A FINITE ELEMENT APPROACH FOR CALCULATING SOUND PRESSURE LEVEL IN A COUPLED STRUCTURAL-ACOUSTIC SYSTEM

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Abstract

In this study, the sound-structure interaction problem in a coupled structural-acoustic system has been investigated. The novelty of this investigation is embedded in using of Finite Element Method to calculate sound pressure level. The governing differential equation for the interaction of the acoustic cavity with the flexible wall (plate) is calculated. The modal analysis of such a problem is possible, an analysis in which the decoupled equations of motion for the cavity and the flexible wall can be obtained separately. In order to accomplish this task, the coupled structural-acoustic system has been decomposed to an acoustic component, a cavity with a rigid wall, and a structural component, a flexible wall (plate). The cavity modes (eigenvalues and eigenvectors) of the cavity with the rigid wall boundary condition and structural modes of the simply supported flexible plate in a vacuum are obtained by using the finite element method. The coupled structural-acoustic equation for the pressure inside the cavity is obtained in terms of the eigenmodes of the cavity and the flexible wall. The coupling method has been successfully implemented into two classical existing problems.

1. Introduction

Interaction between the internal sound pressure field and the flexible wall of an enclosure has been a very popular subject for scientists and researchers. The results of such interaction will induce an acoustic pressure and noise field that poses a major engineering design challenge. Therefore, a good understanding of the mechanisms of noise transmission through an elastic plate allows us to use more effective techniques to control noise.

Consider a fluid in a cavity which has a surface such that part of the surface is flexible (Fig. 1). The flexible wall is excited by external pressure in which interaction between the flexible wall and the fluid in the cavity has an impact on the acoustic pressure within the cavity. Due to vibration of the flexible wall, an acoustic pressure will be generated in the coupled structural-acoustic system.

Various approaches have been developed by different researchers to calculate the interior pressure in a coupled structural-acoustic system[1-7]. In this study, a modal coupling method is adapted to determine the acoustic pressure. In this method, the coupled structural-acoustic system has been decomposed to the acoustic component, a cavity with a rigid wall, and a structural component, a flexible wall (plate) with simply supported boundary condition. The acoustical modes are obtained by mathematically combining the cavity modes and the structural modes along with coupling coefficients to form a non-symmetric eigenvalue problem, which yields coupled acoustical modes. Finally, the interior pressure is obtained in terms of these coupled acoustical modes.

2. Wave Equation in Acoustic Component

The equation of the velocity potential and the associated boundary conditions can be given as

$$\nabla^2\varnothing - \left(\frac{1}{C_0^2}\right)\frac{\partial^2\varnothing}{\partial t^2} = 0 \quad \text{in } V \tag{1}$$

and

$$\frac{\partial\varnothing}{\partial\hat{\boldsymbol{n}}} = 0 \quad \text{on} \quad A_R \qquad\qquad \frac{\partial\varnothing}{\partial\hat{\boldsymbol{n}}} = \frac{\partial w}{\partial t} \quad \text{on} \quad A_F$$

Equation (1), with the rigid wall boundary condition, has the normal mode solutions, $F_n e^{i\Lambda_n t}$, n = 0, 1, 2,… which in turn satisfy

$$\nabla^2 F_n + \frac{\Lambda_n^2}{C_0^2}F_n = 0 \quad (2) \qquad \text{and} \qquad \frac{\partial F_n}{\partial\hat{\boldsymbol{n}}} = 0 \quad \text{on} \quad A = A_F + A_R \quad (3)$$

The solution of the differential equation (Eq. (2)) subjected to the boundary condition (Eq. (3)) can be replaced by an equivalent variational principle as

$$\delta\int_V \frac{1}{2}\left[(\nabla F_n)^2 - \left(\frac{\Lambda_n^2}{C_0^2}\right)F_n^2\right]dV = 0 \tag{4}$$

The finite element method can be used to obtain an approximate solution of this variational principle. Then, the finite element method yields the matrix equation as[8]

$$\left(K_a - \Lambda_n^2 M_a\right)\phi_{\boldsymbol{n}} = 0 \tag{5}$$

where the acoustic stiffness and mass matrices are

$$K_a = \int_V B_a^T B_a dV \qquad M_a = \int_V \frac{1}{C_0^2}N_a^T N_a dV \qquad \text{and} \quad B_a = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} N_a$$

