# AC 2010-842: A LABORATORY METHOD FOR TEACHING ANALOG-TO-DIGITAL AND DIGITAL-TO-ANALOG CONVERSION

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# A Laboratory Method for Teaching Analog-to-Digital and Digital-to-Analog Conversion

## Abstract

Since analog-to-digital converters (ADC's) and digital-to-analog converters (DAC's) are used in such a wide variety of electronic systems, it is important for engineers to have a deep understanding of the distortions caused by ADC's and DAC's such as aliasing and quantization, and to realize that the digital signal is not a perfect representation of the original analog signal. In fact, if the system is set up poorly, the digital signal can be corrupted enough that it is completely unusable. Furthermore, students are often curious how ADC's and DAC's work and benefit from building, testing, and using these important circuits.

This paper proposes a laboratory method for teaching the process of converting between analog and digital signals using circuits that students can construct in a laboratory. The ADC is a 3-bit flash converter made from comparators, a priority encoder, flip-flops, and buffers. The DAC is a simple R-2R resistor network. These circuits work well to demonstrate the effects of aliasing and quantization on various laboratory signals, such as D.C. signals and sinusoids. Furthermore, with the addition of an op-amp adder and an audio amplifier, the students can pass music from a portable music player through the system so they can hear the result, which is definitely recognizable, although it does have noticeable quantization noise.

The advantage of this method of teaching the conversion process is that the students can actively experiment with the system to see and hear the effects of aliasing and quantization in real time. They can measure quantization errors and see the distortion on an oscilloscope that occurs when the analog signal is too big or too small for the input range of the ADC. They can see and hear the distortion caused by aliasing while adjusting the input frequency and the sampling rate. Thus the experiment is more interesting and engaging than a purely theoretical presentation.

## Background

In many electronic systems, analog signals are processed using digital hardware, which makes it necessary to convert between analog and digital signals. Examples of such systems include audio and image processing systems, communication systems, control systems, and instrumentation systems, and many others. However, in practice an analog signal cannot be represented exactly in digital form, and so it is important that engineers that design and use such systems be aware of the distortions that are caused by ADC's and DAC's such as quantization error and aliasing.

Quantization error occurs because the amplitude of an analog signal is represented by a finite number of bits, which causes a range of analog signal values to be represented by a single binary code. This effect is equivalent to adding noise to the signal. The amount of quantization noise can be reduced by using more bits, but since the exact original signal level is not recorded, it is not possible to remove this noise completely. Furthermore, if the amplitude of the input analog signal is not scaled to match the range of the ADC, then the quantization error can be huge regardless of the number of bits the ADC uses.

Aliasing refers to the fact that the sample values of high frequency sinusoids (those above one half of the sampling rate) are identical to those of lower frequency sinusoids. Therefore, high frequency signals can be folded down into lower frequency bands. In most systems, this effect is usually kept to a tolerable level by using a lowpass filter before the ADC to attenuate frequencies that are higher than the range of interest, but this effect cannot be completely eliminated in practice because it requires an ideal lowpass filter.

In order to motivate students to master the theoretical descriptions of quantization and aliasing, it is useful to provide an opportunity for the students to see and hear these effects for themselves.

## A 3-Bit Flash ADC Circuit to Demonstrate Quantization

The ADC circuit shown in Figure 1 is a 3-bit flash ADC which is designed to demonstrate the effect of quantization. The ADC is set up so that input values near 0 Volts are converted to the binary number 000, the input values near 1 Volt are converted to the binary number 001, the input values near 2 Volts are converted to the binary number 010, and so on up to 7 Volts which is converted to 111. The output of this circuit changes immediately after the input crosses one of the levels between the ranges. This circuit does not sample the output at regularly spaced time intervals, and so it cannot be used to demonstrate aliasing. However the appropriate modification to this circuit to demonstrate aliasing is shown in another section in this paper.

