisition and control software has a slightly different form than Equation 1 used by Ziegler and Nichols in that the proportional gain is factored into the integral and derivative time terms changing them from time parameters into gain parameters. The alternate form of the controller equation is shown in Equation 2. A conversion of the tuning rules of Table 1 was made by comparing terms in Equation 1 (repeated following) and Equation 2.

Original form:

$$o(t) = P\left[e(t) + \frac{1}{T_i}\int_{-\infty}^{t} e(t)dt + T_d\frac{de}{dt}\right] \qquad (1)$$

LABTECH NOTEBOOK form:

$$o(t) = Pe(t) + I\int_{-\infty}^{t} e(t)dt + D\frac{de}{dt} \qquad (2)$$

Comparing the two equations:

$$I = \frac{P}{T_i} \quad (3) \quad \text{and} \quad D = P * T_d \quad (4)$$

Using expressions 3 and 4, the controller tuning rules of Table 2-3 are converted into a compatible format for the LABTECH NOTEBOOK form of the controller equation. The values entered into into the PID output control block were determined from Table 2-3 following:

Ziegler-Nichols Tuning Rule Based on Critical Gain P_U and Critical Period T_U (First Method) LABTECH NOTEBOOK Form			
Type of Controller	Proportional Gain P	Integral Gain I	Derivative Gain D
P	$0.5P_U$	0	0
PI	$0.45P_U$	$0.54\left(\frac{P_U}{T_U}\right)$	0
PID	$0.6P_U$	$1.2\left(\frac{P_U}{T_U}\right)$	$0.075(P_U T_U)$

Table 2-3

Test Procedure

The anticipated load range of the system was 5 - 45 lb., with a value of 30 lb. selected to determine the tuning parameters. The ultimate gain P_U and the ultimate period T_U were determined and then used to find the controller constants. The system was then operated at various loads to verify proper operation. The following procedure lists the steps for tuning the PID controller.

1. The LABTECH NOTEBOOK Software was configured for an input voltage channel to read the feedback voltage of the linear potentiometer, an input voltage channel to read the setpoint input and a PID voltage output analog channel to supply the control voltage to the E/P(voltage to pressure converter) unity gain buffer. A block diagram of the test setup is shown in Figure 2-5.

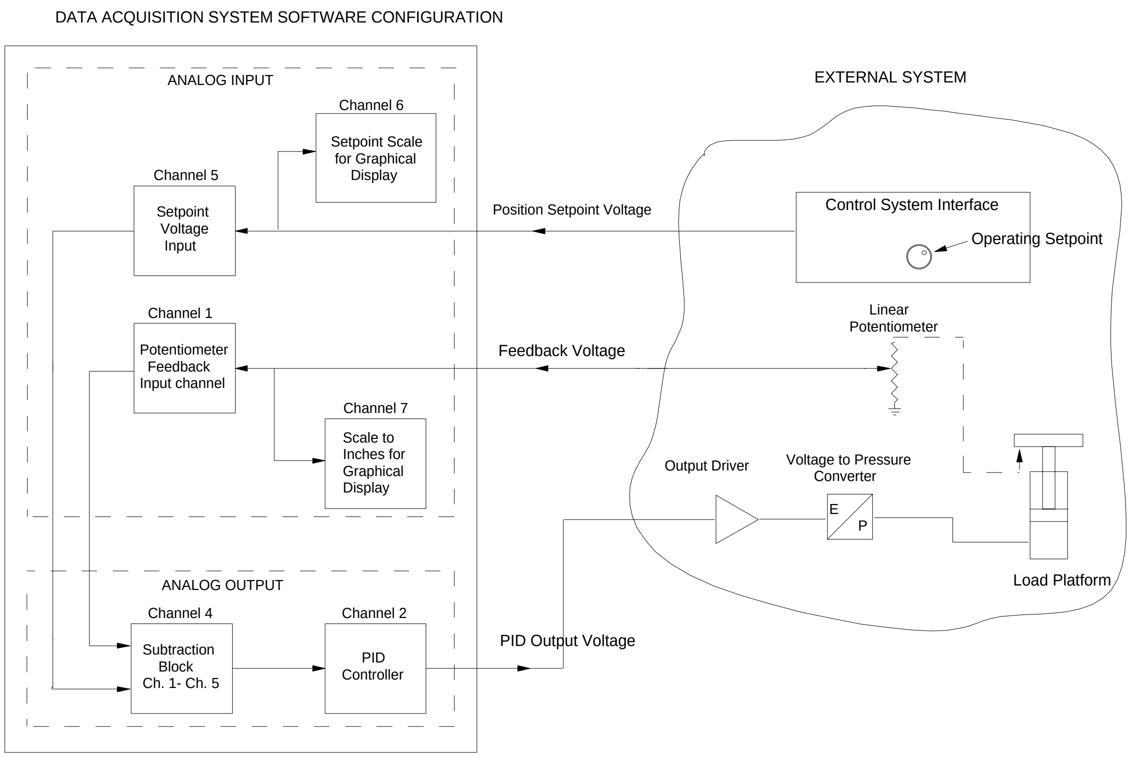


Figure 2-5: LABTECH NOTEBOOK Channels Setup for Load Platform Position Control System

2. The channels setup menu was entered and the *I* and *D* constants in the control loop output menu block to were set to zero. The *P* constant was set to 1.5 for the first test of control loop stability. The set point is was set to 7 volts in the output control block to position the load platform high enough to prevent the cylinder from hitting the bottom bumper during oscillation.

3. The manual regulator was set to 30 psi. This regulator supplies the E/P with source pressure.

4. Three 10 lb. weights were placed on the load platform.

5. With the position control system started, the PC monitor will display an upper window for PID controller output to the E/P driver and a lower window for the PID controller input from the linear potentiometer.

6. The platform raised but did not go into oscillation when disturbed by pushing it downward then quickly releasing. The three 10 lb. weights were removed carefully and the control run terminated.

7. The channels setup menu was returned to and the output block menu modified to have a proportional gain *P* of 1.8.

8. The three 10 lb. weights were replaced.

9. The control loop was started and the load platform allowed to settle into position. With a gentle push in a downward direction and a quick release, the platform attempted to oscillate but movement stopped after a few cycles. The proportional gain P was still not large enough to cause sustained oscillations.

10. The three 10 lb. weights were removed carefully and the control run terminated.

11. The Channels Setup menu was returned to where the output block menu was modified to have a proportional gain P of 1.95.

12. The three 10 lb. weights were replaced.

13. The position control system was started with P = 1.95, the platform went into a slow constant oscillation after a gentle downward push and quick release. This value of P for constant oscillation was recorded as the ultimate gain value of P_U = 1.95

14. The platform was allowed to oscillate for 6 seconds. Ten cycles occurred during the 6 second interval. With this information, the ultimate period T_U was determined by dividing six seconds by the number of cycles: T_U = (6 seconds) / (10 cycles) = 0.6 s

15. With the values of T_U and P_U obtained, the controller constants were calculated in the LABTECH format of the Ziegler - Nichols Tuning Rules for quarter cycle decay response: $P = 0.6\, P_U$ $\quad I = 1.2(P_U\ /T_U)$ $\quad D = 0.075(P_U)(T_U)$
The controller constants were:
P = (0.6)(1.95) = 1.17 $\quad$ I = 1.2 (1.95/0.6) = 3.9 $\quad$ D = 0.075(1.95)(0.6) = 0.088

16. The Ziegler-Nichols PID controller tuning method is an approximation and sometimes it is necessary to make final adjustments after the performance of the system is observed. Operation of this system at a weight near 10 lb. caused the platform to oscillate about the set point due to the rapid ramping of the integrator function. To eliminate this problem, the I constant was reduced to 3.00. The PID constants were set to P = 1.17, I = 3.00 and D = 0.088.