3. Vibration of the Structural Component

The finite element matrix equation of the structural component (plate) in free vibration can be represented as follows:

$$\left(K_s - \lambda_{\tilde{m}} M_s\right)\psi_{\tilde{m}} = 0 \tag{6}$$

where the stiffness matrix K_s and the mass matrix M_s are defined as

$$K_s = \int_{A_F} B_S^T D B_s dA_F \qquad \text{and} \qquad M_s = \int_{A_F} \rho h N_S^T N_S dA_F$$

For the sake of convenience, ψ is usually normalized with respect to the mass matrix so that

$$\psi^T M_s \psi = I$$

4. Coupling Coefficient

The coupling coefficient between the n^{th} cavity rigid wall mode and the $\tilde{m}^{th}$ plate mode is given by[1]

$$L_{n\tilde{m}} = \frac{1}{A_F} \int_{A_F} \phi_n \psi_{\tilde{m}} dA_F \tag{7}$$

In general, if the value of the coupling coefficient in coupled structural-acoustic interaction is very small or equal to zero, the coupling is described as weak coupling. In this case, the problem would be greatly simplified by neglecting these non-significant modes.

$$L_{nm} = \frac{1}{A_F} \int_{A_F} \phi_n \psi_{\tilde{m}} dA_F \neq 0 \qquad m = 1,2,3,\ldots\text{M} \tag{8}$$

Equation (8) is used to identify the participation of the component modes in coupling as well as participated eigenvalues of the structure λ_m. Therefore, non-participated eigenvalues are those not to be satisfied by Eq. (8). L_{nm} is the coupling coefficient between the n^{th} cavity mode and the m^{th} participated in vacuo flexible plate mode in coupling.

5. Acoustic Pressure

The equation of the coupled structural-acoustic interaction problem in matrix form can be written in terms of amplitudes C related to N cavity modes and amplitudes S related to M structure modes as[1,9]

$$I\dot{\boldsymbol{x}}(t) + G\boldsymbol{x}(t) = \boldsymbol{r}(t) \tag{9}$$

where the coefficient matrices I and G are defined as

$$I=\begin{bmatrix} \left[\diagdown \dfrac{V}{A_F C_0^2} \diagdown\right]_{nxn} & 0 & 0 & 0 \\ 0 & \left[\diagdown \dfrac{1}{\rho_0 A_F} \diagdown\right]_{mxm} & 0 & 0 \\ 0 & 0 & \left[\diagdown \dfrac{V\Lambda_n^2}{A_F C_0^2} \diagdown\right]_{nxn} & 0 \\ 0 & 0 & 0 & \left[\diagdown \dfrac{\lambda_m}{\rho_o A_F} + \dfrac{A_F C_0^2 L_{om}^2}{V} \diagdown\right]_{mxm} \end{bmatrix}$$

$$G=\begin{bmatrix} 0 & -[L_{nm}]_{nxm} & \left[\diagdown \dfrac{V\Lambda_n^2}{A_F C_0^2} \diagdown\right]_{nxn} & 0 \\ [L_{nm}]^T & 0 & 0 & \left[\diagdown \dfrac{\lambda_m}{\rho_o A_F} + \dfrac{A_F C_o^2 L_{om}^2}{V} \diagdown\right]_{mxm} \\ -\left[\diagdown \dfrac{V\Lambda_n^2}{A_F C_0^2} \diagdown\right]_{nxn} & 0 & 0 & 0 \\ 0 & -\left[\diagdown \dfrac{\lambda_m}{\rho_o A_F} + \dfrac{A_F C_0^2 L_{om}^2}{V} \diagdown\right]_{mxm} & 0 & 0 \end{bmatrix}$$

and the vector, r is defined as $$\boldsymbol{r}=\begin{Bmatrix} 0 \\ \dfrac{1}{\rho_o A_F} Q_m^E \\ 0 \\ 0 \end{Bmatrix}$$ n=1,2,…N

The state vector and its corresponding components is thoroughly defined in reference[9]. Furthermore, Q_m^E is given as

$$Q_m^E = -\psi_m^T \int_{A_F} N_S^T \boldsymbol{p}^E dA_F$$

where I is a positive definite, real diagonal matrix of order 2(N+M)×2(N+M), G is a skew symmetric real matrix of order 2(N+M)×2(N+M), and $\boldsymbol{x}$ and $\boldsymbol{r}$ are 2(N+M)×1 vectors.

