

## A Laboratory Experiment for Finding the Radiation View Factor Between a Differential Area and a Plane

Aaron Renner, William S. Janna  
University of Memphis

### Abstract

The unit sphere method was developed in order to measure the view factor between a differential and a finite plane area. This method was used to design an experiment for the undergraduate Heat Transfer Laboratory, and is described in this study. Using methods of descriptive geometry, a graphical equivalent of the sphere method was also developed, which is used additionally in the experiment. The apparatus for measuring view factor is described here. Experimental results for one shape are compared to results obtained by equations. Errors in all cases are within a tolerable limit.

### Nomenclature

$A$  = area  
 $A$  = constant in view factor equation  
 $a$  = distance in view factor equation  
 $dA$  = differential area  
 $B$  = constant in view factor equation  
 $b$  = distance in view factor equation  
 $c$  = distance in view factor equation  
 $d_1$  = distance in hemisphere profile  
 $d_2$  = distance in hemisphere profile  
 $F$  = view factor  
 $r$  = radius of circle  
 $r_a$  = actual radius of hemisphere  
 $S$  = distance between areas

### Greek Letters

$\beta$  = angle between  $S$  and normal to  $A$   
 $d\omega$  = solid angle

## Subscripts

1 = source of radiation

2 = receiver of radiation

$s$  = area on hemisphere surface

$b$  = area on base of hemisphere

## I. Introduction

Many problems that today are easily solved with a calculator were at one time rather difficult to complete. The calculation phase of the solution method was very tedious, involving the use of a slide rule or a mainframe computer. Consequently, alternative methods of obtaining a solution to a particular problem were developed. Graphical methods, for example, were developed to a high degree in a number of areas as an alternative or supplement to the calculation method. There are graphical procedures for addition, subtraction, multiplication and division<sup>1</sup>. Graphical methods are available also for solving ordinary differential equations<sup>2</sup>. Graphical solution methods have been devised for solving problems in kinematic analyses in which velocities and accelerations of machine components are determined as illustrated by Hinkle<sup>3</sup> and countless others. Kreith<sup>4</sup> presented what has been popularly called a "Schmidt Plot" into heat transfer analysis of unsteady heat conduction problems. Janna<sup>5</sup> presented the Saul'ev plot also for heat conduction problems. Keith and Janna<sup>6</sup> discuss the use of graphical methods in unsteady viscous flow problems as a means of introducing more advanced fluid mechanics problems to undergraduates.

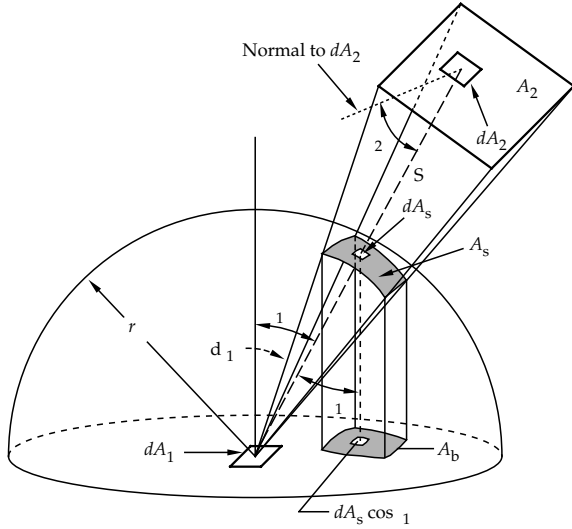
The subject of this paper is the unit sphere method, which is one of the solution techniques that was developed as a supplement to the theoretical method. The unit sphere method for finding the **configuration** or **view factor** between a differential and a finite area was discussed in the early part of this century. The unit sphere method was first introduced by Nusselt<sup>7</sup> as an experimental technique. There is, however, a graphical equivalent<sup>8</sup> based on the theory involved. Alciatore, Janna and Shamburger<sup>10</sup>, present a solution method, suitable for use on a microcomputer, which is the equivalent of the graphical method. Alciatore, Lipp and Janna<sup>11</sup> determined a closed form solution of the general three dimensional radiation configuration factor problem to accompany the microcomputer solution.