There are several methods for converting analog signals to digital, but the flash ADC circuit is probably the easiest to understand, and it can be built with simple components[1,2]. The circuit shown in Figure 1 contains a 3-bit flash ADC that converts an analog input signal $V_{in}$ to a digital signal consisting of a 3-bit binary number. The bits are displayed on three LED's where a logic 1 is indicated by an LED that is lit. The 3-bit binary number is converted back to an analog signal $V_{out}$ by the 3-bit DAC. Although a high quality system such as audio CD uses 16 bits, a 3-bit circuit is sufficient to demonstrate quantization error and can reproduce an audio signal that is recognizable and useable, although the quantization noise will be audible.

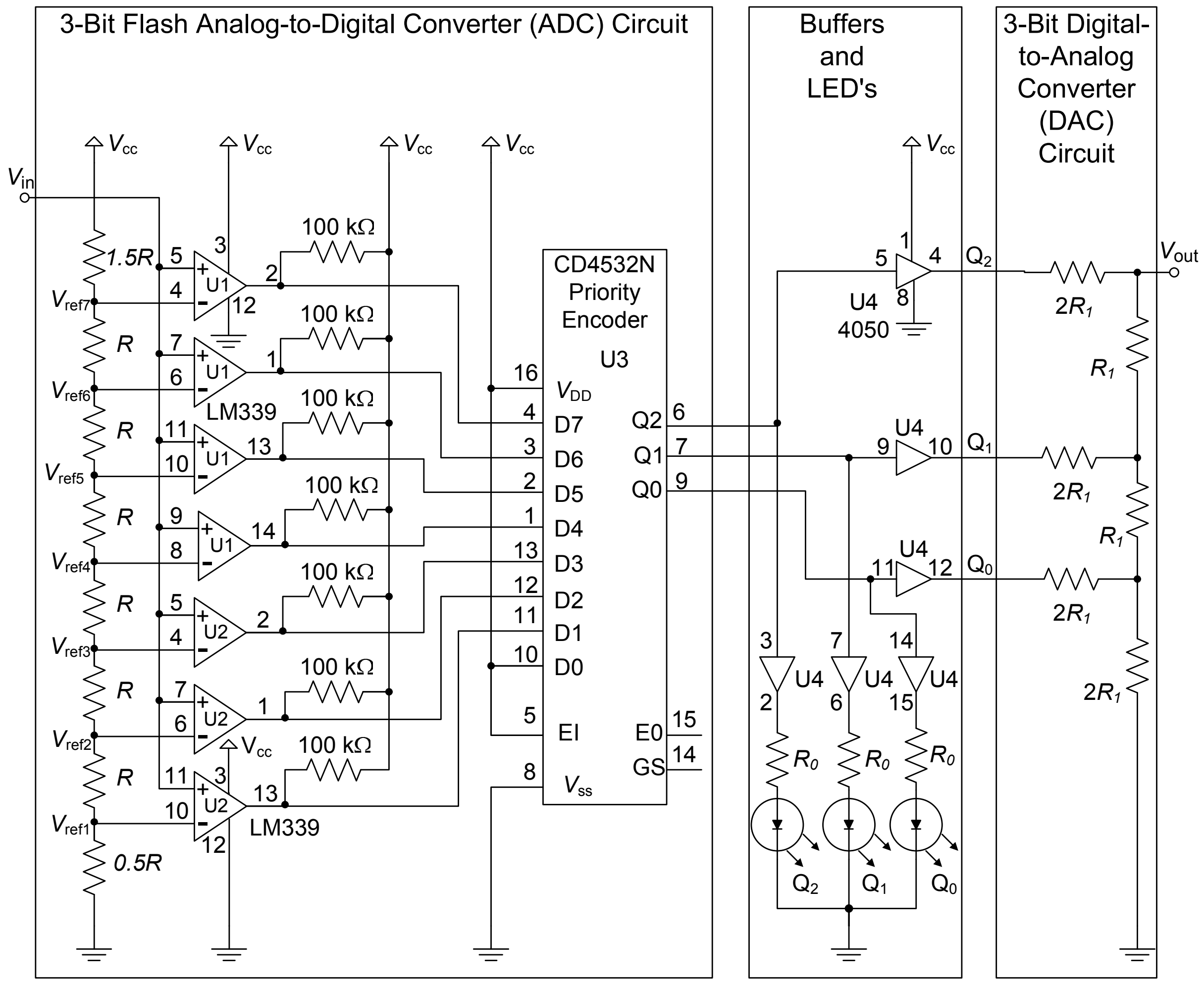

Figure 1: Circuit to Demonstrate Quantization Error

The operation of this circuit is relatively easy for students to understand. The power supply is set to $V_{cc}$ = 8 Volts. It can be shown (using voltage division) that the reference voltages generated by the resistor network on the left of Figure 1 are equal to the values shown in Table 1 regardless of the value of R. However if R is too small the circuit will use excessive power and if R is too big the comparator inputs will load down the circuit causing the reference voltages to drop. The circuit works well with R = 1 kΩ.

Table 1. Reference Voltage Levels

| Reference Levels | Volts |
|---|---|
| $V_{ref1}$ | 0.5 |
| $V_{ref2}$ | 1.5 |
| $V_{ref3}$ | 2.5 |
| $V_{ref4}$ | 3.5 |
| $V_{ref5}$ | 4.5 |
| $V_{ref6}$ | 5.5 |
| $V_{ref7}$ | 6.5 |