In Eq. (9), $\boldsymbol{x}$ vector can be calculated by the expansion theory and Laplace transform. Solution of Eq. (9) yields the nodal acoustic pressure vector within the coupled structural-acoustic system.

$$\boldsymbol{P}^{c} = -\rho_{o}\left(\frac{A_{F}C_{0}^{2}}{V}\sum_{m=1}^{\mathrm{M}} x_{2\mathrm{N}+\mathrm{M}+m}L_{om}\phi_{o} + \sum_{n=1}^{\mathrm{N}} x_{n}\phi_{n}\right) \tag{10}$$

Where $x_{2\mathrm{N}+\mathrm{M}+m}$ and x_n indicate the $(2\mathrm{N}+\mathrm{M}+m)^{\mathrm{th}}$ and $(n)^{\mathrm{th}}$ element of vector $\boldsymbol{x}$, respectively.

6. Numerical Examples

The coupling method developed has been applied to two model examples. The specific details of the cavity and the flexible plate are given below. Guy[10] and Bokil[9] found the experimental and analytical results in works, respectively.

Model I

Model I consisted of a rectangular cube cavity with the following dimensions: length along the x-axis, width along the y axis and the depth along the z axis are 30.48 cm, 15.24 cm, and 15.24 cm, respectively. Also, there is a cavity made of 2.54-cm thick plywood and a 5.08-cm thick layer of concrete surrounding the outside walls. A simply supported aluminum panel was mounted on one of the faces. The dimensions and the material properties of the plate are given as follows: length along the x axis, a is equal to 30.48 cm, width along the y axis, b is equal to 15.24 cm, thickness, h is equal to 1.6256 mm, density, ρ is equal to 2,400 Kg/m^3, modulus of elasticity, E is equal to 7.0×10^{10} N/m^2, and Poisson's ratio, ν is equal to 0.33.

Model II

Mode II consisted of a square cube cavity with the following dimensions: length along the x-axis, width along the y-axis, and depth along the z-axis are 20 cm, respectively. The cavity is made of a 1 cm thick steel box with a simply supported brass panel on one wall. The dimensions and material properties of the plate are given as follows: length along the x axis, a is equal to 20 cm, width along the y axis, b is equal to 20 cm, thickness, h is equal to 0.9144 mm, density, ρ is equal to 8,500 Kg/m^3, modulus of elasticity, E is equal to 10.4×10^{10}N/m^2, Poisson's ratio, and ν is equal to 0.37. The back wall of the box, opposite the brass panel, was a 2.54-cm thick steel piston. As a general form, these two models can be represented by a rectangular cavity as shown in Fig. 2.

In the calculation, the density of the air inside the cavity is assumed to be 1.2 kg/m^3, the nodal external excitation pressure at the center of the back wall is assumed to be 10 N/m^2, and the equilibrium acoustic velocity within the cavity is assumed to be 343 m/sec. The values of the N cavity modes and the M plate modes are assumed to be four and nine, respectively. These values are sufficient to yield results within the required accuracy range. The modal vectors (eigenvectors) and the natural frequency (eigenvalues) of the flexible wall and acoustic wave are calculated by finite element codes. In order to discretize finite element models, linear hexahedral elements for the rigid walled cavity and Kirchhoff rectangular elements for the simply supported plate are being used. In this code, the volume of the cavity is divided to 256 elements, 405 nodes and 405 degrees of freedom and the area of the plate is divided to 64 elements, 81 nodes and 175 degrees of freedom.

The coupling coefficient between the n^{th} cavity rigid wall mode and the m^{th} in-vacuo plate mode is evaluated numerically by Eq. (8). In this study, since the values of N and M are assumed to be four and nine, then only the first nine participated eigenvalues of the plate would be selected in coupling. The participated eigenvalues of the structure are listed in Tables 1 and 2 for Model I and Model II, respectively. Also, the coupled natural frequencies of a coupled structural-acoustic system have been listed in Table 3 for Model I and Model II, to compare with the experimental ones. It can be noted that the numerically calculated coupled natural frequencies are quite close to the experimental ones.