An excellent discussion of view factors, along with various methods of finding it for different geometries, can be found in reference<sup>12</sup>.

In this study, we present the theory, the graphical method itself, and an experimental method suitable for use in the laboratory, for finding the view factor between a differential area and a plane.

## II. Theory

Consider that we are trying to obtain the configuration factor between a differential area  $dA_1$  and the finite area  $A_2$ , as shown in Figure 1. Initially, we identify a differential area  $dA_2$  on  $A_2$ , and draw lines normal to both differential areas. We also draw a line from  $dA_1$  to  $dA_2$ , and this line has a length  $S$ . The angles  $\beta_1$  and  $\beta_2$  are measured from  $S$  to the lines that are normal to  $dA_1$  and  $dA_2$ , respectively. Next, we construct lines from  $dA_1$  to all points on the boundary of  $dA_2$ . These lines form a solid angle denoted as  $d\omega_1$ , which is given by



**FIGURE 1.** Configuration factor between a differential and a finite area (after Siegel and Howell<sup>8</sup>).

$$d\omega_1 = \frac{dA_2 \cos \beta_2}{S^2} \quad (1)$$

The view factor between  $dA_1$  and  $dA_2$  is found as

$$F_{dA_1-dA_2} = \frac{\cos \beta_1 \cos \beta_2 dA_1 dA_2}{\pi S^2} \quad (2)$$

Integrating over the area  $A_2$  results in the configuration factor that we are trying to evaluate:

$$F_{dA_1-A_2} = \int \int_{A_2} \frac{\cos \beta_1 \cos \beta_2 dA_2}{\pi S^2} \quad (3a)$$

or in terms of solid angle, with Equation 1, we get

$$F_{dA_1-A_2} = \frac{1}{\pi} \int_{A_2} \int \cos \beta_1 d\omega_1 \quad (3b)$$

where the integration limits on  $A_2$  extend over the portion that can be viewed by  $dA_1$ . Now suppose that we construct a hemisphere over the area  $dA_1$ , again referring to Figure 1. The projection of  $dA_2$  onto the surface of the hemisphere is  $dA_s$  which is given by

$$dA_s = r^2 d\omega_1$$

from the definition of solid angle. Combining with Equation 3b gives

$$F_{dA_1-A_2} = \frac{1}{\pi r^2} \int_{A_s} \int \cos \beta_1 dA_s \quad (4)$$

where the integral is to be evaluated over  $A_s$ . The term  $dA_s \cos \beta_1$  is the projection of the area  $dA_s$  onto the base of the hemisphere, and if this term is integrated over  $A_s$ , we obtain the area  $A_b$ . As shown in Figure 1, the area  $A_b$  is the projection of  $A_s$  onto the base of the hemisphere. Equation 4 therefore becomes

$$F_{dA_1-A_2} = \frac{A_b}{\pi r^2} \quad (5)$$

This equation states that the configuration factor between  $dA_1$  and  $A_2$  equals the ratio of the projection of  $A_2$  onto the base of the hemisphere ( $= A_b$ ) to the area of the base of the hemisphere:

$$F_{dA_1-A_2} = \frac{\text{projection of } A_s \text{ onto the base of the hemisphere}}{\text{area of a circle with radius of the hemisphere}}$$

A plan view of Figure 1 would reveal the two areas. Measurements or calculations could then be performed to find the view factor.