Each of the comparators compares the input voltage $V_{in}$ to one of the reference voltages, and if $V_{in}$ is greater than the reference voltage then the comparator's output is 8 Volts (logic 1), otherwise it is 0 Volts (logic 0). So for example, if $V_{in} < 0.5$ Volts, then $V_{in}$ is smaller than all of the reference voltages, so all of the comparator outputs will be logic 0. If $0.5 < V_{in} < 1.5$ Volts, then the output of the comparator at the bottom of Figure 1 will be logic 1, and the output of the rest of the comparators will be logic 0. If $1.5 < V_{in} < 2.5$ Volts, then the output of the two bottom comparators will be logic 1, and the output of the rest of the comparators will be logic 0 (see Table 2). The 100 kΩ resistors connected to the comparator outputs are pull-up resistors that are required because the LM339 comparators have open collector outputs.

Table 2. Input Voltage and Output of Comparators, Priority Encoder, and DAC.

| Analog Input $V_{in}$ (Volts) | Comparators Output/ Priority Encoder Input $D_7D_6D_5D_4D_3D_2D_1D_0$ | Priority Encoder Output $Q_2Q_1Q_0$ | Analog Output $V_{out}$ (Volts) |
|---|---|---|---|
| $V_{in}<0.5$ | 0 0 0 0 0 0 0 1 | 0 0 0 | 0.0 |
| $0.5< V_{in}<1.5$ | 0 0 0 0 0 0 1 1 | 0 0 1 | 1.0 |
| $1.5< V_{in}<2.5$ | 0 0 0 0 0 1 1 1 | 0 1 0 | 2.0 |
| $2.5< V_{in}<3.5$ | 0 0 0 0 1 1 1 1 | 0 1 1 | 3.0 |
| $3.5< V_{in}<4.5$ | 0 0 0 1 1 1 1 1 | 1 0 0 | 4.0 |
| $4.5< V_{in}<5.5$ | 0 0 1 1 1 1 1 1 | 1 0 1 | 5.0 |
| $5.5< V_{in}<6.5$ | 0 1 1 1 1 1 1 1 | 1 1 0 | 6.0 |
| $V_{in}>6.5$ | 1 1 1 1 1 1 1 1 | 1 1 1 | 7.0 |

The logic function required to convert the output of the seven comparators into the appropriate 3-bit binary number is performed by the priority encoder. The priority encoder generates a 3-bit binary number that represents the highest numbered input $D_i$ that has a logic 1 (see Table 2). The input $D_1$ is connected directly to logic 1. The output of the priority encoder is a 3-bit unsigned binary number that indicates which range the input voltage is in, and this binary number can be displayed on the LED's.