A computer program has been written to numerically calculate the transmission loss and the sound pressure level. The flowchart of the computer program is shown in Fig. 3. The nodal acoustic pressure at the center of the back wall was calculated by Eq. (10). The transmission loss expressed in decibel was, in turn, calculated as the ratio of the rms nodal external excitation pressure to the rms nodal acoustic pressure at the center of the back wall. The numerical data obtained by the finite element code and the experimental data of the transmission loss are plotted as a function of frequency of the external excitation pressure for Model I and Model II in Figs. 4 and 5. The comparison of the numerically calculated transmission loss with the experimental values shows generally good agreement.

7. Conclusion

The development of technology and the increasing application of computers in industries require our education system to provide students with superior knowledge in computational methods. Today, individuals in the field utilize computational methods such as finite element analysis to analyze complex problems. In this study, finite element models have been used to predict eigenvalues and eigenvectors of a rigid walled cavity and simply supported plate. The proposed method can be applied to any complex structure acoustic interaction problem. A problem of this kind can be assigned as a project in an applied acoustics course. The problem can be decomposed to a structure and a cavity in which a student-developed finite element code or any commercially available codes, such as NASTRAN[11] and COMET[12], can be used to determine eigenmodes of the structure and cavity. The acoustic pressure can be, in turn, calculated in terms of eigenmodes of cavity and structure.

Nomenclature

A_F	flexible wall (plate) area
A_R	rigid wall area
B_s	strain displacement matrix of the structural component (plate)
C_0	acoustic velocity within the cavity
D	material matrix of the structural component (plate)
F_n	eigenfunction of the acoustic component (cavity)
h	thickness of the plate
K_s	stiffness matrix of the structural component (plate)
K_a	stiffness matrix of the acoustic component
L_{nm}	coupling coefficient
M_s	mass matrix of the structural component (plate)
M_a	mass matrix of the acoustic component
N_a	interpolation matrix of the acoustic component
N_s	interpolation matrix of the structural component
nt	Newton
$\boldsymbol{p}^c$	nodal acoustic pressure vector within the cavity
$\boldsymbol{p}^E$	nodal external excitation pressure vector
t	time
V	volume of the acoustic component (cavity)
w	elastic displacement of the flexible wall (plate)
$\boldsymbol{x}$	extended generalized coordinates vector

Greek Symbols

Λ_n	eigenvalue of the acoustic component (cavity)
$\lambda_{\tilde{m}}$	eigenvalue of the plate
ρ_o	acoustic (air) density
ρ	density of the structural component (plate)
ϕ_n	eigenvector of the acoustic component (cavity)
$\psi_{\tilde{m}}$	eigenvector of the structural component (plate)
$\varnothing$	velocity potential within the cavity

Table 1 Participated Eigenvalues of Plate and Cavity in Coupling (Model I)

Eigenvalues of rectangular plate		Eigenvalues of cavity	
Eigenmode number	Eigenvalue (rad/sec)	Eigenmode number	Eigenvalue (rad/sec)
1	1393.76	1	7041.3
3	3639.25	2	7041.3
8	8237.18	3	9891.2
10	10428.0	4	13985.2
14	11923.5		
17	15360.4		
18	15620.3		
25	20966.5		
27	22078.6		

Table 2 Participated Eigenvalues of Plate and Cavity in Coupling (Model II)

Eigenvalues of square plate		Eigenvalues of cavity	
Eigenmode number	Eigenvalue (rad/sec)	Eigenmode number	Eigenvalue (rad/sec)
1	481.35	1	5345.9
5	2417.10	2	10693.2
6	2417.10	3	10693.2
11	4136.44	4	10693.2
16	6347.57		
17	6347.57		
21	7723.24		
22	7723.24		
31	10826.2		

Table 3 Natural Frequencies of Coupled Structural-Acoustic Systems

Coupled natural frequency of Model I (HZ)		Coupled natural frequency of Model II (HZ)	
Numerical value	Experimental value	Numerical value	Experimental value
229.0	234.4	85	91
577.8	588.5	384	397
1126.6	1126.6	658	730
1311.1	1316.8	859	864
1593.3	1594.1	1010	1034
1660.5	1680.5	1715	1729

