A Heat Transfer Experiment to Illustrate Dimensional Analysis

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Overview

Dimensional analysis is a technique used in many fields of engineering to facilitate correlation and interpretation of experimental data. It provides a means of combining the many parameters of an experiment into a lesser number of dimensionless groups. This technique greatly reduces the amount of experimental work needed to determine the effect of parameter variation on the dependent parameter of the experiment.

As an example from heat transfer, consider the heat transfer to a fluid flowing across a heated horizontal cylinder. The convective heat transfer coefficient depends on 6 parameters – the diameter of the cylinder, the velocity of flow across the cylinder, and four properties of the fluid (density, specific heat, viscosity, and thermal conductivity). The problem therefore involves 7 parameters – the heat transfer coefficient, which is the dependent parameter, and the other 6 parameters which are the independent parameters. To fully investigate the heat transfer phenomenon, it would appear that one would have to vary all 6 independent parameters separately to see how a change in a given parameter would affect the value of the heat transfer coefficient. This would indeed be a formidable undertaking involving many different fluids. Voluminous data would be generated that would be hard to correlate and interpret. Through the technique of dimensional analysis, the problem can be reduced from one involving seven parameters to one involving three dimensionless groups, one group of which contains the dependent parameter. By reducing the number of variables in the problem from seven to three, the necessary experimental work is greatly reduced, and correlation of the experimental data is greatly facilitated.

In the mechanical engineering curriculum, dimensional analysis is typically taught in the fluid mechanics and heat transfer lecture courses, where the Buckingham-Pi theorem is used to determine the appropriate dimensionless groups for a given problem. [1, 2, 3]. The procedure used for determining the dimensionless groups is generally straight-forward, but tedious. It is believed that a laboratory experiment specifically designed to illustrate dimensional analysis would increase the students' interest in the subject and would significantly enhance the students' comprehension of the technique.

Accordingly, we have developed an experiment dealing with natural and forced convection from heated horizontal cylinders. There are three cylinders of different diameters, each having an internal electric heater. The heat transfer rates from the cylinders to the surrounding fluid can be changed by varying the power input to the heaters. Steady-state measurements are made of the power input to the heater, the temperature of the cylinder, and the temperature of the fluid. For forced convection operation, the velocity of the fluid flow across the cylinder is also measured.

Initially, the intent was to have the experiment as a demonstration in the heat transfer lecture class. Students would participate in the performance of the demonstration and the gathering of data. They would also prepare graphs using software such as Excel or MathCad. These graphs would clearly indicate the usefulness of dimensional analysis in the correlation of the obtained data. The writer strongly believes that the use of such demonstrations in lecture classes significantly increases student participation and active learning and leads to greater student comprehension of the topics discussed. [4, 5]

Unfortunately, it turned out that the time response of the cylinders to changes in heat input was such that only a limited number of experimental runs could be performed during a lecture class session. Indeed, not enough runs could be performed to adequately illustrate the topic of dimensional analysis. Hence, the experimental apparatus is better suited for the engineering laboratory, with its longer class period.

The experimental apparatus and procedures are described below in considerable detail. Examples of data obtained from the experiment are presented. Plans for future work are outlined.

In summary, this paper describes a heat transfer experiment to illustrate the topic of dimensional analysis. The experiment should greatly increase the students' interest in the topic and their comprehension of the use of dimensional analysis in the planning of experimental programs and the correlation of experimental data.

I. Objectives

The main objective was to design and construct a heat transfer experiment to illustrate the concept and usefulness of dimensional analysis. An accompanying benefit of the effort was the addition of an experiment to the mechanical engineering laboratories.

II. Experimental Apparatus

The apparatus has been designed to be low cost, relatively easy to construct by lab technicians, and readily transportable from storage location to place of use.

The three cylindrical specimens each consist of an aluminum rod four (4) inches long. The rods are of different diameters (1/2 inch, 3/4 inch, and 1 inch), and are mounted in a U-shaped Lucite holder.

Figure No. 1 shows the three cylindrical specimens.

Figure No. 2 is a close-up view of the smaller specimen.