## Buffers and LED's

The outputs of the priority encoder cannot deliver much current, so buffers are used to drive the LED's and the DAC circuit. The output of the ADC can be displayed on LED's so that the students can easily verify the circuit's operation. The value of $R_0$ is chosen to limit the current through the LED's to a safe value. A typical value is $R_0 = 1\ \text{k}\Omega$.

## A 3-Bit DAC Circuit

In order to demonstrate that quantization changes the value of the signal, the output of the ADC is converted back to an analog voltage by a R-2R resistor network that forms a 3-bit DAC circuit as shown in Figure 1. It can be shown (using source transformation) that the output $V_{out}$ is given by the following equation.

$$V_{out} = 0.5Q_2 + 0.25Q_1 + 0.125Q_0$$

The value of $R_1$ drops out of the equation, but if $R_1$ is too small the circuit loads down the buffer outputs and if $R_1$ is too big then anything connected to the output $V_{out}$ loads down the output voltage. The value of $R_1 = 10\ \text{k}\Omega$ works well in this circuit.

The DAC input voltages $Q_2$, $Q_1$, and $Q_0$ are approximately 8 Volts for logic 1 or 0 Volts for logic 0, which causes the output voltage to be the equivalent value as the three bit unsigned binary number $Q_2Q_1Q_0$. For example, if $Q_2$ is 0 volts, and $Q_1$ and $Q_0$ are both 8 volts, then the equivalent unsigned binary number is $011_2$ which is equivalent to the decimal number $3_{10}$, and this causes the output to be $V_{out} = 3$ Volts (see Table 2). In practical DAC's, the output voltage is proportional to the equivalent binary value, but they usually are not set up to have the output voltage to be equal to the equivalent value since that would required generating very large voltages. However, having this equivalence makes verifying that the circuit is operating correctly much easier for the students.

## Lab Experiment

The circuit in Figure 1 has been used in the sophomore level circuits laboratory at the author's institution for about 5 years. In addition to demonstrating ADC and DAC circuits, it is used as an example circuit to practice D.C. analysis techniques. The students calculate the value of the reference voltages $V_{ref1}$ through $V_{ref7}$, determine the output of the comparators and priority encoder for each range, and calculate the output voltage $V_{out}$ as shown in Table 2.

The students then use a D.C. voltage as the input to the circuit to verify that the circuit works and to demonstrate the operation of the circuit. A potentiometer such as the one shown in Figure 2 can be used to generate any input voltage between 0 and 8 Volts, or alternatively a power supply can be employed. The students can verify the operation of the circuit by observing the output of the circuit for each of the input voltage ranges in Table 2. This exercise demonstrates that the exact value of the input is lost due to quantization, and the ADC generates only the approximate value of the input signal. The students can also observe that the maximum error is one half of the distance between the steps, or 0.5 Volts for this ADC. The maximum error can be reduced by adding more steps, which requires more bits.

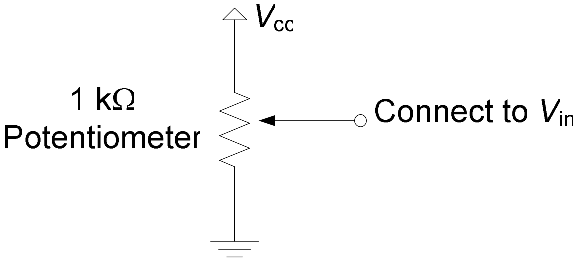

Figure 2: Circuit to Generate a D.C. Input Signal

A sinusoidal input can also be used to demonstrate how the signal is distorted if the input analog signal is not properly scaled for the ADC input range. A function generator can be connected to $V_{in}$ instead of the potentiometer, and an oscilloscope can be used to display the input and output voltages. If the function generator is set to generate an input that is properly scaled, which in this case would be between 0 and 7.5 Volts, then the output $V_{out}$ is a stair step approximation to the input as shown in Figure 3. In Figure 3 (and all of the following oscilloscope displays) the input $V_{in}$ is on channel 1 displayed with a blue line, and the output $V_{out}$ is on channel 2 displayed with a red line, and the display is adjusted so that zero volts is on the bottom line. The vertical scale of each channel is displayed on the bottom of the figure after Ch1 and Ch2. Since the vertical scale for both channels is 1 volt per division in Figure 3, the top line is 8 volts. Note that the output is a stair step approximation to the input.