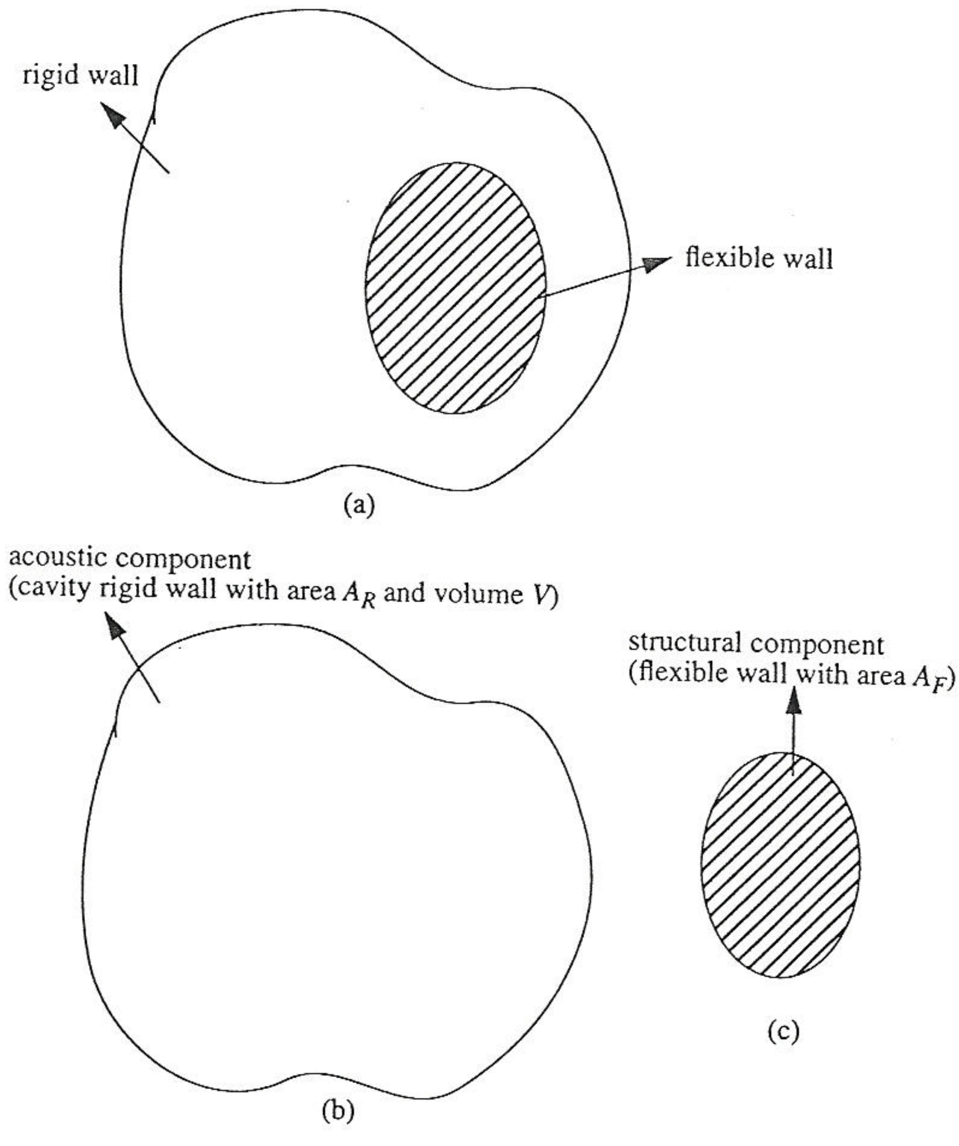

Figure 1 (a) Structural-Acoustic System (b) Acoustic Component: Cavity with Rigid Wall