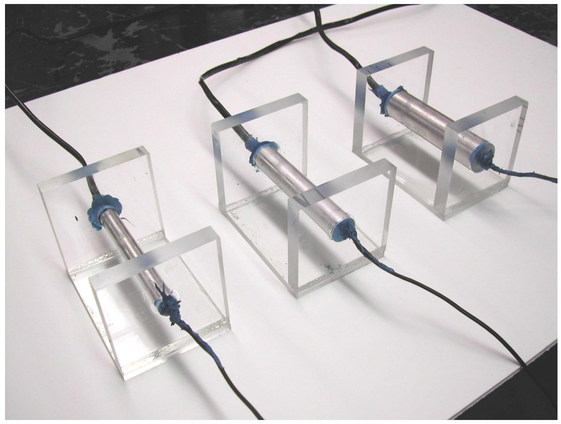

Figure No. 1 - The Three Cylindrical Specimens

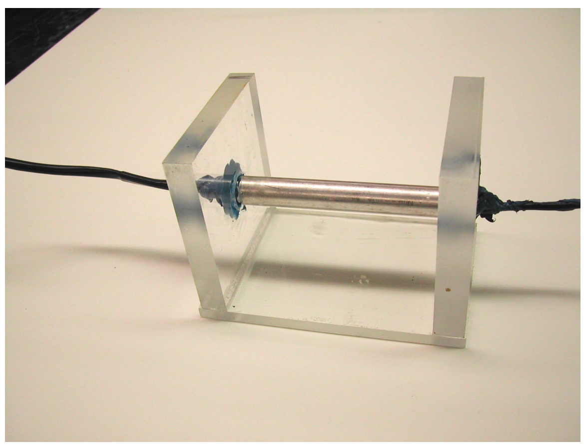

Figure No. 2 - Close-Up of the Small Specimen

For each cylinder, the construction was as follows: A 1/4 inch diameter, approximately 3-1/4 inch deep hole was drilled on the centerline of the cylinder. An electric cartridge heater was inserted into the hole and the open end was sealed with Permatek RTV adhesive. On the other end of the aluminum cylinder, a small diameter hole (about 1/4 inch deep) was drilled on the outside of the end of the cylinder. A Type-K thermocouple bead was pressed into the hole and sealed with RTV. The power leads to the heater and the thermocouple leads were passed through the 1/2 inch thick Lucite end plates. Additional RTV adhesive was used to seal the hole in the Lucite end plates. Finally, shrink insulation was installed on the power and thermocouple leads to seal the leads from moisture and electrically insulate the power leads from the surrounding medium when the cylinders were operated submersed in liquid. Holes of the same diameter as the cylinder, 1/4 inch deep, were cut in the Lucite end plates, and the plates were attached to the ends of the cylinder. Since a cylinder is 4 inches long and the recesses in the end plates were 1/4 inch deep, the length of cylindrical surface exposed to the surrounding fluid is 3-1/2 inches. (Note: The electrical insulating technique was not completely successful. When immersed in water, there was some electrical leakage. Repairs have to be made before the experiment is safe and acceptable for student use! Also, the obtained experimental data indicates that the heater for the small cylinder is not perfectly sealed against water intrusion. Temperature values for natural convection in water were abnormally low, and the data obtained for this mode of operation were disregarded. The small cylinder will be cleaned and resealed shortly to permit collection of accurate data.)

Figure No. 3 shows the temperature measuring instrumentation. The digital thermometer on the left is connected to the thermocouple on the cylindrical specimen. The digital thermometer in the center, with attached probe, measures the temperature of the fluid surrounding the specimen.