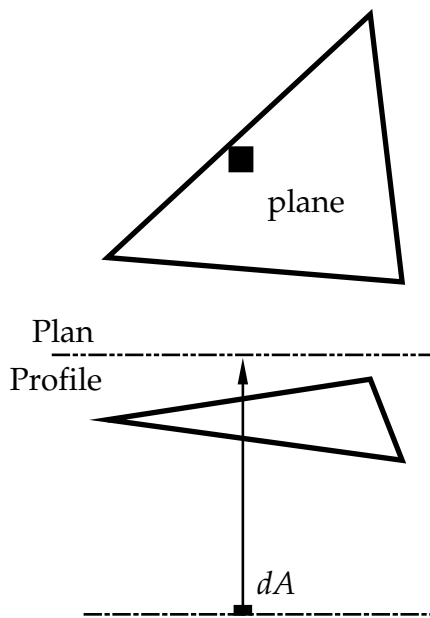
What has been described in the preceding paragraphs is a method for finding the view factor between a differential element and a finite area. At one time, this method

was called the Unit Sphere Method, because the intent was to use a hemisphere that had a radius of 1. The advantage was that in Equation 5, the view factor was merely the projected area divided by  $\pi$ . The method can be used with any radius hemisphere, however.

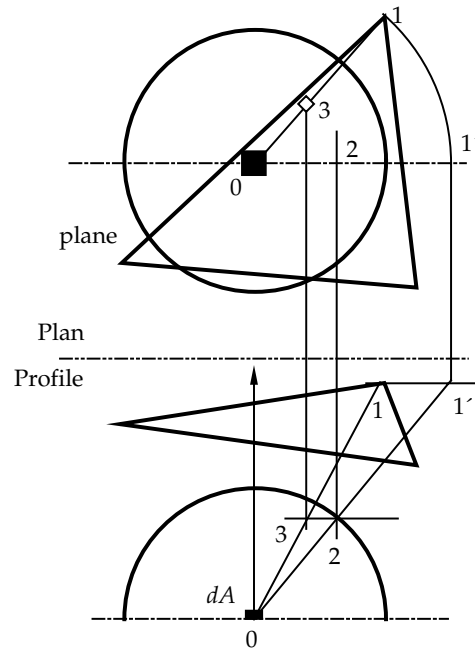
The sphere method has a graphical equivalent, as described in the following section. The sphere method can also be used with a photographic technique, which is described later in this paper. Both methods used together make up a worthwhile experiment in the undergraduate heat transfer laboratory. In some geometries, there are theoretical solutions which have been developed and catalogued<sup>12</sup>. One such case has been investigated with the sphere method, and the experimental results are compared to those obtained by equation.

### III. Graphical Solution to a Specific Problem

Figure 2 shows the plan and profile views of a differential area located at the origin, and a plane, selected to illustrate the method. We will use results obtained from the sphere method to obtain a graphical solution for the configuration factor between the differential area  $dA$  and the plane.



**FIGURE 2.** Plan and profile views of a differential area and a plane.



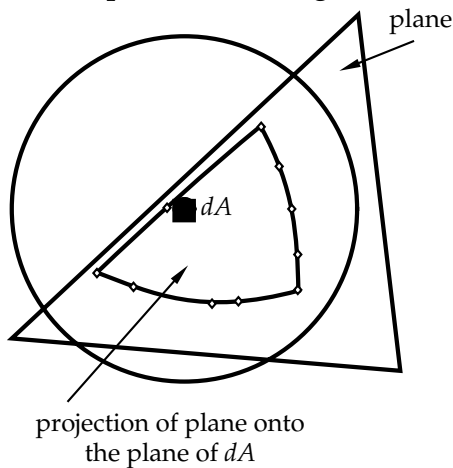
**FIGURE 3.** Plan and profile views of a differential area, a plane, and a hemisphere.

The graphical solution method first involves drawing a hemisphere of radius  $r$  in both views. The hemisphere is constructed with its center at the area  $dA$ , and the radius of the hemisphere is selected arbitrarily. The hemisphere can encompass the entire plane, or pass through it or exclude it. Next lines are constructed from the origin to every point on the edges of the plane. These lines form a solid angle with a unitless dimension of steradian (sr). The lines connecting the origin with the edges of the plane intersect the hemisphere and form an area that we have identified in Figure 1 as  $A_s$ .