(c) Structural Component: Flexible Wall

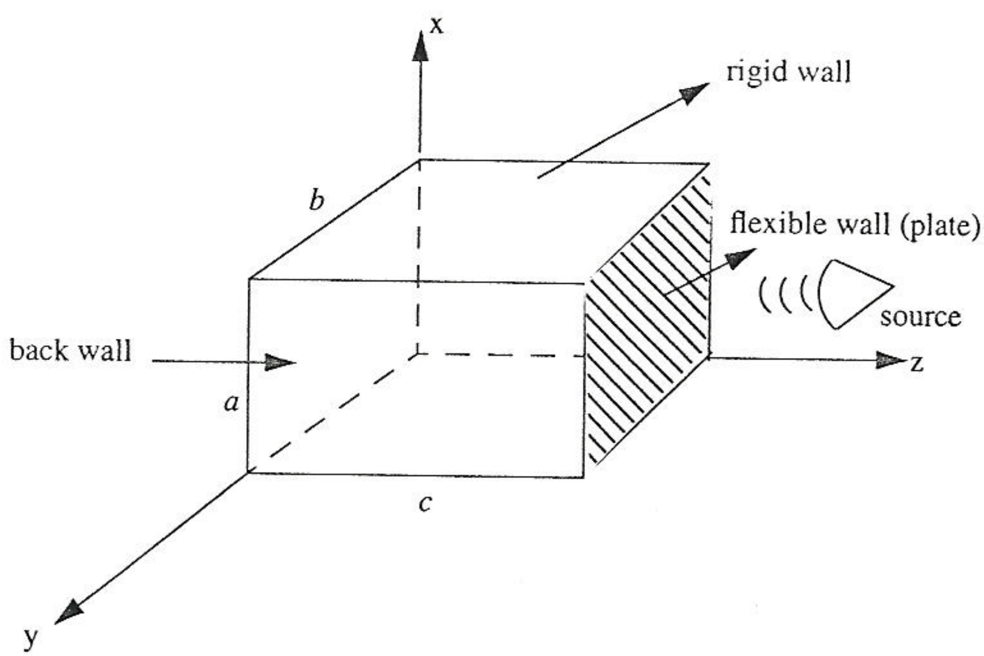

Figure 2 Rectangular Cavity with a Flexible Plate on One Face

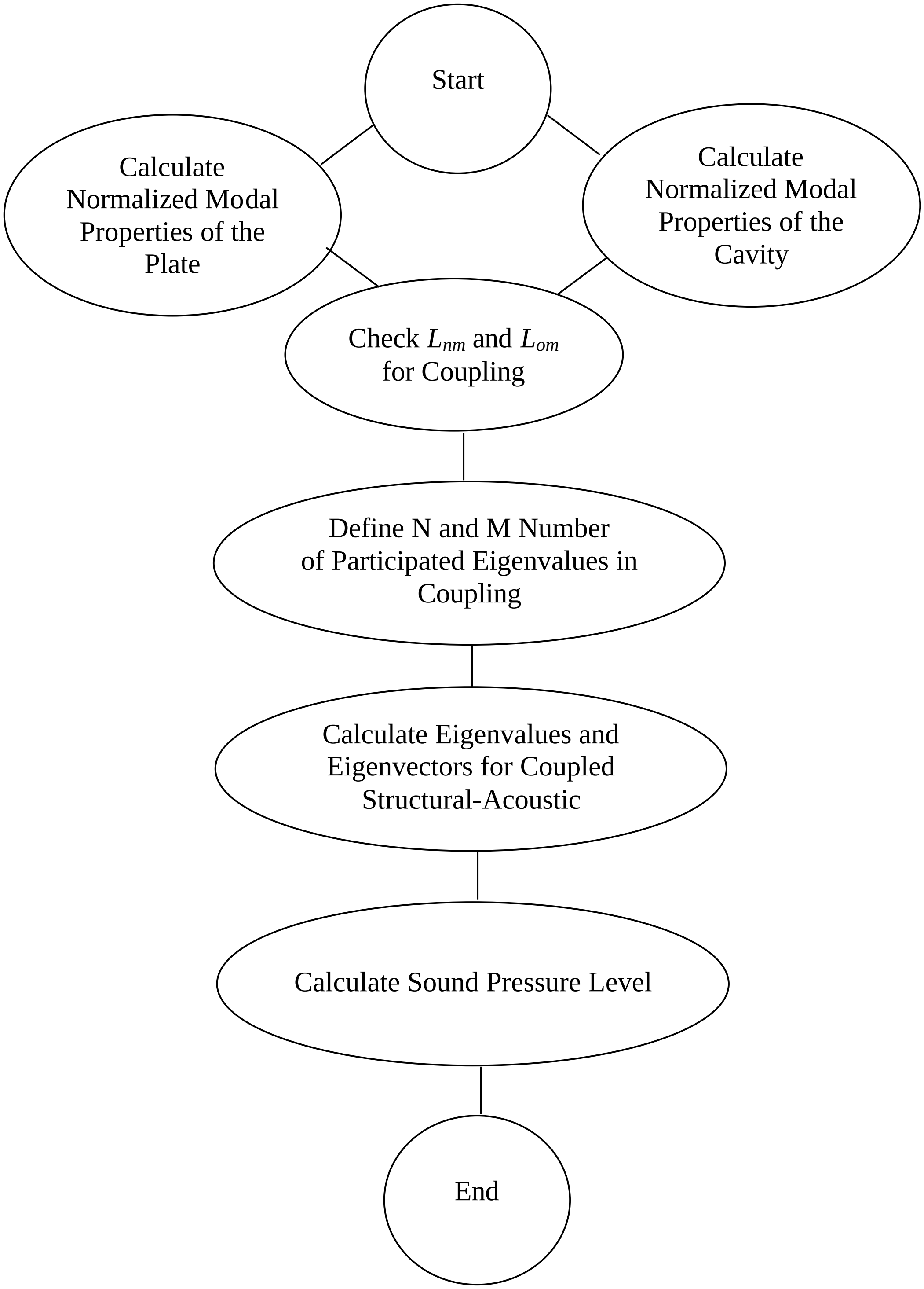