If the input signal is not scaled properly, the maximum error caused by the ADC can be very large. If the input goes up to 12 Volts, for example, the output is limited to the maximum value of 7 Volts, and the top of the signal is clipped as shown in Figure 4. In Figure 4 the vertical scale is 2 volt per division, and so the top line is 16 volts. If the input is negative, it is clipped as well.

If the input signal is too small, the amount of quantization error can be large compared to the signal and severe distortion can result even if the ADC has a large number of bits. For example, the sinusoidal input signal in Figure 5 (with vertical scale of 1 volt per division) is small enough that it only crosses two levels, and so the input sinusoid signal is converted to a square wave output. In the extreme case, if the input is so small that it never crosses a level, it will be converted to a constant value, and the signal is completely lost as shown in Figure 6. This circuit can help students understand that the input analog signal needs to be properly scaled when setting up an ADC or the resulting quantization error can be excessive regardless of how many bits the ADC uses.

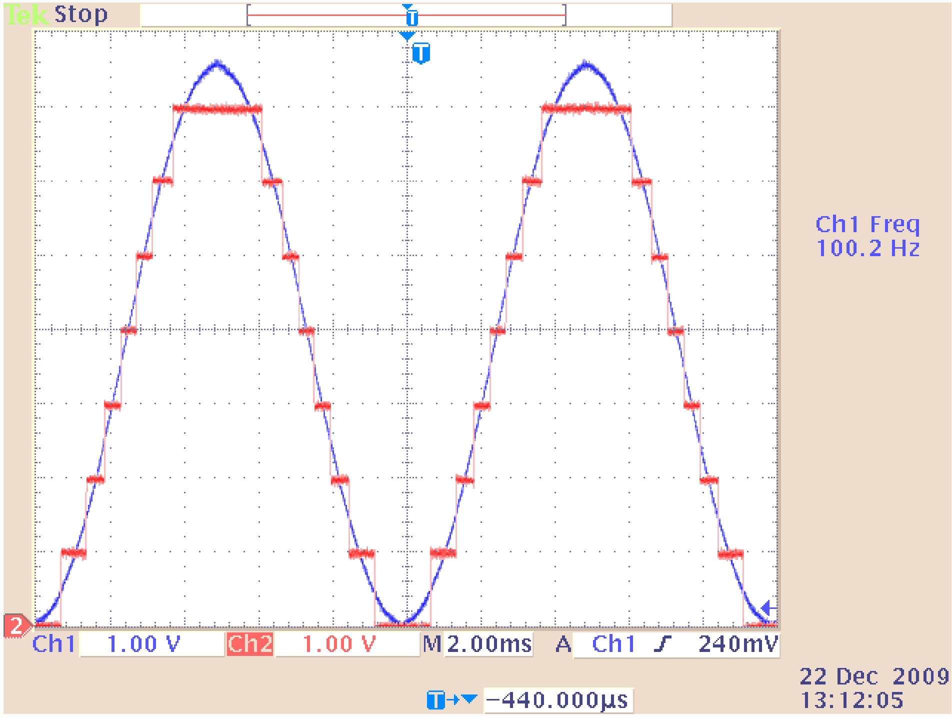

Figure 3: Effect of Quantization (with Input Properly Scaled)