17. The position control system was tested with loads from 5 - 45 lb. and remained stable while returning to the desired operating point.

III. Analysis of System

In the previous section, the PID controller constants required for stable operation of the pneumatic position control system were found using the Ziegler-Nichols Tuning Rules. The "ultimate gain" Pu, the value of proportional gain that causes oscillation of the closed loop system was found. The "ultimate period" Tu, the period of the oscillation at the gain Pu was noted and then Tu and Pu were used in the Ziegler-Nichols tables of tuning rules to find the optimum PID controller constants. In this section, the dynamic behavior of the system components will be modeled using experimental data and Laplace Transforms. The components will be combined into a block diagram model of the system, and then simplified by block diagram algebra into a single transfer function that is the ratio of two polynomials. This open loop transfer function was then analyzed using MATLAB's root-locus analysis program "rlocus" to determine the gain for oscillation *Pu* and the frequency of oscillation ω from which the ultimate period *Tu* will be derived.(The variable names *Tu* and *Pu* are used to represent the same variables in the actual and simulated system although their values are not necessarily the same for each system) In terms of a root-locus diagram, *Pu* is the value of gain *K* that causes the roots of the closed-loop transfer function equation to enter the right half plane. Using SIMULINK, the closed-loop system will be simulated, using the theoretical gain for oscillation *Pu*, to determine the closed-loop frequency of oscillation corresponding to the value of gain in the root-locus analysis. As with the actual system, the Ziegler-Nichols tuning rules use *Pu* and *Tu* to find the PID controller constants for stable operation of the simulated system.

A piping and instrumentation drawing of the load platform control system is shown in Figure 3-1. The components modeled in the analysis are the E/P converter, load platform air cylinder and feedback potentiometer. The characteristics of the E/P converter that will be determined are its internal resistance and voltage to pressure gain ratio. Parameters modeled in the load platform air cylinder are: the air flow generated by the movement of the air cylinder piston and capacitance of the volume of air underneath the piston, and the force balance relationship between piston force and the load force. The linear potentiometer, in the feedback loop, is modeled as a gain with a zero offset.

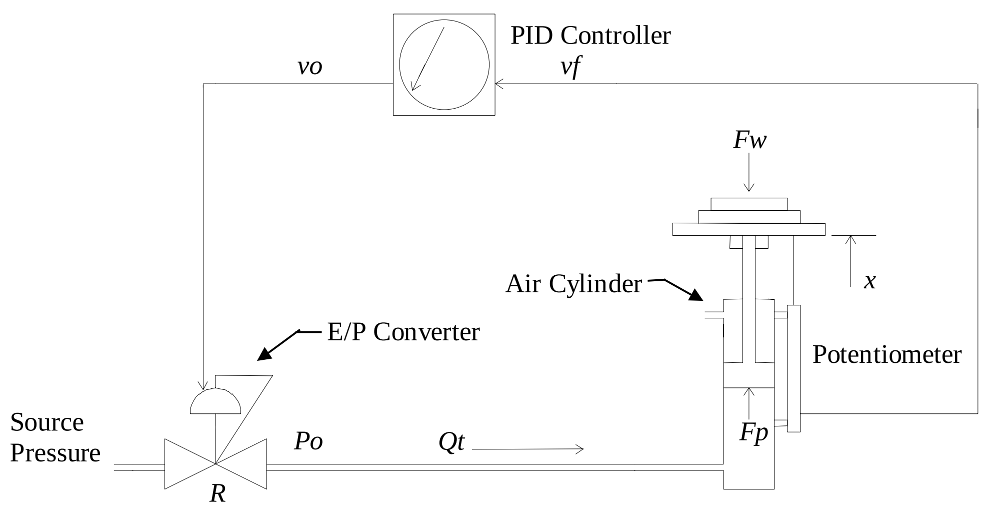


Figure 3-1: Piping and Instrumentation Diagram

The load platform position control system was modeled and analyzed using lumped-parameter analysis. This method of analysis applies to linear systems that can be represented by ordinary differential equations and their corresponding Laplace Transforms.

Description of Block Diagram

A block diagram representation of the system is shown in Figure 3-2, where the blocks represent constants or Laplace Transforms, will be constructed to realize the dynamic characteristics of components and the contribution of each in the overall performance of the system. The system transfer function is derived by block diagram reduction.

The block *Kep* represents the gain in $psi/volt$ of the E/P converter for no flow conditions. The input of the block is the voltage output of the PID controller Vo. During high flow conditions, the internal resistance *R* of the E/P converter becomes a significant factor in the pressure output KVo of the E/P because it causes a pressure drop and affects the amount of pressure *Pp* available to drive the air cylinder piston. The relationship between commanded E/P pressure and the piston pressure is shown below.

$$Pp = KepVo - QtR$$

where

Pp = piston pressure
Kep = voltage to pressure gain of E/P
Vo = PID output voltage
Qt = total air flow into air cylinder
R = internal resistance of E/P

Total flow *Qt* is caused largely by the sweeping volume of the air cylinder as it moves in the *x* direction. This contribution to flow is designated as *Qp* and is expressed as

$$Qp = A\frac{dx}{dt}$$

where

A = area of piston

A secondary contribution to flow is the compression of the air in the air cylinder. The compressed air stores energy and can be modeled as a capacitance. The flow of the pneumatic capacitor is expressed by

$$Qc = C\frac{dp}{dt}$$

Where *C* is the capacitance of the air cylinder for small changes in volume at cylinder mid-stroke and $\frac{dp}{dt}$ is a small change in pressure at cylinder mid-stroke.

The total flow during a change in platform position is then:

$$Qt = A\frac{dx}{dt} + C\frac{dp}{dt}$$

The summing point of the three flows occurs at the third summer in the block diagram. The output sum of this block is the variable *Qc* which is applied to the gain block $\frac{1}{C}$ to produce the piston pressure *Pp*.

The basis of the model is the force balance relationship between the force caused by the pressure *Pp* on the piston face with area *A* and the force caused by the combined weight of the load and platform. Any unbalance in the force balance relationship *Fw* = *Fp* will cause the platform to move up or down along the x direction. The force balance is expressed as:

$$Fp - Fw = m\frac{dx^2}{dt}$$

and is illustrated on the block diagram by fourth summing node were *Fw* and *Fp* are added resulting in an output $m\frac{dx^2}{dt}$.