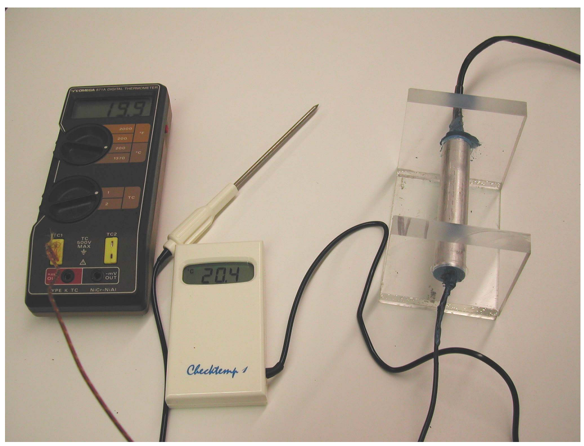

Figure No. 3 - Temperature Instrumentation

For forced convection experiments in air, a normal table fan is used to blow air across the specimen. The air velocity is measured by a hot-wire anemometer.

For all experimental runs, the power input to the heater is varied using a Variac autotransformer. The Variac is connected to a power analyzer which has a direct digital readout of the power input to the heater.

Figure No. 4 shows the table fan used in the forced convection in air experiments. It also shows the temperature measuring instrumentation and the specimen.

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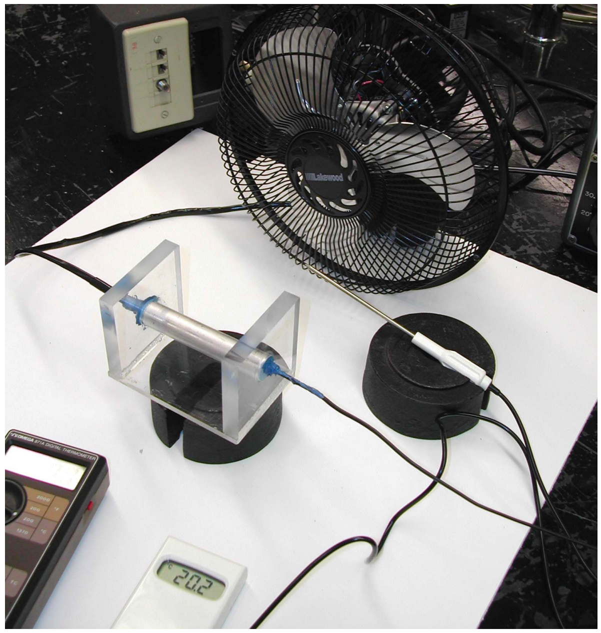


Figure No. 4 - Fan for Forced Air Runs

Figure No. 5 shows the air velocity instrumentation in the foreground and the Variac and power analyzer in the background.

Figure No. 5 - Air Velocity and Power Instrumentation

For natural convection in liquid experiments, the specimen is immersed in liquid in a plastic container about 1-1/2 feet wide, 2 feet long and 6 inches deep. The volume of liquid is sufficient so that the temperature of the liquid remains constant during the experimental runs.

A hydraulic channel in the engineering laboratories is used to provide a controlled flow of water over the specimens for the forced convection in water experiments. A digital paddlewheel flowmeter indicates the volumetric flowrate of the water down the channel and the average velocity of flow over the specimen can be estimated through measurement of the cross sectional flow area of the water. Alternatively, a pitot static tube can be used to determine the flow velocity of the water over the cylindrical specimen.

Figure No. 6 shows the temperature and power instrumentation used for the forced convection in water experiments. In the back of the figure is the specimen mounted to the bottom of the hydraulic channel.

Figure No. 6 - Instrumentation and the Specimen Mounted in the Hydraulic Channel

Figure No. 7 shows the water flowing over a mounted specimen during a forced convection in water run.