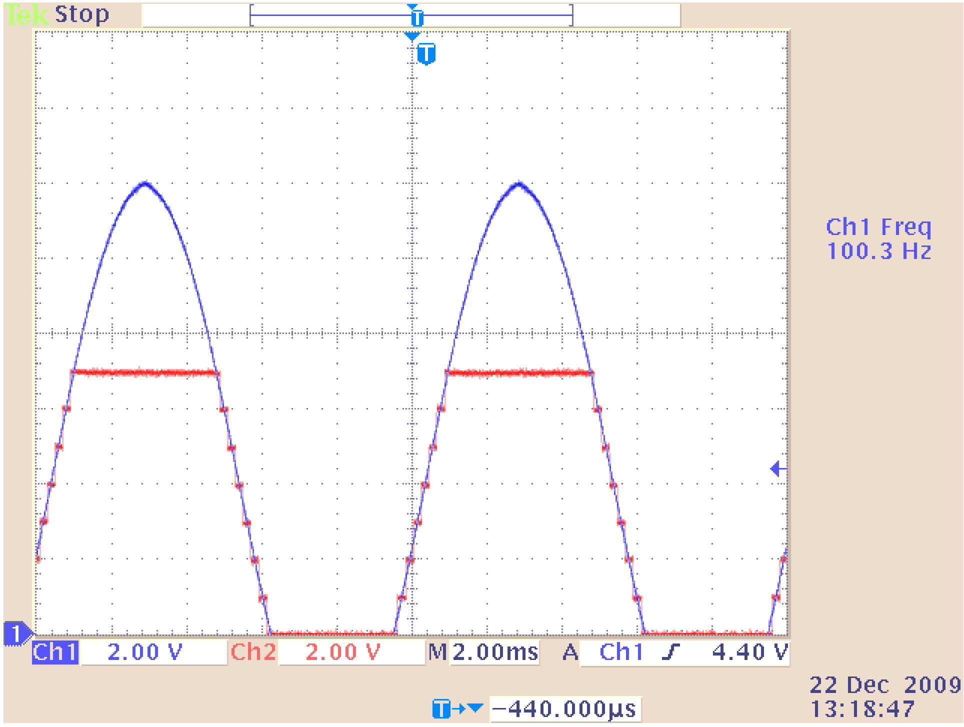

Figure 4: Clipping Occurs if the Analog Input is Too Big

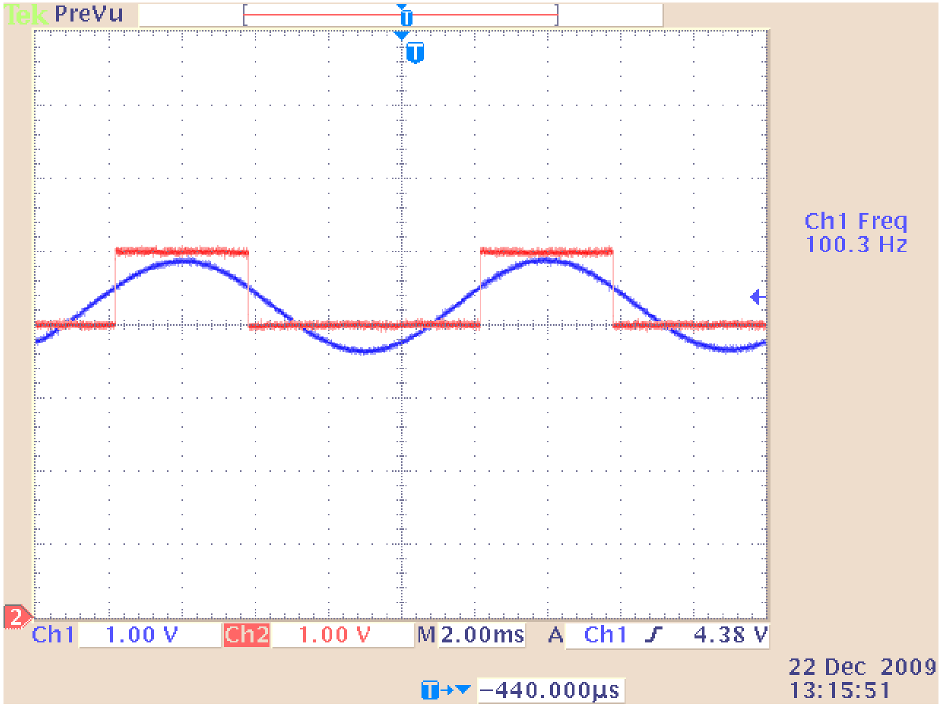

Figure 5: Severe Distortion Occurs if the Analog Input is Too Small

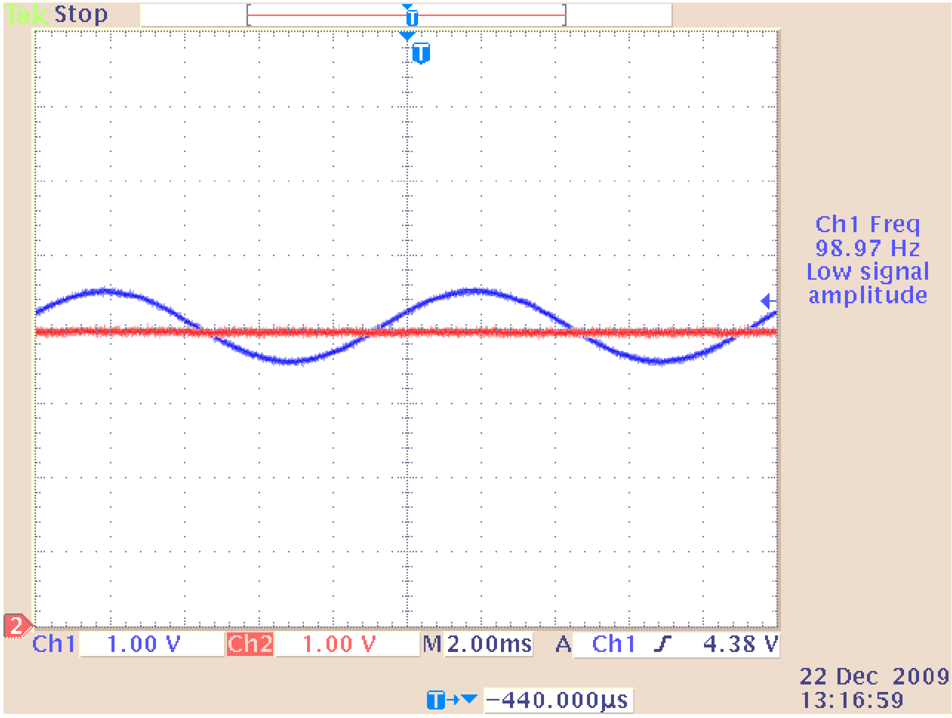

Figure 6: The Signal is Completely Lost if the Analog Input is Much Too Small

## ADC Circuit to Demonstrate Aliasing

The sample values of high frequency sinusoids taken at regularly spaced time intervals are identical to lower frequency sinusoids, which can lead to an effect called aliasing[3]. In order to demonstrate this effect, it is necessary to modify the ADC so that the output is captured at regular intervals. The circuit in Figure 7 is identical to that in Figure 1 except for the addition of the Flip Flops and Clock Circuits section in between the ADC section and the Buffers and LED's section. The D flip-flops capture the output of the priority encoder at regular intervals that are determined by the clock circuit based on the 555 timer. The sampling frequency in Hz is determined by the values of $R_2$, $R_3$, and C according to the following equation[1].

$$f_s = \frac{1}{0.693(R_2 + 2R_3)C}$$

For example, with $R_2 = 3.3$ kΩ, $R_3 = 6.6$ kΩ, and C = 0.1 μF, the sampling rate will be $f_s =$ 874.5 Hz, which means that the output of the priority encoder will be captured by the D flip flops 874.5 times a second as shown in Figure 8.

If the input frequency is increased to a value greater than one half of the sampling rate, then aliasing occurs which causes the output signal to have a lower frequency than the input signal as shown in Figure 9 where the input frequency is 812.9 Hz, but the output frequency is about 100 Hz.