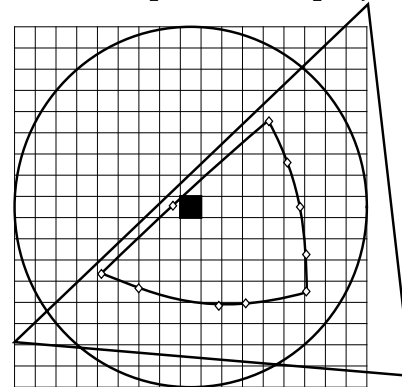
As mentioned previously, after drawing a hemisphere, we then draw lines from the origin to a number of points on the edges of the plane. The lines will intersect the hemisphere on the edge of the projection plane. In order to find a point of intersection, the true length of the line must be constructed in the profile view.

The true length of the line is found here by a rotation of the line as illustrated in Figure 3. We draw line 0-1 in both views. Line 0-1 is rotated in the plan view to the point labeled 1'. The line from 0 to 1' is drawn to locate point 2 in the profile view. Point 2 is projected back to the plan view and rotated back to line 0-1 to locate point 3. Thus 3 is the point where the line 0-1 intersects the hemisphere.

This process of finding intersecting points is repeated for many points on the periphery of the plane until something like Figure 4 results. Shown is the plan view of the completed drawing, which includes the areas  $dA$ , the plane, and projection of



**FIGURE 4.** *Plan view of the completed drawing.*



**FIGURE 5.** *Plan view of the completed drawing with grid imposed for determining area.*

the plane onto the base of the hemisphere. The view factor is the ratio of the projected area to that of the circle whose radius equals that of the hemisphere.

Figure 5 again shows the solution but with a grid imposed. The projected area can be determined either by counting squares or by some numerical technique. This is quite suitable for the heat transfer laboratory.

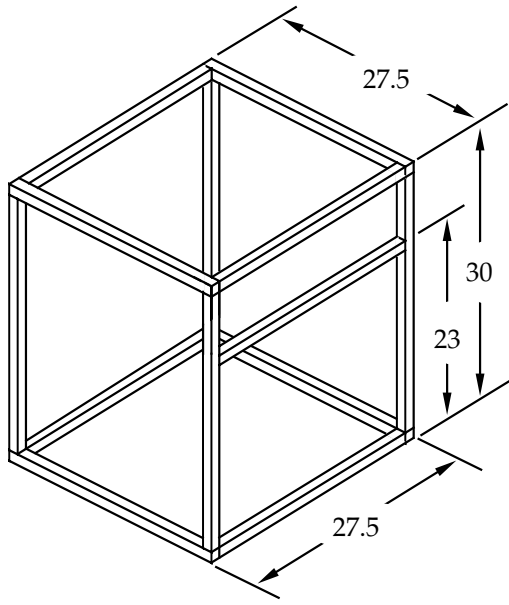
#### IV. Photographic Method

The photographic method is based on the concepts discussed in the theory section. Regarding the problem solved graphically in the preceding section, we can use a reflecting hemisphere which is placed over the area  $dA$ , and view directly above  $dA$ . The reflection of the plane in the surface of the hemisphere will appear like that shown in Figure 4. So a photograph taken in this way can be used to obtain the areas, and the configuration factor can then be calculated.

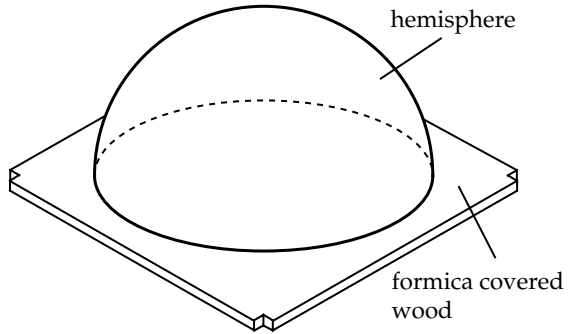
A frame was built and a hemisphere was purchased to construct an apparatus for the photographic method. Figure 6 shows an isometric view of the frame itself. It consists of 1 x 1 square tubing welded together into a boxlike shape. An additional piece of tubing is welded in place so that various planes can be bolted onto the frame. The dimensions were selected based on the hemisphere that was available.