Figure 3. Flowchart of the Computer Program to Analyze the

Coupled Structural-Acoustic System

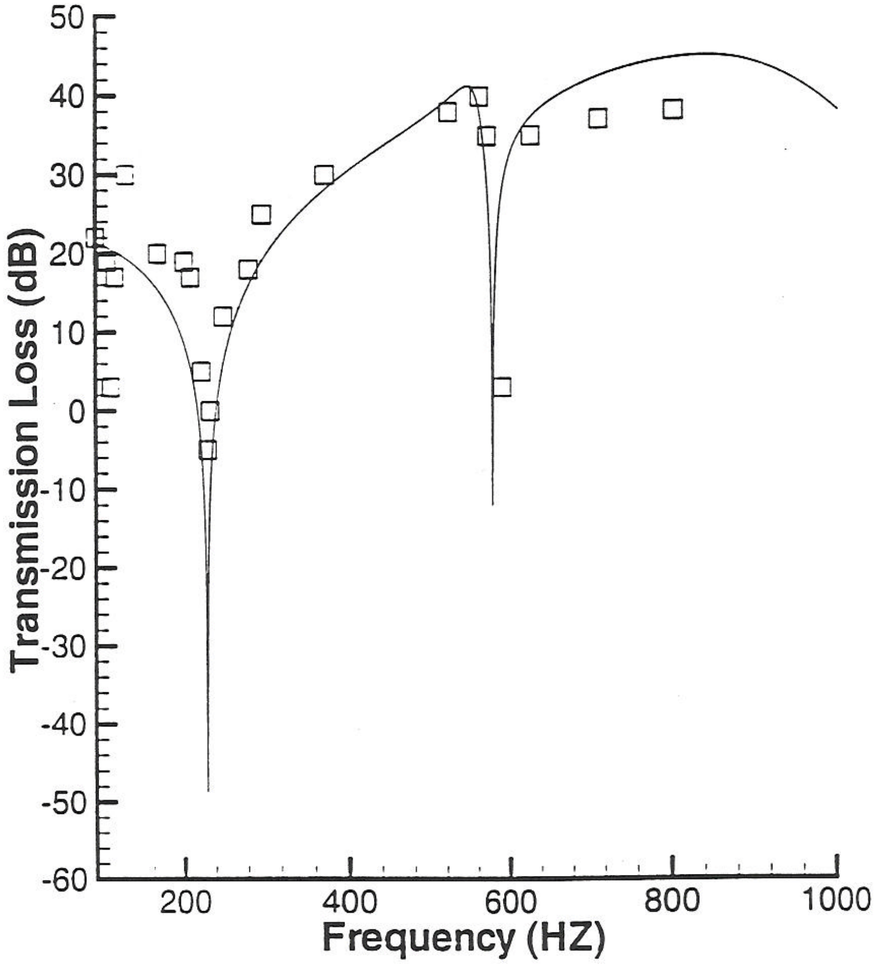

Figure 4 Transmission Loss in Model I
________ Numerical Method
□□ Experiment Data (Guy)