To model the position *x* output of the system, the expression for force $m\frac{dx^2}{dt}$ was divided by the gravity constant *m* and then passed through two integrators. First, the force term was passed through a gain block $\frac{1}{m}$ leaving the acceleration term $\frac{dx^2}{dt}$. This term was integrated once and used in the air-flow velocity feedback path through block *A* on the top of the diagram. Then the velocity term $\frac{dx}{dt}$ was integrated to produce the desired *x* output. To supply the controller with information about the position *x* of the platform, a linear potentiometer was used. This potentiometer was represented by the factor *Kp* and an input labeled *Vos.*

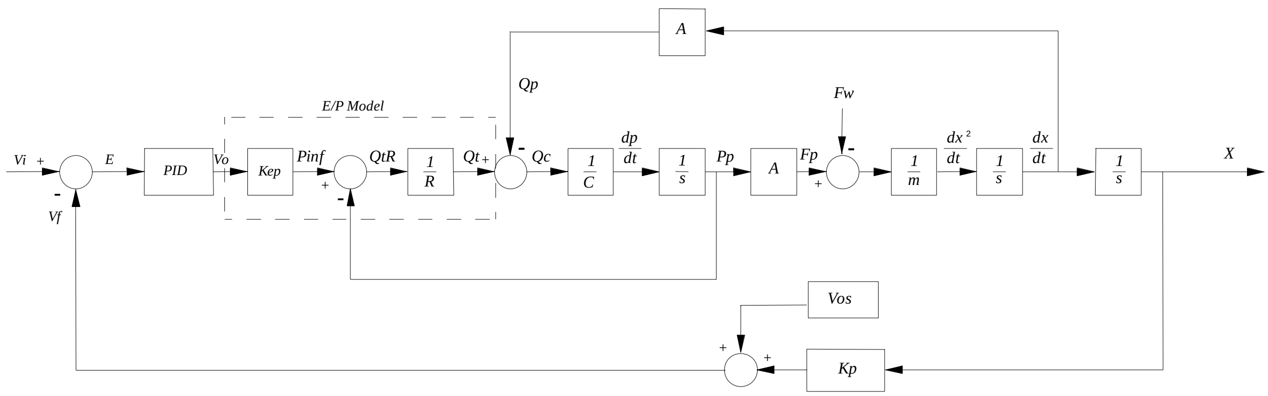


Figure 3-2: Block Diagram of Load Position Control System

Determination of System Transfer Function

The block diagram of Figure 3-2 was used to derive a transfer function by block combination and simplification. The first step in simplification was eliminate the *Fw* input by holding it constant and including its mass contribution into the model. Any inputs can be ignored since they are not of concern when finding the transfer function. The PID controller was set to unity gain during response testing so it may also be removed to simplify the model. The resulting simplified model that will be used for analysis is shown below in Figure 3-3.

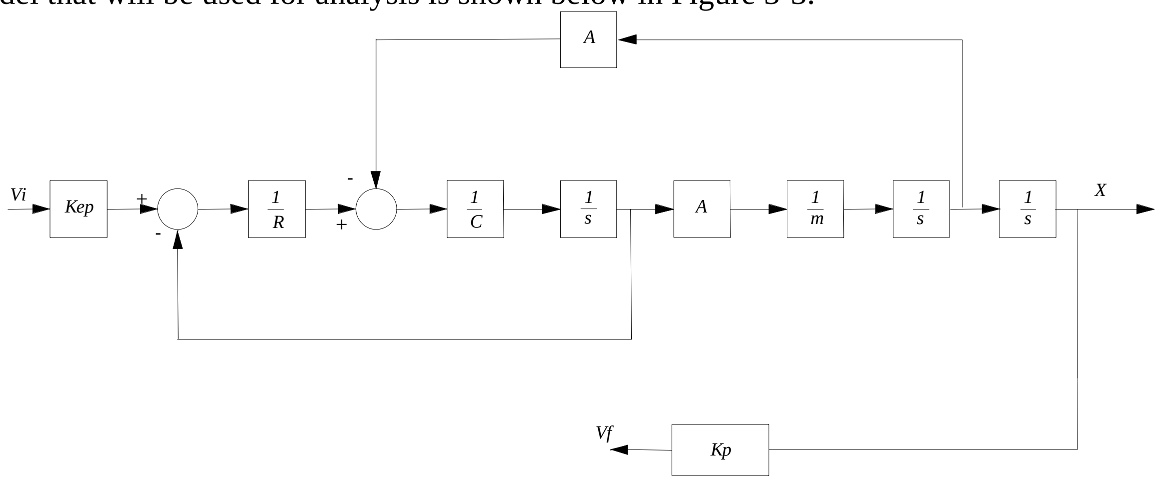


Figure 3-3: Reduction Step 1, System Model Used for Derivation of System Transfer Function

The second step in simplifying the block diagram was to combine adjacent blocks and move $\frac{1}{R}$ through the outer summer to eliminate it.

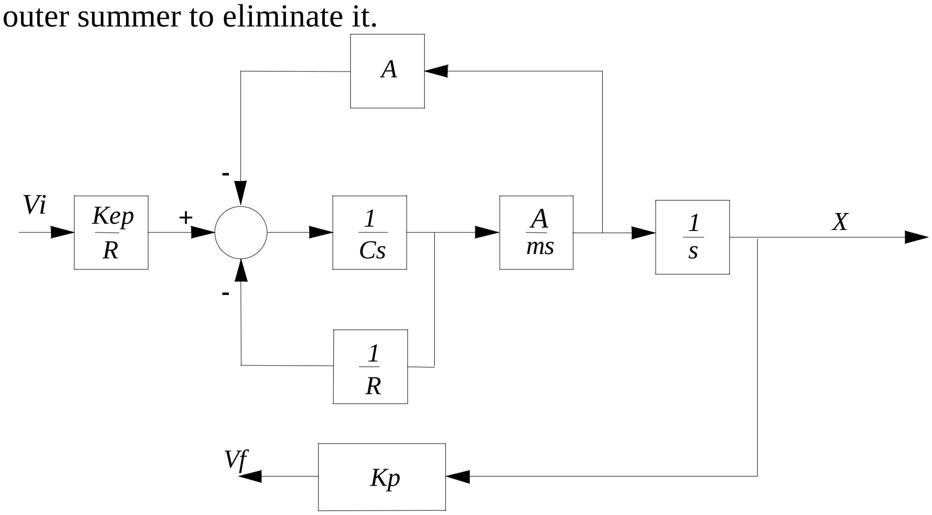


Figure 3-4: Reduction Step 2

The third step was to find the equivalent gains of the inner loops then and combine them into a single block.

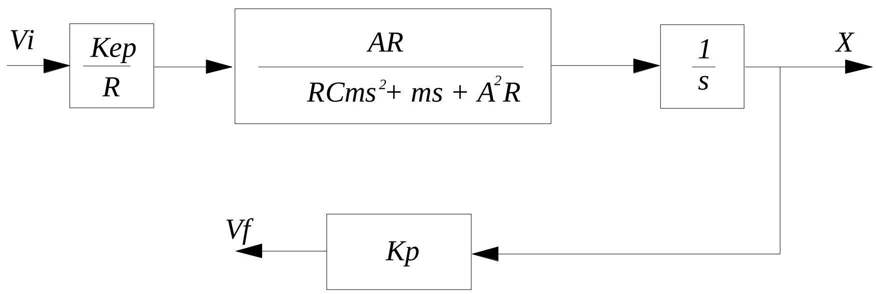


Figure 3-5: Reduction Step 3

The last step in simplification was to combine adjacent blocks resulting in a single block representing the transfer function of the system and a feedback block representing the linear potentiometer.

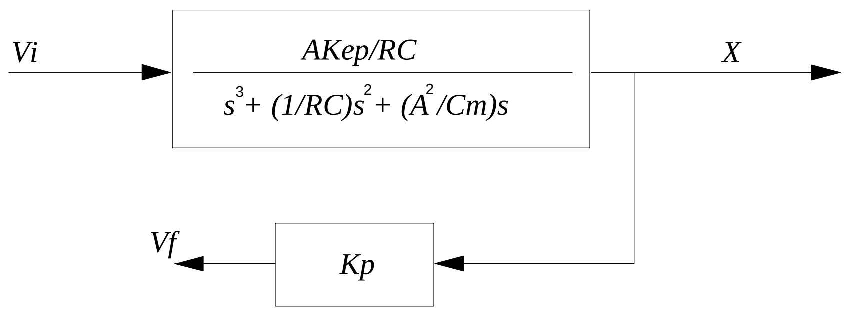


Figure 3-6: Reduction Step 4

Combining the above blocks and putting in transfer function form results in:

$$\frac{Vi}{Vf} = \frac{AKepKp/RCm}{s^3 + \left(\frac{1}{RC}\right)s^2 + \frac{A^2}{Cm}s}$$

This transfer function will be used for root locus analysis of the system to find the maximum gain for instability and the frequency of the resulting oscillation.