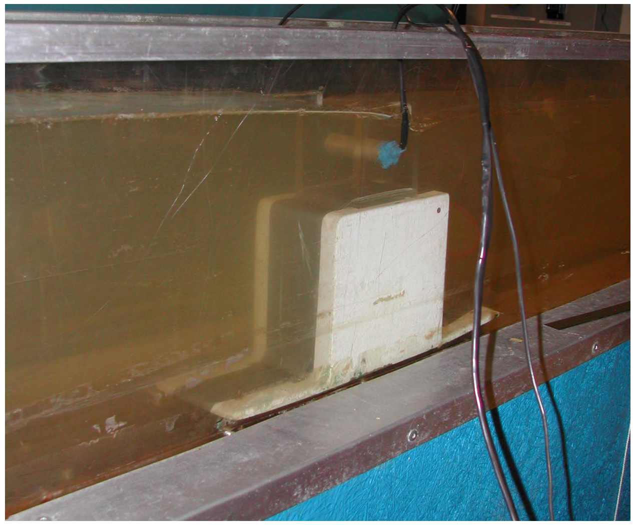


Figure No. 7 - Water Flow Over the Specimen

The cylindrical speciments were constructed by Mr. John Legault, Supervisor of the Hofstra Engineering Labs. All of the instrumentation and power supply equipment items used in this experiment were available in the Hofstra Engineering Laboratories. The following is a list of such equipment items, with approximate current costs:

Temperature Measurement
Omega Engineering Model 781A Digital Thermometer, with Type K thermocouple ($ 250) (For measurement of specimen temperature.)

Hanna Instruments Pocket Thermometer Checktemp 1C. MSC Industrial Supply Item 86491008 ($ 35) (For measurement of fluid temperature.)

Power Supply and Measurement
Variac Autotransformer; General Radio Co. Type W5LMT3; Load 0-120 V, 7.1 A ($ 200) (For setting the power input to the specimen heater.)

Extech Power Analyzer / Data Logger Model 380803 ($ 700) (For measurement of the power input.)

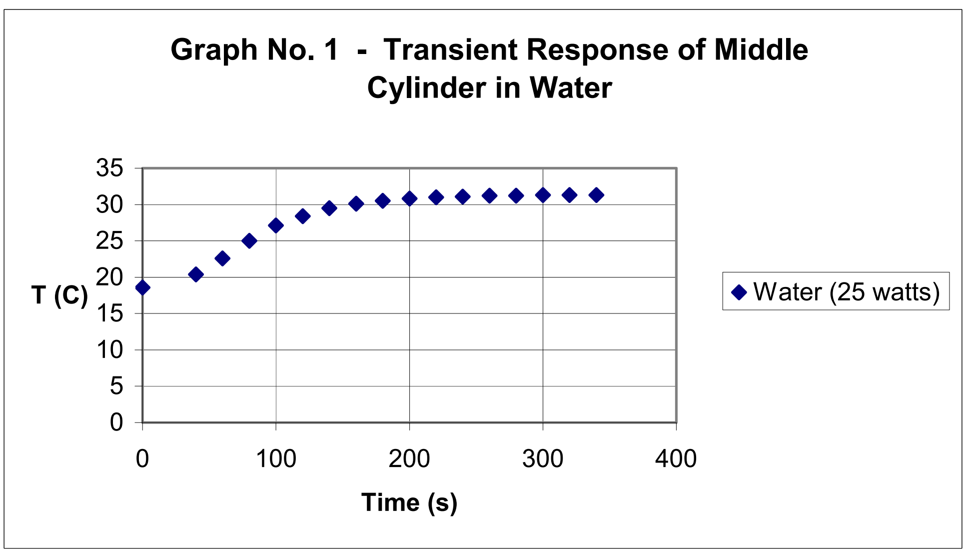
Air Velocity Measurement
Omega Engineering Model HHF615 Hot Wire Anemometer ($ 1400) (For measurement of air velocity for the forced convection in air runs.)