Figure 7 is a sketch of the hemisphere (made of smoked plexiglas) which is bolted onto a piece of black, formica-topped plywood. The plywood, with notched corners, fits directly onto the base of the frame. The hemisphere selected was one that is used primarily in security surveillance operations (available from W. W. Grainger, Inc.; Item #1M840, see-through smoked full dome, 24 in.). A problem encountered with this hemisphere is that it is not truly hemispherical. Its diameter is 22 in., and so at its center it should be 11 in. tall. It is actually only  $9\frac{3}{4}$  in. tall, which affects the





**FIGURE 6.** *Isometric view of the frame. (Dimensions in inches.)*



**FIGURE 7.** *Isometric view of the hemisphere mounted on a wooden base.*

results of the experiment. However, this deficiency is easily corrected, as will be shown in the results section.

Figure 8 is a sketch of the assembled apparatus. Shown is the relative mounting of all pieces. The plane area is mounted on the frame, and a digital camera is used to take a photograph in the viewing direction.

Various plane areas were made of 16 gage sheet metal, with edges that have been smoothed and polished for safety reasons. These planes included a disk ( $11\frac{7}{8}$  in. in diameter), a square (10 in. x 10 in.), two octagons (17 in. and  $13\frac{1}{8}$  in. across flats), and an equilateral triangle with each side  $12\frac{1}{4}$  in long. All plane areas were painted a glossy white so they would reflect effectively.

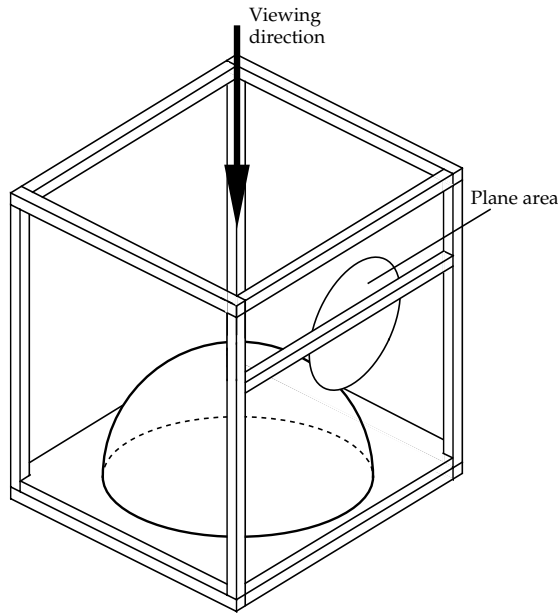
## V. Experiment

The students are given one of the plane areas and they are to mount it on the apparatus. They take a digital photograph and transfer the result to a computer. They are then given two options:

1. They can print a copy of the photograph, superimpose a grid, and count squares to determine the areas.
2. They can use a program (such as AutoCAD) to determine the areas.

Next, the students are to draw the system and use the graphical method to obtain a

solution. Again, they can determine the areas in any convenient way. Finally, they are to compare the results of the two methods.



**FIGURE 8.** *Sketch of the assembled apparatus.*

For some cases, a mathematical solution has been obtained for the view factor when the plane area is a disk, or when it is a square (or rectangle). In these cases, the students can make a third comparison.

