2006-2363: A HYDRODYNAMIC WHEATSTONE BRIDGE FOR USE AS A TEACHING TOOL IN INSTRUMENTATION LABORATORY COURSES

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Abstract

Undergraduate engineering students often find systems composed of electrical circuits difficult to grasp because variables such as current, voltage, resistance, capacitance, and inductance are not easily visualized as their analogs in mechanical systems. Thus, a Hydrodynamic Wheatstone bridge, using the analogy of flow in pipes, was developed to serve as a teaching tool in the classroom. A series of tests were performed to simulate ¼, ½, full, and shunted bridge circuits, where the increase or decrease in resistance in the strain gage is analogous to partially closing or opening a valve in a pipe network. The difference in head potential (i.e., ΔV) was measured with manometers located between the valves. The results agree with the ¼, ½, full, and shunted Wheatstone bridge circuits. Future enhancements include the addition of flow meters to relate water flow to current flow, scaled full turn valves to more accurately represent changing resistances, and a flexible tube section, in place of a valve, to replicate a strain gage in tension.

Introduction

In 1994, Civil Engineering professors at the University of Florida developed an undergraduate course, "INSTRUMENTATION FOR ENGINEERS", in which students are exposed to the fundamentals of circuitry through basic analysis of DC and AC circuits. Exercise problems are routinely performed in the classroom and given to the students for reinforcement through independent practice. Accompanying the course is a weekly two hour lab which provides the students hands on practice with civil engineering instrumentation: e.g., load cells, displacement transducers (LVDT), accelerometers, and pressure transducers. Since the Wheatstone bridge circuit is frequently used in these instruments, analysis of it is paramount to the students’ success in the course. Unfortunately, this topic is one that causes the most angst (perhaps second only to Thevenins!), and thus we felt it would be helpful to develop a device that would allow the students to observe what happens to water pressures when flowing in a “bridge circuit” pipe network.

Hence, using the analogy of flow in pipes, a bridge circuit was developed as a teaching tool. The similarity between hydrodynamics (flow, pressure, and valve position) to electricity (current, voltage, and resistance) provides a useful analogy. For example, a strain gage in a bridge circuit can be modeled with a needle valve in a pipe network so that an increase or decrease in resistance (via tension or compression) in the strain gage is analogous to closing or opening a valve in a pipe system. This will, in turn, create a ΔV (assuming a ¼ bridge) that can be represented by a difference in water heights in manometers located at the mid-points of the “bridge”.

Similarly, ½, full, and shunted bridge circuits can be modeled depending on the valve settings. For example, the students can 'balance' the bridge by adjusting the valves until the water heights in the manometers are equal. Then, by opening (less resistance = compression of a gage) one or
more valves, they can see the effect this has on the water levels. Stopping the flow at the outlet also shows that regardless of the valve(s) position, the water levels are equal for no-flow conditions.

**Instrument Description**

The Hydrodynamic Wheatstone Bridge (HWB) shown in Figure 1, was designed for use as a hands-on lab instrument. It utilizes a 1000 mL graduated cylinder filled with water and placed adjacent to the bridge to create the applied pressure (voltage) to the pipe network (circuit). Swagelok valves represent the four strain gages in the legs of the circuit. The valves are connected by ¼" OD tubing and 45° brass fittings. Manometer tubes, (Fig. 2) are located in between the valves, allowing the students to measure the water column height or head (voltage) and the changes caused by opening or closing one or more valves. The valves can be adjusted to a prescribed setting, simulating the straining of the gage(s) (∆R). The manometer tubes serve as the “voltmeter” to measure the difference in voltage across the two legs of the bridge.

In electrical circuits, Wheatstone bridges are often used to measure medium resistance values (1). Very small resistances are difficult to measure because of thermoelectric voltages generated at the junctions of dissimilar metals and thermal heating effects. Very large resistances are difficult to measure accurately because of leakage currents. There is no direct analog to these two phenomena in the hydrodynamic Wheatstone bridge. While the opening and closing of valves is not a perfect analogy for electrical resistors, the hydrodynamic circuit does serve as an accurate analog as long as the valves are not completely closed or open.