III. Experimental Runs

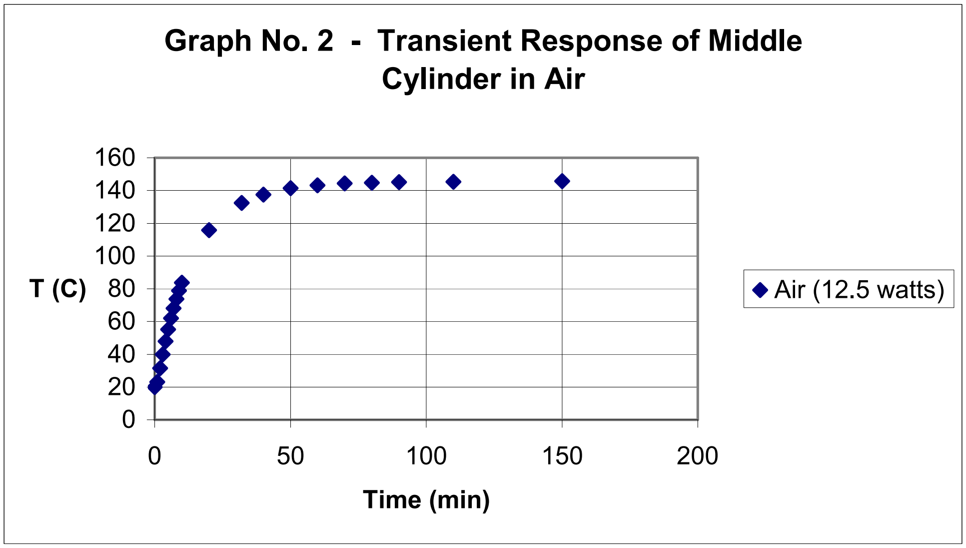
Runs have thus far been made of forced and natural convection from the heated horizontal cylinders to air and water for a variety of power inputs. The experimental procedure is as follows:

The cylindrical specimen is positioned as desired. For forced convection runs, flow of the fluid over the specimen is established and the velocity of flow is determined by the instrumentation. The power supply to the specimen heater is turned-on and adjusted to the desired level by the Variac. Timing begins and periodic measurements are taken of the power input, specimen temperature, and fluid temperature. When steady-state is reached, the final temperature and power readings are taken.

Graph No. 1 shows the temperature-time response of the 3/4 inch diameter cylinder for a natural convection in water run with a power input of 25 watts. The specimen is at an initial temperature of about 18C when the power is turned-on, and the specimen reaches a steady-state temperature of about 31 C. It takes about 3-1/2 minutes for the specimen to reach steady-state.



Graph No. 2 shows the temperature-time response of the 3/4 inch diameter cylinder for a natural convection in air run with a power input of 12.5 watts. The specimen has an initial temperature of about 20C , and reaches a final temperature of about 145C. It takes over an hour for the specimen to reach steady-state.



The response times for the other two cylinders are of the same order of magnitude. The steady-state times for the water runs are short enough for several experimental runs to be performed during the approximately 1-1/2 hour lecture class. But, the response time for the air runs is too long to permit multiple runs during the lecture class. To adequately illustrate dimensional analysis, several runs should be performed and data should be collected for more than one fluid. Hence, the current experiment is more suitable for a laboratory session of 2-1/2 to 3 hours than for a lecture session.

However, a possible use of the experiment in a lecture class could be as follows: A couple of experimental runs could be made with data collection by the students. This would acquaint the students with the apparatus and the procedure. Then the instructor could provide the experimental results of many previously performed runs, and the students could discuss the use of dimensional analysis in correlating all the data.

IV. Heat Transfer and Dimensional Analysis Theory [1, 2, 3, 6]

Under steady-state conditions, the electric power input to the specimen heater is equal to the rate of heat transfer from the specimen to the surroundings. This heat transfer includes several components: convection from the specimen's cylindrical surface to the surrounding medium, conduction through the electrical wires, conduction to the plastic end plates, and, if the surrounding medium is a gas, heat radiation to the room surfaces. Estimates have been made of the magnitudes of the contributions of these different components to the total heat transfer. For the power inputs and temperatures of the current experiment, the only significant mode of heat transfer is by convection from the surface of the specimen to the surrounding medium.

The rate of heat transfer by convection from a surface to a fluid is given by the equation:

$$Q_{conv} = h \quad A \quad (T_{surf} - T_{\infty})$$

In this equation, "Q_{conv}" is the rate of heat transfer by convection, "h" is the convective coefficient, "A" is the surface area in contact with the surrounding fluid, "T_{surf}" is the surface temperature of the specimen, and "T_{∞}" is the temperature of the surrounding fluid.

The purpose of experimental investigations in convection heat transfer is usually to determine the convective heat transfer coefficient "h". This coefficient depends on many parameters.