## VI. Correction Factor

As mentioned earlier, the hemisphere used is not quite a true hemisphere; it has been truncated, as shown in Figure 9. The actual known dimensions, referring to the figure, are radius  $r_a$  and height  $d_2$ . In order to make appropriate calculations, it is necessary to determine the true radius  $r$ . For the dimensions of Figure 9, we can write:

$$r^2 = r_a^2 + d_1^2$$

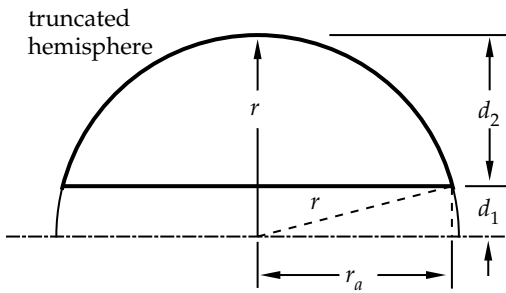
in which only  $r_a$  is known. We can also write

$$d_1 + d_2 = r$$

The preceding two equations contain two unknowns ( $d_1$  and  $r$ ). Solving simultaneously, we get

$$r = \frac{r_a^2 + d_2^2}{2d_2} \quad (6)$$

and  $d_1 = r - d_2$ . For this experiment, we measured  $r_a = 11$  in., and  $d_2 = 9.75$  in. The radius we would use in calculations involving this hemisphere then, from Equation 6, is  $r = 11.08$  in. Also,  $d_2 = 1.33$  in.



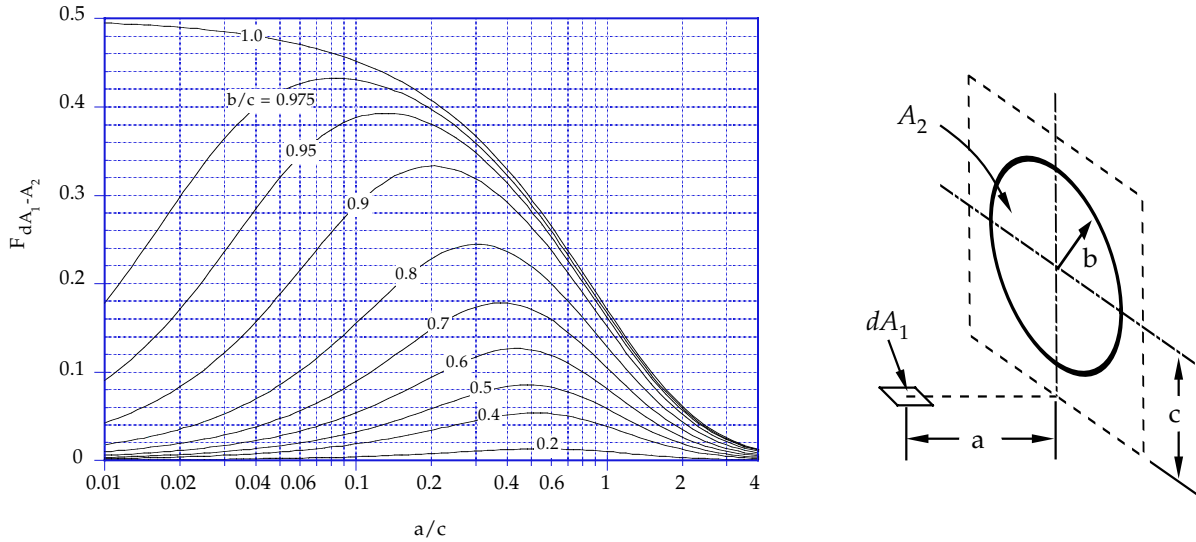
**FIGURE 9.** Sketch of the truncated hemisphere used in this experiment.

## VII. Comparison of Results for a Disk

When the plane area is a disk oriented in a certain way, an exact expression can be used to find the view factor. Figure 10 shows a drawing of two surfaces. One is a differential area  $dA_1$  and the other is a disk  $A_2$ . It is desired to find the view factor  $F_{dA_1-A_2}$ . The exact solution to this problem is given by<sup>8, 9</sup>:

$$F_{dA_1-A_2} = \frac{A}{2} \left( \frac{1 + B^2 + A^2}{\sqrt{(1 + B^2 + A^2)^2 - 4B^2}} - 1 \right) \quad (10)$$

where  $A = a/c$  and  $B = b/c$ . A graph of this equation is shown also in Figure 10.