The water in the hydro-bridge circuit is pumped into the graduated cylinder from a reservoir located below the circuit (Fig 1). Outflow from the circuit is channeled back into the reservoir to maintain a constant flow condition. Tap water with food dye serves as the fluid to help observe water heights in each manometer (Fig. 2).
Figure 1. Hydrodynamic Wheatstone Bridge
Test Setup

After construction, a series of tests were conducted to verify the concept. The tests were planned to simulate $\frac{1}{4}$, $\frac{1}{2}$, Full, and shunted bridge circuits. Table 1 provides the details of each test, i.e., valve settings.

Figure 2. Hydrodynamic Bridge Circuit
Table 1. Test Simulations

<table>
<thead>
<tr>
<th>Bridge Configuration</th>
<th>$\frac{1}{4}$</th>
<th>$\frac{1}{2}$</th>
<th>Full</th>
<th>Shunt (Valve 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valve Settings</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Valve 1</td>
<td>$\frac{1}{2}$ Open</td>
<td>$\frac{1}{4}$ Open</td>
<td>$\frac{1}{4}$ Open</td>
<td>$\frac{1}{2}$ Open</td>
</tr>
<tr>
<td>Valve 2</td>
<td>$\frac{1}{2}$ Open</td>
<td>$\frac{1}{2}$ Open</td>
<td>$\frac{3}{4}$ Open</td>
<td>$\frac{9}{16}$ Open</td>
</tr>
<tr>
<td>Valve 3</td>
<td>$\frac{1}{4}$ Open</td>
<td>$\frac{1}{4}$ Open</td>
<td>$\frac{1}{4}$ Open</td>
<td>$\frac{1}{2}$ Open</td>
</tr>
<tr>
<td>Valve 4</td>
<td>$\frac{1}{2}$ Open</td>
<td>$\frac{1}{2}$ Open</td>
<td>$\frac{3}{4}$ Open</td>
<td>$\frac{1}{2}$ Open</td>
</tr>
</tbody>
</table>

The readings of the manometer tubes for the different valve configurations are the measured water column heights. Setting all the valves to the same open position (e.g. $\frac{1}{2}$ open) represents a balanced bridge in which all the strain gages are oriented in the same direction with equal resistances. This scenario produces zero voltage difference between the legs, hence $V_A - V_B$ or $\Delta V = 0$ (Refer to Figure 3. below). A balanced Wheatstone bridge condition can be described by Equation 1.

Figure 3. Wheatstone Bridge Circuit

\[
\frac{R_1}{R_4} = \frac{R_2}{R_3} \quad \text{Eq. 1}
\]

In the case of the Hydrodynamic bridge, setting valves 1 and 4 to the same setting and valves 2 and 3 to a different, but equal setting, also results in a balanced bridge ($\Delta V = 0$). It was anticipated that manometers $H_A$ and $H_B$ would be equal and half of the initial manometer ($H_i$)
water height. Opening each valve to a different setting results in an unbalanced bridge ($\Delta V \neq 0$), in which case manometer heights $H_A$ and $H_B$ would be different.

For a $\frac{1}{4}$ bridge, a single gage is strained in compression or tension. The straining increases or decreases the resistance ($\Delta R$), which if higher order terms are neglected, produces a difference in voltage, described by equation 2.

$$\Delta V = \frac{GF}{4} \varepsilon V_o$$  \hspace{1cm} \text{Eq. 2}

where: $GF$ is the gage factor, $\varepsilon$ the strain and $V_o$, the excitation voltage

For the $\frac{1}{2}$ bridge circuit, two gages (R₁ and R₃) are strained in compression or tension, resulting in a voltage difference twice that of the $\frac{1}{4}$ bridge. The change in voltage is described by equation 3.

$$\Delta V = \frac{(GF) \varepsilon}{2} V_o$$  \hspace{1cm} \text{Eq. 3}

Therefore, it was expected that the difference in readings between $H_A$ and $H_B$ would produce similar results, assuming equal lengths of the tubing in the circuit.