<u>For forced convection from horizontal cylinders</u>, the coefficient depends on 6 parameters – the diameter of the cylinder, the velocity of flow across the cylinder, and four properties of the fluid (density, specific heat, viscosity and thermal conductivity). By dimensional analysis, this problem involving 7 parameters can be reduced to one involving three dimensionless groups. This greatly reduces the amount of experimental work needed, and facilitates correlation of the experimental data.

The convective coefficient "h" is part of the Nusselt number "Nu", one of the dimensionless groups.

$$Nu = (h \ D / k)$$

In this definition of "Nu", "D" is the diameter of the cylinder and "k" is the thermal conductivity of the fluid.

The two other dimensionless groups relevant to forced convection are the Reynolds number "Re" and the Prandtl number "Pr". Their definitions are as follows:

$$Re = (\rho \ V \ D \ / \ \mu) \quad \text{and} \quad Pr = (C_p \ \mu \ / \ k)$$

In these equations, "ρ" is the density of the fluid, "V" is the velocity of the flow across the cylinder, "μ" is the viscosity of the fluid, and "C_p" is the specific heat of the fluid. "D" and "k" are as previously defined.

For natural convection from horizontal cylinders, the velocity of flow is no longer a major parameter. Rather, the buoyancy of the surrounding fluid is important. Experimental data can still be correlated using three dimensionless groups. The Nusselt and Prandtl numbers are still pertinent. The Reynolds number, however, is replaced by the Grashof number, "Gr", which contains the buoyancy related parameters of the acceleration of gravity and the volumetric coefficient of expansion.

The definition of the Grashof number is

$$Gr = \left(\left(g \ \rho^2 \ \beta \ D^3 \ (T_{surf} - T_\infty) \right) / \mu^2 \right)$$

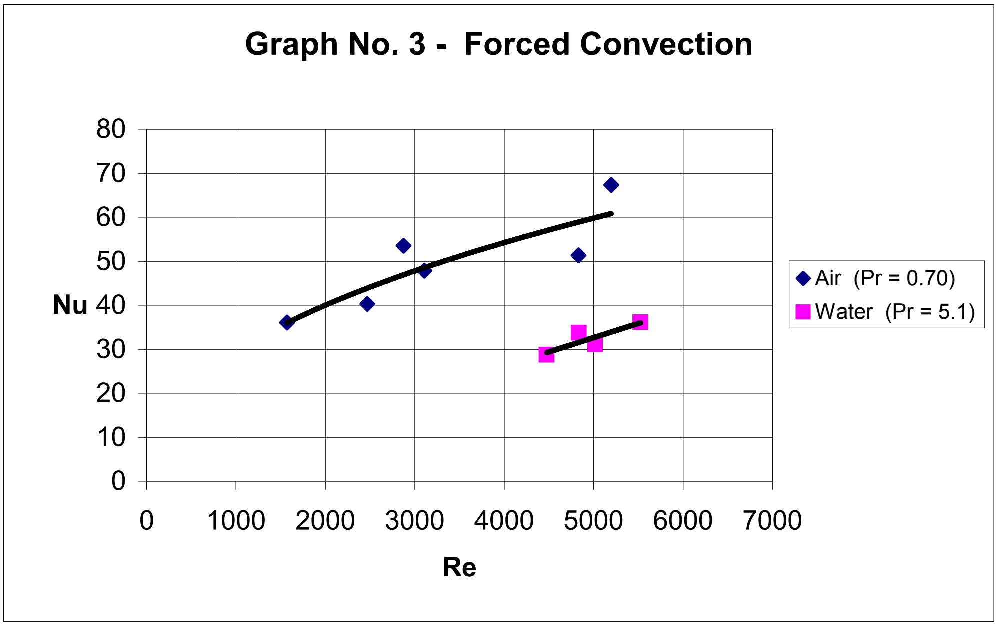
In this equation, the items not previously defined are the acceleration of gravity "g", and the volumetric coefficient of expansion "β".

In some natural convection investigations it is observed that the Grashof and Prandtl numbers are not independent parameters. In these cases, it is customary to express the Nusselt number as a function of the product of the two other numbers. This product is called the Rayleigh number "Ra".