**FIGURE 10.** A graph of Equation 10 for the view factor between a differential area and a disk.

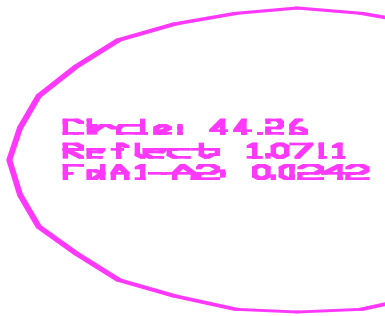
This problem was set up in the laboratory using a hemisphere, as in Figure 8. A photograph was taken from the top and the result is shown in Figure 11. A grid was superimposed on Figure 11 and squares were counted in order to find the area of the circle and of the ellipse (which is the reflection of the disk in the hemisphere). The results are shown in the following table:

	Square Counting	AutoCAD
Circle	941 squares	44.26 units
Ellipse	24.5 squares	1.0711 units
Ratio of ellipse to circle = $F_{dA_1-A_2}$	0.026	0.0242
% error	7.1%	13.6%

It is interesting to compare these results to those obtained with Equation 10. The raw data are as follows:  $a = 11.25$  in.;  $b = 5.94$  in.;  $c = 18.75$  in. These data must be corrected to account for the fact that the dome is not a perfect hemisphere. The affected dimension is  $c$ , which must be increased by 1.33 in. ( $= d_1$  in Figure 9). Thus  $c = 18.75 + 1.33 = 20.08$  in. Substituting into Equation 10 gives  $F_{dA_1-A_2} = 0.028$ . The percent error entries in the previous table are based on a comparison with this result. This problem was not solved graphically for this study.

The drawing of Figure 11 was obtained from the photograph taken in the laboratory. (The photograph would not produce very well for this publication.) The photo was

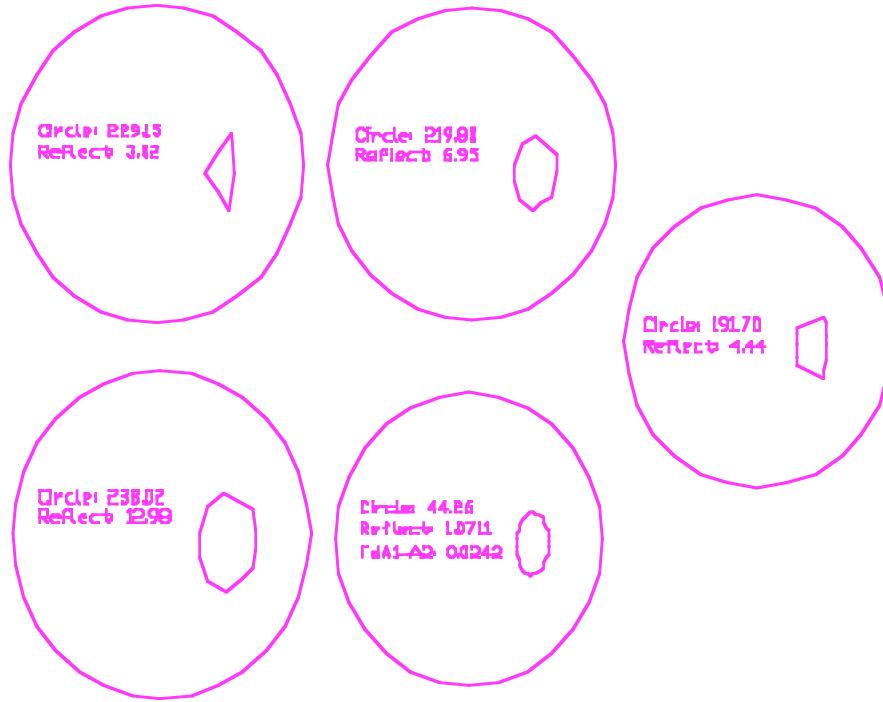
then imported into AutoCAD and the outlines of both the circle and the ellipse were traced out. Figure 12 is an AutoCAD drawing of all the shapes prepared for this study.