Determination of the Transfer Function Coefficients

The physical constants of the system were determined by experimentation, measurement and inspection of the component data sheets. Each component was modeled using standard units when finding the system transfer function. To enhance understanding of the system, units were converted back to standard units for simulation and demonstration. The components modeled were the E/P, air cylinder and load platform, and linear potentiometer.

E/P Gain

For the E/P, the transfer function was taken from response data. The plot below shows the input vs. output with the output port of the E/P sealed.

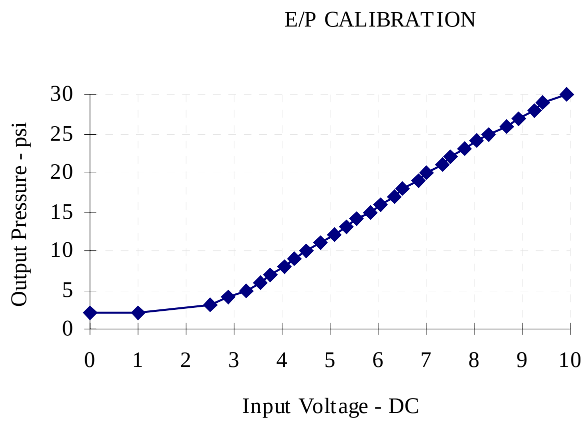


Figure 3-7

The transfer function of the system was modeled for a load of 36.81 lb. The air cylinder must generate an equal and opposite force to position the platform. The air pressure amplified by the surface area of the piston results in the generated force. With a surface area of 1.767 in^2 the operating pressure must be 20.83 psi. At this point on the graph, the voltage required to produce this pressure is approximately 7 volts DC. Therefore the gain at this point is 20.83psi /7 V or approximately 3 psi/volt. This value converted to standard units is

$$Kep = 432 \text{ lb./ft}^{\cdot}\text{/V}$$

E/P Pneumatic Resistance

Using a first order approximation, another characteristic of the E/P was determined, its pneumatic resistance. By observing how long it took to fill a known volume and estimating the capacitance of that volume the resistance of the E/P was found from the time constant. The E/P output resistance was determined indirectly by finding a system's time constant, when using the E/P to pressurize a tank of known volume. An assumption was made that the system was first order. In a first order system, the output will reach 63% of its final value in one time constant when subjected to a step input. Using the data acquisition system, the system's response was recorded. It was used to extract the 63% of final value and the time constant. Using the time constant value, and an estimate of the pneumatic capacitance of the volume of the tank, the E/P output resistance can be calculated.

The E/P input was stepped from 7.0151 volts to 7.7870 volts, raising the pressure of the tank from 20.624 psi. to 23.828 psi. This pressure range was selected because it is within the pressure operating range of the load platform control system during PID tuning. Figure A-1 shows a plot of the tank pressure while the step input was applied. From the data collected (Table 3-1), the start of the step was found at t = 28.5000 seconds and the time at which the output pressure settled to a steady state value was at t = 47.399s. The point where the output equaled 63% of the change from the initial pressure to the final would be

$$p = [(23.828 - 20.624) \times 0.63] + 20.624 = 22.65\, psi$$

inspection of the data table yields the closest 63% point of 22.636 psi at t= 33.2 seconds.

PRESSURIZATION OF 1296 CU IN TANK THROUGH E/P

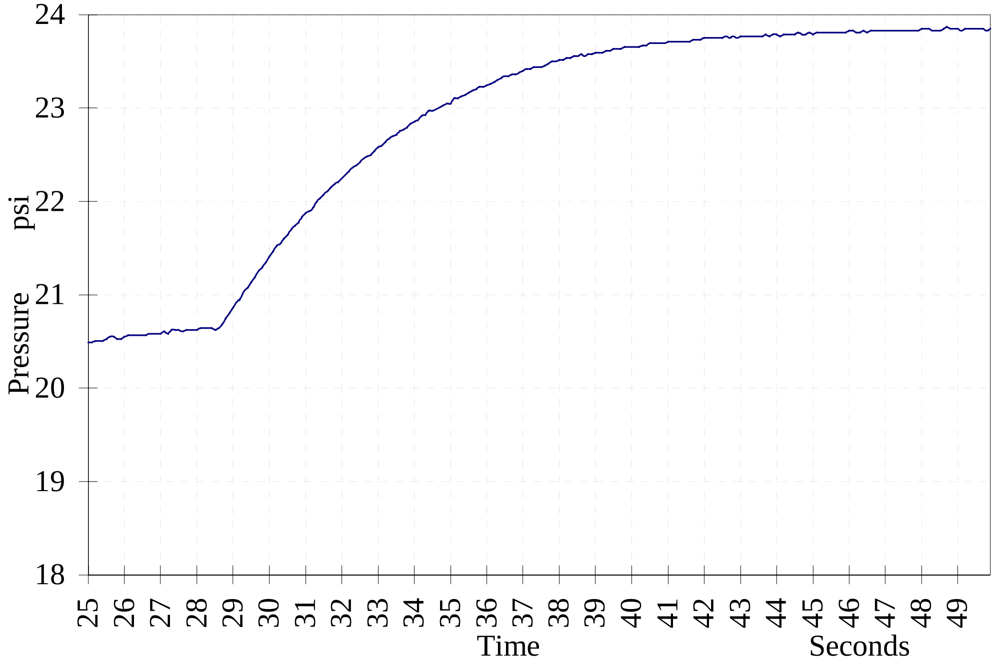


Figure 3-8

The time constant τ can be determined by subtracting the time of the start of the E/P input step from the time where the pressure output is at 63%.

$$\tau = 33.2 - 28.5 = 4.7\,s$$

Pneumatic Capacitance of Test Tank

For a constant-temperature pressure system (isothermal), the pneumatic capacitance can be determined by: [Ref.5 p. 91]

$$C = \frac{\overline{V}}{n\overline{p}}$$

where:

$\overline{V}$ = average volume

n = polytropic constant

$\overline{p}$ = average absolute pressure

The polytropic constant n is assumed to be 1.

The average absolute pressure is

$$\overline{p} = \frac{(20.624 + 14.1)\,psi + (23.828 + 14.1)\,psi}{2} = 36.3\,psia$$

or in standard units

$$\bar{p} = 36.3\frac{lbf}{in^2} \times \frac{in^2}{\left(\frac{1}{12}\,ft\right)^2} = 5.2 \times 10^3\ \frac{lbf}{ft^2}$$

If the volume of the tank is 1296 cu. in., then the pneumatic capacitance is

$$C = \frac{1296\,cu.in}{36.3\,psia} = 35.7\,\frac{cu.in.}{psia}$$

or in standard units

$$C = \frac{750 \times 10^{-3}\ ft^3}{5.2 \times 10^3\ \frac{lbf}{ft^2}} = 144.2 \times 10^{-3}\ \frac{ft^3}{lbf/ft^2}$$

<u>Pneumatic Resistance of the E/P</u>

The time constant of the first order tank system was expressed as $\tau = RC$. Solving this expression for resistance and using the values determined above for the time constant and pneumatic capacitance, the E/P resistance is

$$R = \frac{\tau}{C} = \frac{4.7\,s}{35.7\,\frac{cu.in.}{psia}} = 131.65 \times 10^3\ \frac{s}{\frac{cu.in.}{psia}}$$

or in standard units

$$R = \frac{\tau}{C} = \frac{4.7\,s}{144.2 \times 10^{-3}\ \frac{ft^3}{lbf/ft^2}} = 32.6 \times 10^3\ \frac{s}{\frac{ft^3}{lbf/ft^2}}$$

Note: The actual value for pneumatic resistance used in the model for simulation was slightly less than the value shown due to error during an initial calculation of the resistance. The value actually used was

$$R = \frac{\tau}{C} = 31.44 \times 10^3\ \frac{s}{\frac{ft^3}{lbf/ft^2}}$$

$$\%error = \frac{31.44 - 32.6}{32.6} \times 100\% = -0.036\%$$

The percentage error is less than 1% and will not significantly affect the accuracy of the modeling and simulation.