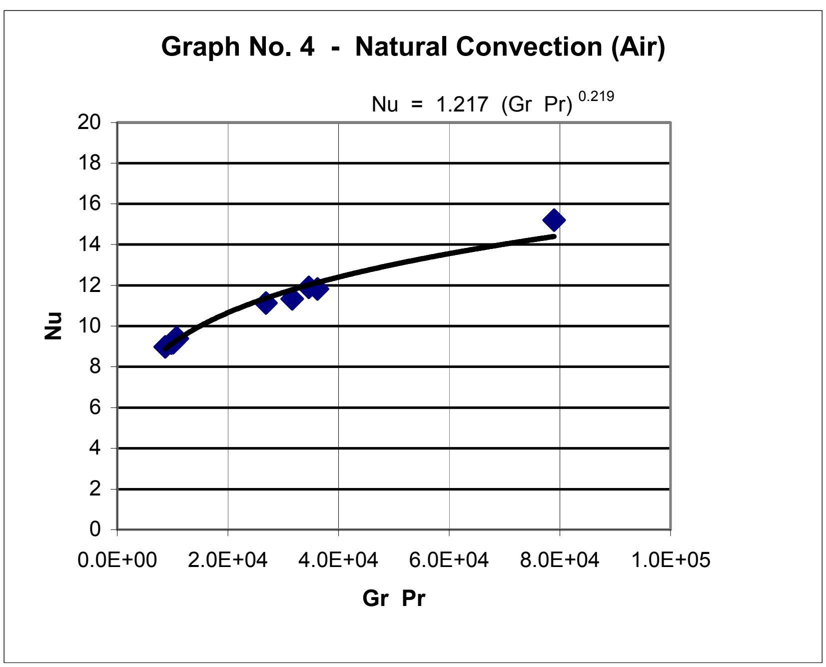
$$Ra = Gr \ Pr$$

V. Sample Experimental Results

Graph No. 3 shows results of forced convection runs in both air and water. The upper trendline is for the air results and the lower trendline is for the water results. For the air runs, there was essentially no variation in Prandtl number, and the upper line is for a Prandtl number of about 0.70. The variation of the Prandtl number for the water runs was also small with an average Prandtl number of 5.1. The upper line on Graph No. 3 is therefore the line for a Prandtl number of 0.70 and the lower line is for a Prandtl number of 5.1. The data agrees with the dimensional analysis conclusion that, for forced convection, the Nusselt number is a function of Prandtl and Reynolds numbers.



Graph No. 4 shows results of forced convection runs in air. The data includes runs for all three specimens and for a variety of power inputs. Consistent with dimensional analysis and heat transfer theory, it is seen that the Nusselt number is a function of the Rayleigh number (Gr * Pr). Fitting the data to a power relation using Excel, we obtained the result that the Nusselt number was proportional to the Rayleigh number to the 0.219 power. This value corresponds closely to the value of 0.25 given in heat transfer references. [7]



VI. Planned Future Work

As noted above, we plan to repair the small cylinder to eliminate the water intrusion that occurs during natural convection runs. The above noted electrical leakage problems for immersed specimens will be eliminated through better electrical insulation.

Plans are to collect additional data for water and air and also to collect data for another liquid, preferably one that has a Prandtl number between that of air and water.

The current work clearly indicates that experiments in forced and natural convection are beneficial in illustrating the use of dimensional analysis to correlate experimental data. The main problem with the current apparatus is its long time constant, which limits the usefulness of the apparatus as a demonstration in a lecture class. Work will be done on modifying the current design (probably through use of smaller cylinders) to reduce the response time of the apparatus. Currently, the hydraulic channel in the engineering labs is used as a water flow system for the forced convection in water experiments. We plan to work on a portable water flow system that could be easily transported to classrooms. This would permit the runs to be made outside of the engineering labs.

VII. Conclusion

A heat transfer experiment in forced and natural convection has been designed and constructed to illustrate the topic of dimensional analysis. The experimental apparatus has performed successfully and has provided results which clearly show the application and usefulness of dimensional analysis in correlating experimental data.

VIII. Bibliography

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