**FIGURE 11.** *AutoCAD drawing of the disk and hemisphere as taken from the photograph.*

### VIII. Discussion and Conclusions

The experiment described in this study was very inexpensive to put together. It is easy to perform and it gives the students a good background in learning about view factors. The results of calculations with the equations and the figures show acceptable agreement with regard to laboratory work. The sources of error can be related to errors in measurements of length, especially lengths  $a$  and  $c$  of Figure 10, because of the presence of the hemisphere.



**FIGURE 12.** AutoCAD drawing of all shapes prepared for this experiment, as taken from photographs.

This is a worthwhile experiment to perform in the heat transfer laboratory, and one of the few radiation experiments not found in commercially available apparatus.

#### IX. Acknowledgements

The authors wish to express their gratitude to Ms Shelley Dean for her help in obtaining data and in taking measurements.

## Bibliography

1. Earle, J. H., *Drafting Technology*, Addison-Wesley Pub. Co., 1982.
2. Woodworth, F., *Graphical Simulation*, International Textbook Co., 1967.
3. Hinkle, R. T., *Kinematics of Machines*, 2ed, Prentice-Hall, Inc., 1960.
4. Kreight, F., *Principles of Heat Transfer*, 2ed, International Textbook Co., 1969.
5. Janna, W. S., *Engineering Heat Transfer*, PWS Engineering, 1986.
6. Keith, T. G. and W. S. Janna, "Graphical Solution of Unsteady Viscous Flow Problems", *Int J Mech Engrg Educ*, 5, 2, 1977, ppg. 97--105.
7. Nusselt, W., "Graphische Bestimmung des Winkelverhältnisses bei der Wärmestrahlung", *VDI Z* 72, 1928, pg. 673.
8. Siegel, R. and J. R. Howell, *Thermal Radiation Heat Transfer*, McGraw-Hill Book Co., 1972.
9. Sparrow, E. M. and R. D. Cess, *Radiation Heat Transfer*, Hemisphere Publishing Co., 1978.
10. Alciatore, D., W. S. Janna and E. S. Shamburger, "Microcomputer Solution to the Mathematical Equivalent of the Graphical Method for Finding Shape Factors" presented at the *Engineering Design Graphics Session* of the ASEE Annual Winter Meeting, Austin, TX, January 1987, and published in *ASEE Journal of Engineering Design Graphics*, vol. 52, no. 1, Winter 1988, p. 45.
11. Alciatore, D., Stephen Lipp and W. S. Janna, "Closed Form Solution of the General Three Dimensional Radiation Configuration Factor Problem with Microcomputer Solution" presented by Professor C. Ray Wimberly at the 1989 National Heat Transfer Conference--Radiation Heat Transfer Session, August 6-9, 1989, Philadelphia, PA.
12. Hamilton, D. C. and W. R. Morgan, *Radiant-Interchange Configuration Factors*, NACA Tech Note 2836.

## AARON RENNER

Aaron Renner is currently a MS student in Mechanical Engineering at the University of Memphis. He received his BS degree in Mechanical Engineering also at the University of Memphis. He teaches the fluid mechanics laboratory course, and is interested in conducting research on reducing the drag characteristics of various wing shapes by using an air injection technique. He serves the profession as Chair of the student section of ASME.

## WILLIAM S. JANNA

William S. Janna is currently a Professor of Mechanical Engineering at the University of Memphis. He has written three textbooks, and conducts research in the area of phase change; melting, and sublimation. He is currently serving the ME Division of ASEE as webmaster and editor of Mechanical Engineering News (<http://www.me.memphis.edu/menews/menews.html>). He received BS, MS, and PhD degrees from the University of Toledo.