TABLE 3-1: Data Table for pressurization of 1296 cu in tank through E/P

Time (s)	Pressure Transducer (volts)	Pressure Eng. Units (psi)
25	0.5205	20.488
25.1	0.5205	20.488
.	.	.
.	.	.
.	.	.
28.4	0.5244	20.644
28.5	0.5239	20.624 **(Step Input)**
.	.	.
.	.	.
.	.	.
32.3	0.5674	22.364
32.4	0.5679	22.384
32.5	0.5688	22.42
32.6	0.5698	22.46
32.7	0.5703	22.48
32.8	0.5708	22.5
32.9	0.5718	22.54
33	0.5728	22.58
33.1	0.5732	22.596
33.2	0.5742	22.636 **(pressure at 63% of final value)**
33.3	0.5752	22.676
33.4	0.5757	22.696
.	.	.
.	.	.
.	.	.
45.9999	0.604	23.828
46.0999	0.604	23.828
46.1999	0.6035	23.808
46.2999	0.6035	23.808
46.3999	0.604	23.828
46.4999	0.6035	23.808
46.5999	0.604	23.828
46.6999	0.604	23.828
46.7999	0.604	23.828
46.8999	0.604	23.828
46.9999	0.604	23.828
47.0999	0.604	23.828
47.1999	0.604	23.828
47.2999	0.604	23.828

Time (s)	Pressure Transducer (volts)	Pressure Eng. Units (psi)
47.3999	0.604	23.828 **(Final Pressure)**
47.4999	0.604	23.828
47.5999	0.604	23.828
47.6999	0.604	23.828
47.7999	0.604	23.828

Air Cylinder Pneumatic Capacitance

The capacitance of the air in the air cylinder at cylinder midpoint with a combined load and platform weight of 36.81 lb. was found using the following relationship:

$$C = \frac{\overline{V}}{n\overline{p}}$$

where:

$\overline{V}$ = average volume

n = polytropic constant

$\overline{p}$ = average absolute pressure

This relationship is valid for a small change in volume and pressure, and constant temperature. The polytropic constant is assumed to be 1 in this isothermal process.

For the air cylinder at midstroke: $\overline{V} = 3.07 \text{ x } 10^{-3}\,\text{ft}^3$

polytropic constant: $n = 1$

average absolute pressure: $\overline{p} = \left(14.1 \text{ psi} + \dfrac{36.81 \text{ lbf}}{1.767\text{in}^2}\right) \text{x} \left(\dfrac{\text{in}^2}{\frac{1}{12}\text{ft}^2}\right) = 5.03\text{x}10^3\,\dfrac{\text{lbf}}{\text{ft}^2}$

Then: $C = 610.3 \text{ x } 10^{-9} \text{ ft.}^3 \text{/lb./ft.}^2$

The area of the air cylinder piston for the 1.5 inch air cylinder was

$$A = 12.27 \text{ x } 10^{-3} \text{ ft.}^2$$

The combined weight of the load and load platform, piston rod, and piston was 36.81 pounds. The mass associated with this force was

$$m = 36.81 \text{ lb.} / (32.2 \text{ ft/sec}^2) = 1.143 \text{ slugs}$$

The gain of the 8” linear potentiometer was calculated for a bias of 10 volts as

$$Kp = 15 \text{ V/ft}$$

Substituting the constants into the transfer function of the system yields:

$$\frac{Vi}{Vf} = \frac{AKepKp/RCm}{s^3 + \left(\frac{1}{RC}\right)s^2 + \frac{A^2}{Cm}s}$$

$$= \frac{3625}{s^3 + 52.12s^2 + 215.82s}$$

or $$\frac{3625}{s(s + 4.54)(s + 47.58)}$$

Root-Locus of Transfer Function

The open loop transfer function was entered into MATLAB to plot the root-locus. The root-locus plot is a graphical representation of how system gain affects the location of the transfer function roots in a complex plane. For the system to remain stable, the root-loci must remain in the left half plane. The gain *K* that causes the open loop root-locus to cross the imaginary axis will also be the closed loop system gain *Pu* that causes continuous oscillation. The frequency of oscillation at the gain crossover point ω will be used to determine the ultimate period *Tu*. Figure 3-9 shows the root locus plot of the open loop transfer function.

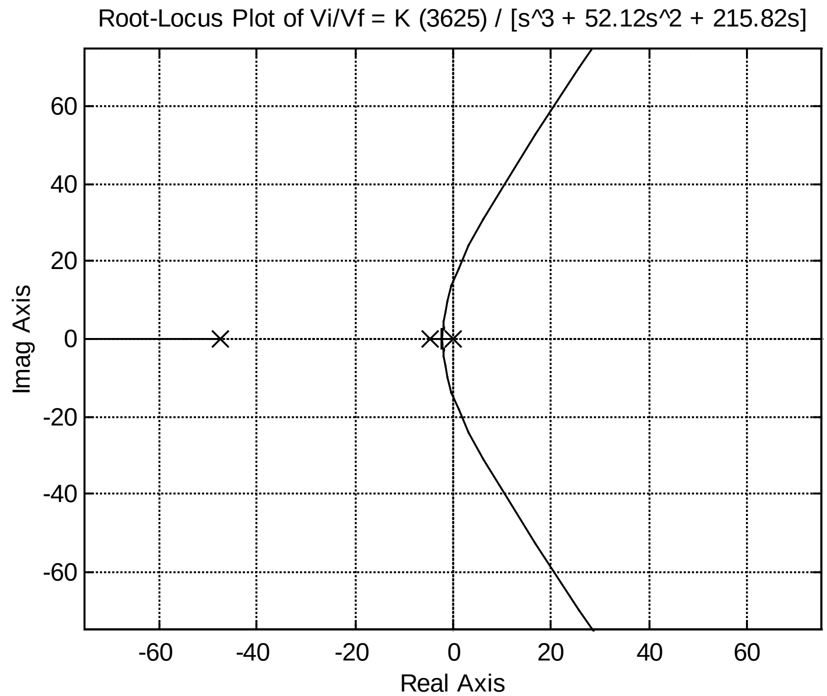


Figure 3-9

From plot above and the pole-zero form of the open loop transfer function *Vi /Vf* = $\frac{3625}{s(s + 4.54)(s + 47.58)}$ the poles originating at s = 0, s = -4.54 and s = -47.58 are identified. There are no zeros. As *K* is varied from 0 to infinity: the pole at s = -47.58 moves towards negative infinity on the real axis and will not cause system instability. To improve observation of the roots around the imaginary axis, the plot was expanded in Figure 3-10.