To simulate a shunted circuit, we adjusted the valve setting to replicate the parallel combination of $R_2$ and a shunt resistor, $R_S$. The high resistance of $R_S$ combined with the resistance of $R_2$ results in a value that is slightly less than $R_2$. Therefore, valve 2 was opened slightly more than the other valves. The setting for the other valves is described in Table 1. As a result, there is a small increase in $V_B$ and $\Delta V$ (i.e., $V_A - V_B$). This was observed in the manometer readings of $H_A$ and $H_B$ as well.

**Test Results**

Tests were performed on the HWB to simulate $\frac{1}{4}$, $\frac{1}{2}$, full, and shunted bridge circuit configurations. The results of each test are presented in Tables 2-5.

**$\frac{1}{4}$ Bridge**

The difference in water heights of $H_A$ and $H_B$ was 4.1 cm. Considering Ohm's law ($V = IR$), for constant current, as the resistance increases so does the voltage drop or difference. For these tests, the water flow was maintained at a constant, low rate. In the case of the HWB, head loss due to the flow path (pipe friction, fitting losses etc. from the source to the first manometer) was negligible. We measured a 0.5 cm. difference (or loss) between these two points. It is important to note that the flow rate had to be adjusted in order for the bridge to work properly. This was done by trial and error and once set, the remainder of the tests worked properly. While we did not have a flowmeter sensitive enough to measure it, we plan to incorporate a rotameter to monitor the flow in the next model.
Table 2. Results of the $\frac{1}{4}$ Bridge Simulation

<table>
<thead>
<tr>
<th>Source Head (36 cm)</th>
<th>Valve Settings</th>
<th>Manometer</th>
<th>Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valve 1</td>
<td>$\frac{1}{2}$ Open</td>
<td>$H_I$</td>
<td>35.5 cm</td>
</tr>
<tr>
<td>Valve 2</td>
<td>$\frac{1}{2}$ Open</td>
<td>$H_B$</td>
<td>30.2 cm</td>
</tr>
<tr>
<td>Valve 3</td>
<td>$\frac{1}{4}$ Open</td>
<td>$H_F$</td>
<td>0 cm</td>
</tr>
<tr>
<td>Valve 4</td>
<td>$\frac{1}{2}$ Open</td>
<td>$H_A$</td>
<td>26.1 cm</td>
</tr>
</tbody>
</table>

Figure 4. Manometer $H_I$
\section{\(\frac{1}{2}\) Bridge}

The expected results of the \(\frac{1}{2}\) bridge simulation was a difference in head between \(H_A\) and \(H_B\) approximately twice that of the \(\frac{1}{4}\) bridge. Table 3 presents the test results. We see that \(\Delta H\) was close to 2\(x\) the \(\frac{1}{4}\) bridge setup.

\begin{table}[h]
\centering
\caption{Results of \(\frac{1}{2}\) Bridge Simulation}
\begin{tabular}{|l|c|c|c|}
\hline
Source Head (36 cm) & Valve Settings & Manometer & Reading \\
\hline
Valve 1 & \(\frac{1}{4}\) Open & \(H_I\) & 35.7 cm \\
\hline
Valve 2 & \(\frac{1}{2}\) Open & \(H_B\) & 30.4 cm \\
\hline
Valve 3 & \(\frac{1}{4}\) Open & \(H_F\) & 0 cm \\
\hline
Valve 4 & \(\frac{1}{2}\) Open & \(H_A\) & 22.5 cm \\
\hline
\end{tabular}
\end{table}