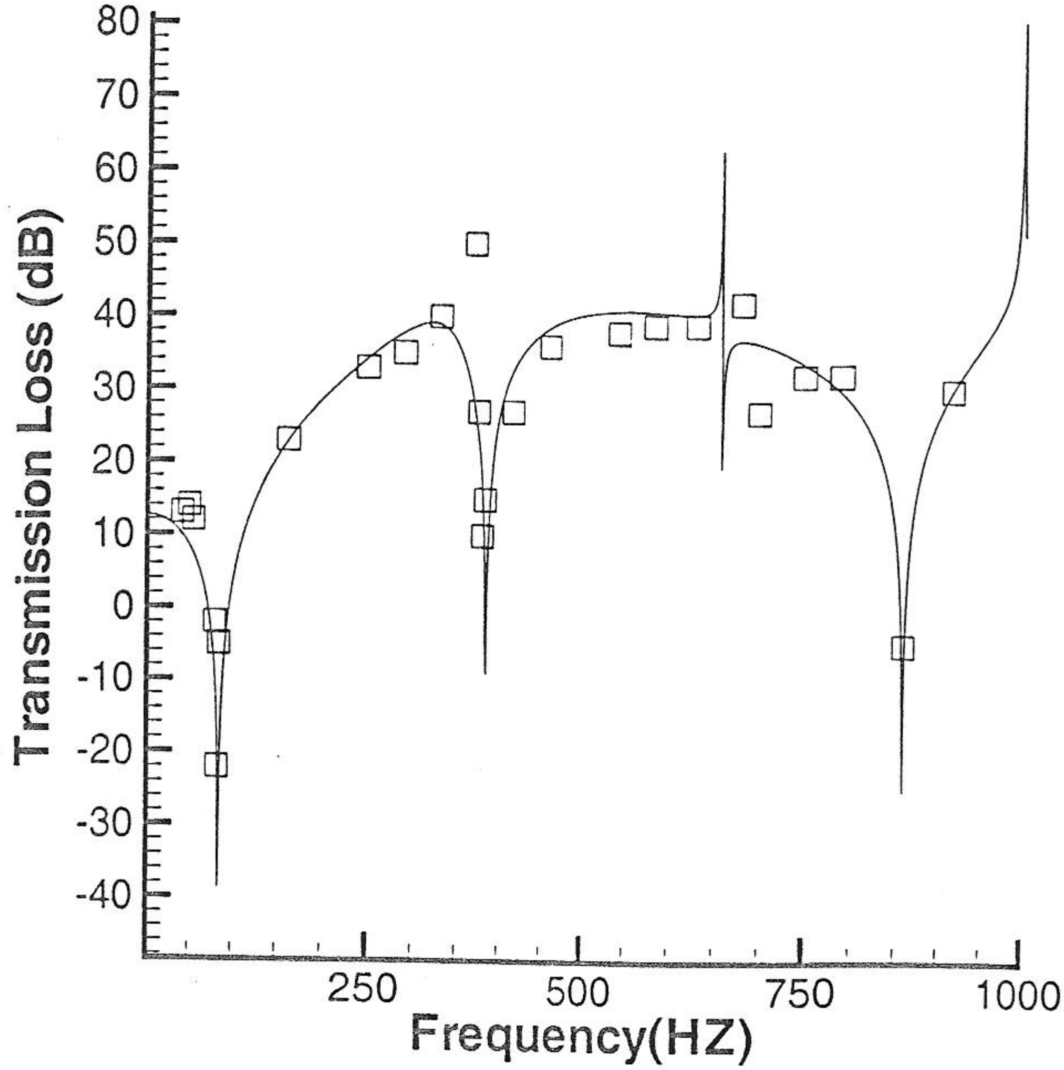

Figure 5 Transmission Loss in Model II
________ Numerical Method
□□ Experiment Data (Guy)

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