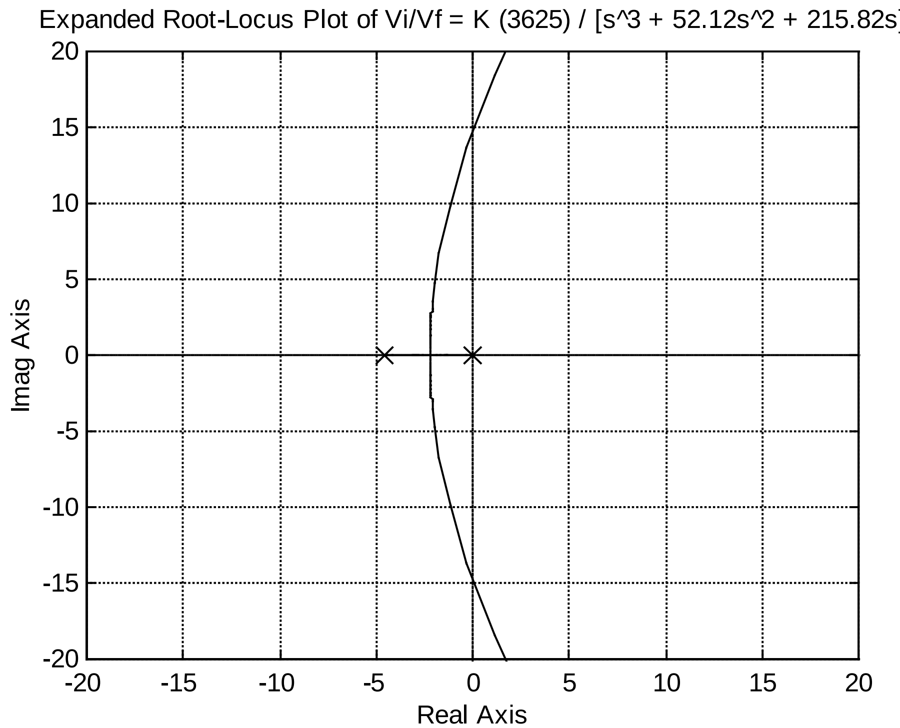


Figure 3-10

The poles starting at s = 0 and s = -4.54 meet on the real axis near s = -2.4 where the pole originating at s = -4.54 moves along the negative imaginary axis and the pole originating at s = 0 moves along the positive imaginary axis. To determine the values of K and ω at the gain crossover point, the root-locus was calculated in MATLAB for a range of K near the previously known crossover point of 1.95 of the actual system. Through trial runs, the crossover point for the theoretical system was found at K = 3.1000 with an ω of 14.6841 rad/s. The data tables as copied from MATLAB are shown following.

Table 3-2: Data Listing From MATLAB for Gain *K* of Root-Locus Near Gain Crossover Point

```
» K = [2:0.1:3.5]

K=

  Columns 1 through 7

    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000

  Columns 8 through 14

    2.7000    2.8000    2.9000    3.0000    3.1000    3.2000    3.3000

  Columns 15 through 16

    3.4000    3.5000
```

Table 3-2: Data Listing From MATLAB for Root-Locus Near Gain Crossover Point

```
» r = rlocus(num,den,K)

r =
1.   -50.6841        -0.7179 +11.9385i      -0.7179 -11.9385i
2.   -50.8208        -0.6496 +12.2217i      -0.6496 -12.2217i
3.   -50.9560        -0.5820 +12.4968i      -0.5820 -12.4968i
4.   -51.0899        -0.5150 +12.7643i      -0.5150 -12.7643i
5.   -51.2225        -0.4488 +13.0248i      -0.4488 -13.0248i
6.   -51.3538        -0.3831 +13.2787i      -0.3831 -13.2787i
7.   -51.4838        -0.3181 +13.5265i      -0.3181 -13.5265i
8.   -51.6126        -0.2537 +13.7684i      -0.2537 -13.7684i
9.   -51.7403        -0.1899 +14.0049i      -0.1899 -14.0049i
10.  -51.8667        -0.1266 +14.2361i      -0.1266 -14.2361i
11.  -51.9920        -0.0640 +14.4625i      -0.0640 -14.4625i
12.  -52.1162        -0.0019 +14.6841i      -0.0019 -14.6841i
13.  -52.2394         0.0597 +14.9014i       0.0597 -14.9014i
14.  -52.3614         0.1207 +15.1144i       0.1207 -15.1144i
15.  -52.4824         0.1812 +15.3234i       0.1812 -15.3234i
16.  -52.6024         0.2412 +15.5286i       0.2412 -15.5286i
```

The root-locus that crosses the imaginary axis was taken as line 12 where s = -0.0019 ± 14.6841i The frequency is in radians/second, converting to cycles per second yields

$$f = \frac{\omega}{2\pi} = \frac{14.681 \text{ rad / sec}}{2\pi} = 2.337 \text{ Hz}$$

and

$$T = \frac{1}{2.337 \text{ Hz}} = 0.428 \text{ seconds}$$

To see what K was at this point, inspection of the data table for K at column 12 shows a gain value of 3.1000.

Load Platform Position Control System Model

The system model was entered into MATLAB's toolbox SIMULINK to verify the K = 3.1 value for constant oscillation and the frequency of oscillation ω = 14.681 rad/s found from root-locus analysis. Figure 3-11 is the Simulink block diagram representation of the load platform position control system shown in Figure 3-2. The closed loop testing procedure for the simulated system used the same test parameters as those in the actual system. The operating setpoint was 7 volts, platform load was 36.81 lb. and the simulation time was six seconds.

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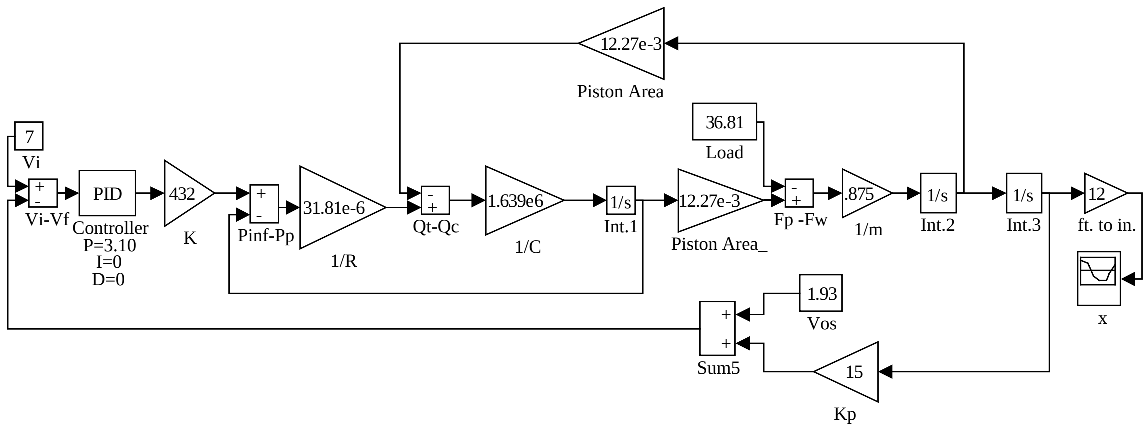


Figure 3-11: Closed Loop SIMULINK System Model

First, to show that the crossover gain determined in the root locus is actually a value that has magnitude that is just enough to place the system at the edge of instability, a K value slightly less than the crossover gain will be used to show that the system has a damped oscillation that will not oscillate continuously. Figure 3-12 shows the system output plotted by the graph block in Simulink.