\section{Full Bridge}

In Table 4 the results of the full bridge simulation are shown. As expected, the height difference was approximately 16.9 cm or 4.12 times the \(\frac{1}{4}\) bridge difference (as opposed to 4). Again, the flow rate has a substantial effect on the heights and it was necessary to reduce it to a minimum in order to maintain the correct bridge output effects.

\begin{table}[h]
\centering
\caption{Results of the Full Bridge Simulation}
\begin{tabular}{|l|c|c|c|}
\hline
Source Head (36 cm) & Valve Settings & Manometer & Reading \\
\hline
Valve 1 & \(\frac{1}{4}\) Open & \(H_I\) & 34.6 cm \\
\hline
Valve 2 & \(\frac{3}{4}\) Open & \(H_B\) & 28.6 cm \\
\hline
Valve 3 & \(\frac{1}{4}\) Open & \(H_F\) & 0 cm \\
\hline
Valve 4 & \(\frac{3}{4}\) Open & \(H_A\) & 11.7 cm \\
\hline
\end{tabular}
\end{table}
**Shunted Bridge**

Table 5 presents the results of the shunt simulation for valve 2. The readings of $H_A$ and $H_B$ indicate a small difference, approximately 0.4 cm. This agrees with the trend when shunting a bridge with a large kilo ohm resistor in parallel with one of the standard gages (120 or 350 ohms). As previously discussed, the difference between $V_A$ and $V_B$ was small due to the parallel combination of $R_2$ and $R_S$.

<table>
<thead>
<tr>
<th>Source Head (36 cm)</th>
<th>Valve Settings</th>
<th>Manometer</th>
<th>Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valve 1</td>
<td>½ Open</td>
<td>$H_I$</td>
<td>34.3 cm</td>
</tr>
<tr>
<td>Valve 2</td>
<td>9/16 Open</td>
<td>$H_B$</td>
<td>14.6 cm</td>
</tr>
<tr>
<td>Valve 3</td>
<td>½ Open</td>
<td>$H_F$</td>
<td>0 cm</td>
</tr>
<tr>
<td>Valve 4</td>
<td>½ Open</td>
<td>$H_A$</td>
<td>14.2 cm</td>
</tr>
</tbody>
</table>

**Conclusions**

The results from these Hydrodynamic bridge tests agree reasonably well with ¼, ½, full, and shunted Wheatstone bridge circuits. While this instrument is a prototype, it does perform as anticipated. We plan to utilize it this semester and see if it aids the students’ understanding of bridge circuits.

Future enhancements will include adding a flowmeter to each leg in order to measure water flow and attempt to relate it to current flow. In addition, the valves will be replaced with full turn valves and scaled numbers representing gage resistances. This will allow students to set the valves more accurately and also see how changing the “resistance” from 120 $\Omega$ to 119.5 $\Omega$ affects the water level (voltage balance).

Finally, we plan to insert a flexible tube section to replace one of the valves. This will allow us to stretch it and hopefully replicate a strain gage wire in tension (Fig. 5). In the case of wire, the resistance changes due to its deformation (described in Equation 4 below). For longitudinal straining (tension) of a wire, it is apparent that the area becomes smaller as $\delta L$ increases. This results in increased resistance. We will attempt to utilize this effect to form a ¼ bridge “active gage”. It is doubtful that we will be able to mimic compression with this setup.
\begin{equation}
R = \frac{\rho L}{a}
\end{equation}

where: \( R \) = resistance, \( \rho \), the resistivity of the material,
\( L \), length and \( a \), cross sectional area

It is the goal of the authors to further develop this and other instruments for classroom and lab use. For example, we are designing an “AC” water flow circuit and hope to simulate capacitors and inductors. Flow analogies in engineering education are valuable because of the transparency and relationship to electricity. We believe basic electrical circuit knowledge is essential for practicing engineers and hence, whatever teaching tools help to achieve this understanding is warranted. The contribution of these instruments will hopefully increase students’ performance in the classroom and that the concepts will remain long after they have graduated.

Bibliography