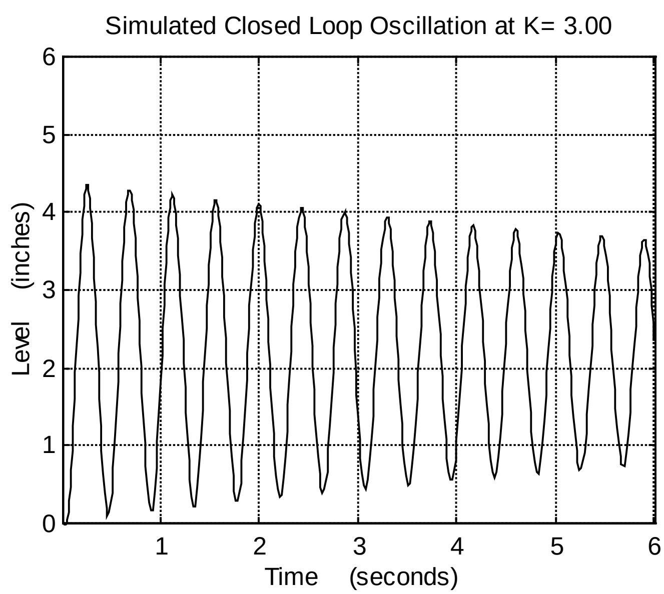


Figure 3-12

Next, the system simulation was run with a K value at the ultimate gain magnitude K= 3.100. The system oscillated continuously as shown in Figure 3-13.

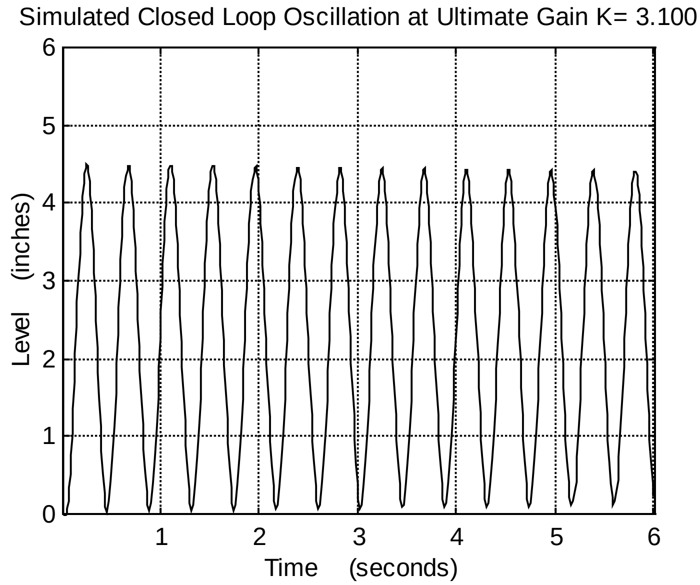


Figure 3-13

To find the ultimate period Tu, the total number of cycles of Figure 3-13 was divided into the six second simulation time.

The total number of cycles shown in the graph is 14. The period for one cycle was then calculated as

$$Tu = \frac{6 \text{ seconds}}{14 \text{ cycles}} = 0.429 \text{ seconds}$$

and

$$F = \frac{1}{Tu} = 2.333 \text{ Hz}$$

Table 3-4: Determination of *Pu* and *Tu* by Root-Locus and Closed Loop Simulation

	Ultimate Gain *Pu*	Ultimate Period *Tu* (seconds)	Frequency *F* (Hz)
Root-Locus Method	3.100	0.428	2.337
Closed Loop Simulation	3.100	0.429	2.333

The table shows that the results from the root-locus and closed loop simulation are practically identical. The values *Pu* and *Tu* from the closed loop simulation will be used to calculate the

PID controller constants using the modified form of the Ziegler-Nichols Tuning rules used in the LABTECH NOTEBOOK Software.

Table 3-5: Ziegler-Nichols Tuning Rule Based on P_U and T_U - LABTECH NOTEBOOK Form			
Type of Controller	Proportional Gain *P*	Integral Gain *I*	Derivative Gain *D*
P	$0.5P_U$	0	0
PI	$0.45P_U$	$0.54\left(\frac{P_U}{T_U}\right)$	0
PID	$0.6P_U$	$1.2\left(\frac{P_U}{T_U}\right)$	$0.075(P_U T_U)$

With the values of T_U = 0.429 and P_U = 3.100 obtained, the controller constants were calculated in the LABTECH format of the Ziegler - Nichols Tuning Rules for quarter cycle decay response:

$$P = 0.6\, P_U \qquad I = 1.2(P_U\ /T_U) \qquad D = 0.075(P_U)(T_U)$$

The controller constants were:

$$P = (0.6)(3.100) = 1.860 \qquad I = 1.2(3.100/0.428\ ) = 8.692$$

$$D = .075(3.100)(0.428) = .0995$$

<u>Summary</u>

The Load Platform Position Control System was modeled using Laplace Transforms and block diagrams. The transfer function of the system was derived by block diagram reduction and then used in root-locus analysis to find the of transfer function gain *K* which caused the root-locus to cross the imaginary axis. This crossover point, *K* is equal to the ultimate gain *Pu* for continuous oscillation. The period of this oscillation was found using the value of ω at the gain crossover point. The values of *Pu* and *Tu* were then used in the Ziegler-Nichols tuning rules to obtain the PID controller constants.

A closed loop simulation using SIMULINK verified that the value of *Pu* obtained in the root-locus method was of magnitude such that it just caused continuous oscillation of the closed loop system with the period *Tu.*

IV. Conclusion

The paper presents a feedback control system for use in an Engineering Technology Laboratory Course. The feedback control system was to control the position of a load platform. The design task included finding the appropriate PID controller constants for a software based PID controller. Two approaches were used to find the PID controller constants.

1. The system was tested with a load of 30 lb. for the closed loop gain *Pu* that caused continuous oscillation and the period *Tu* of that oscillation. *Pu* and *Tu* were then used in the Ziegler-Nichols tuning rules to find the PID constants.
2. A model was generated for the system and the resulting transfer function analyzed using root-locus analysis. *Pu* and *Tu* were obtained from the root-locus plot and then verified by computer simulation of the closed-loop system. *Pu* and *Tu* were then again used in the Ziegler-Nichols Tuning Rules to find the PID constants.

The gain for oscillation *Pu* in the actual system was 60% lower than in the theoretical system and the period for oscillation *Tu* in the actual system was 30% higher than the theoretical system. These differences in *Pu* and *Tu* reduced the magnitude of the PID constants in the actual system. When the theoretical values were used in the actual system violent oscillation of the load platform resulted. These results suggest that a refinement of the system model is necessary. The parameters of the actual and theoretical system are compared in Table 4-1.

	Pu	*Tu*	P	I	D
Actual System	1.95	0.600	1.170	3.900	0.088
Theoretical System	3.10	0.428	1.860	8.692	0.0995

Table 4-1: Comparison of Actual and Theoretical System Parameters

To compare the performance of the actual and theoretical system, the closed loop response to 5 lb. and 10 lb. load disturbances was plotted in Figure 4-2. The data for the actual system was obtained by storing data from a channel that scaled the output voltage of the linear potentiometer to inches. To duplicate the test in SIMULINK, the system modeled in Chapter 3 was modified to include the varying mass of the platform. Step inputs were programmed according to the loading profile intervals observed in the time column of the data acquisition file taken while operating the actual system. The modified SIMULINK system model is shown in Figure 4-1.

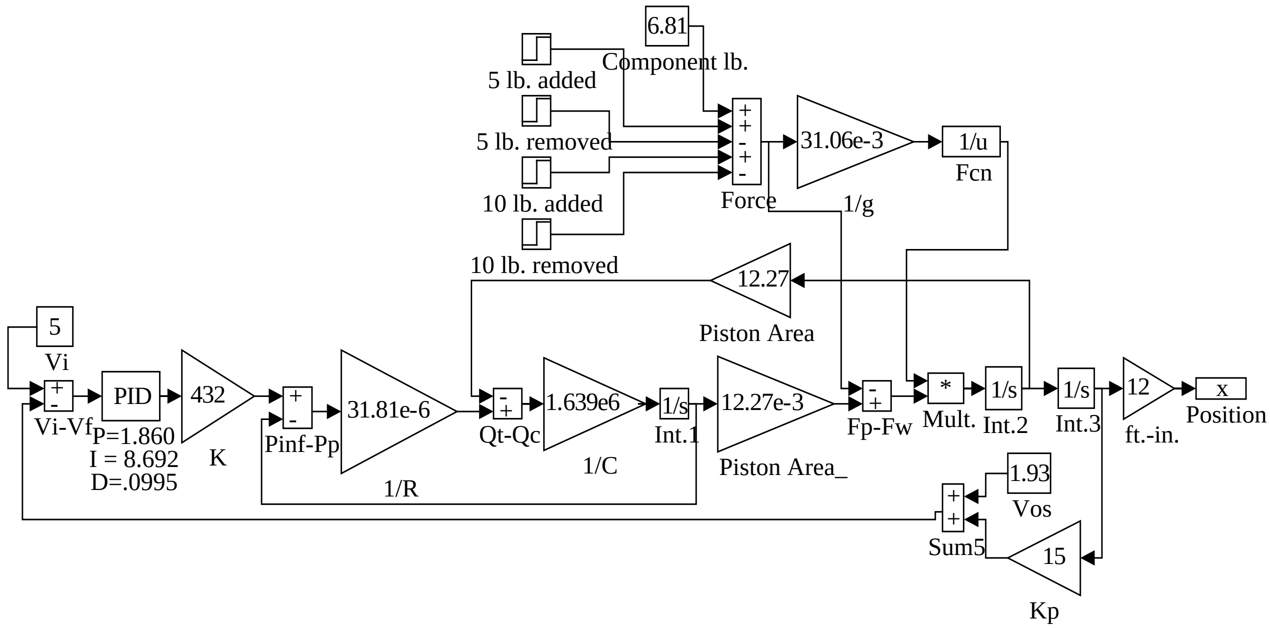
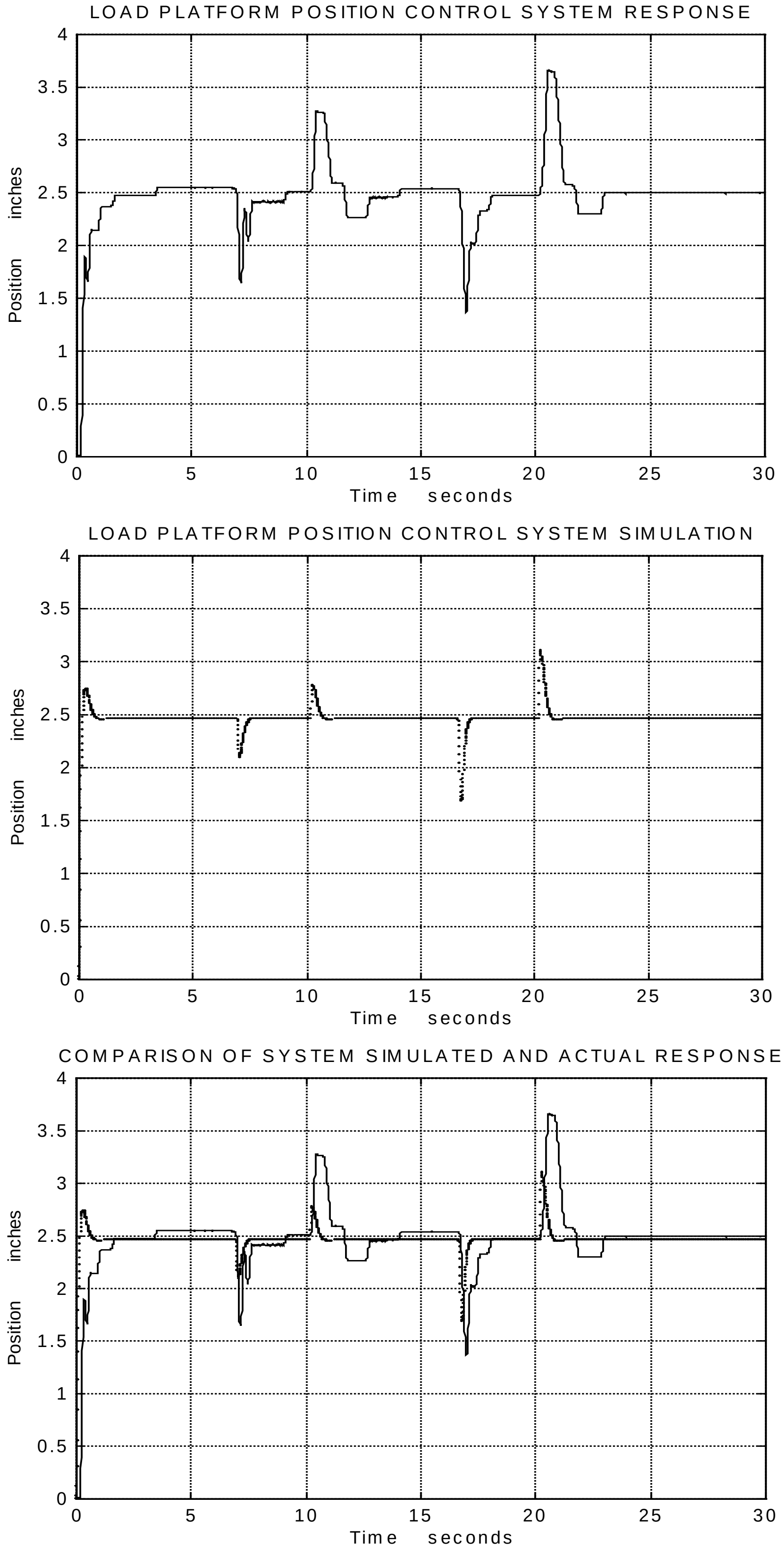


Figure 4-1: SIMULINK Representation of the Multiple-Loads Profile



a)

b)

c)

Figure 4-2: System responses: a) actual system, b) simulation, c) actual and simulation superimposed

Simulation data was generated by storing the output of the system of Figure 4-1 into a MATLAB workspace matrix. The data from the actual system was imported into MATLAB, and then stored in time and position matrices. The simulation and actual system response is shown in Figure 4.2. The lower overshoot in the plot of the simulation indicates that this system has higher damping than the actual system. This agrees with the fact that the root-locus of the simulated system showed a higher allowable gain K before the system went into sustained oscillation.

The difference in the plots of the actual system and model is caused by stiction and the estimation of the capacitance of the air in the air cylinder. The stiction of the air cylinder was ignored in the model but is quite evident by the "jaggedness" seen in the plot of the actual system. The more highly damped response of the model is due to the fact that the capacitance of the air in the air cylinder was theoretically higher than it should have been because it was modeled for a load of 36.81 lb. This gave the simulated system an unrealistically high time constant for the loads of 5 lb. and 10 lb.

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