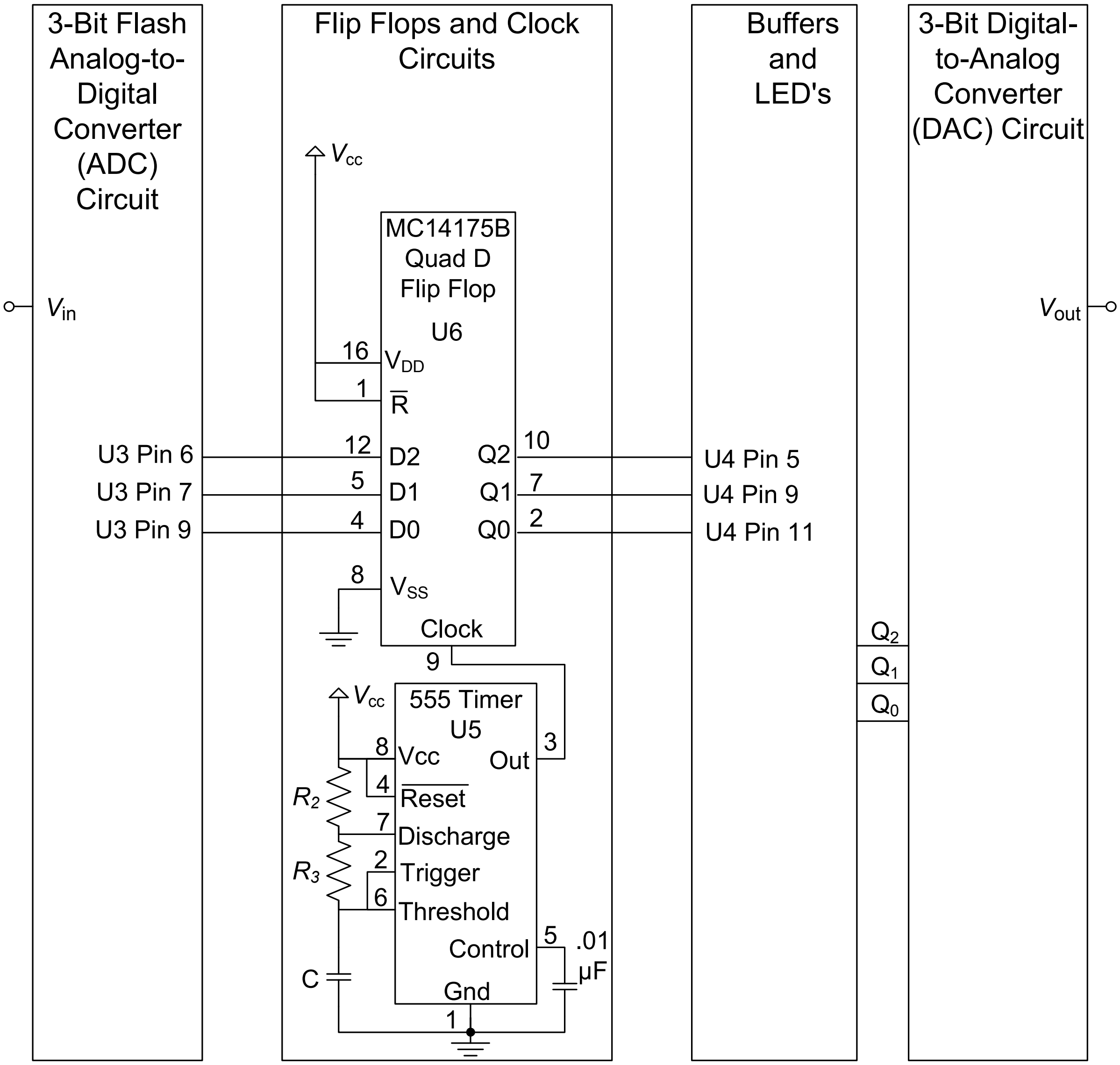

Figure 7: Circuit to Demonstrate Aliasing and Quantization

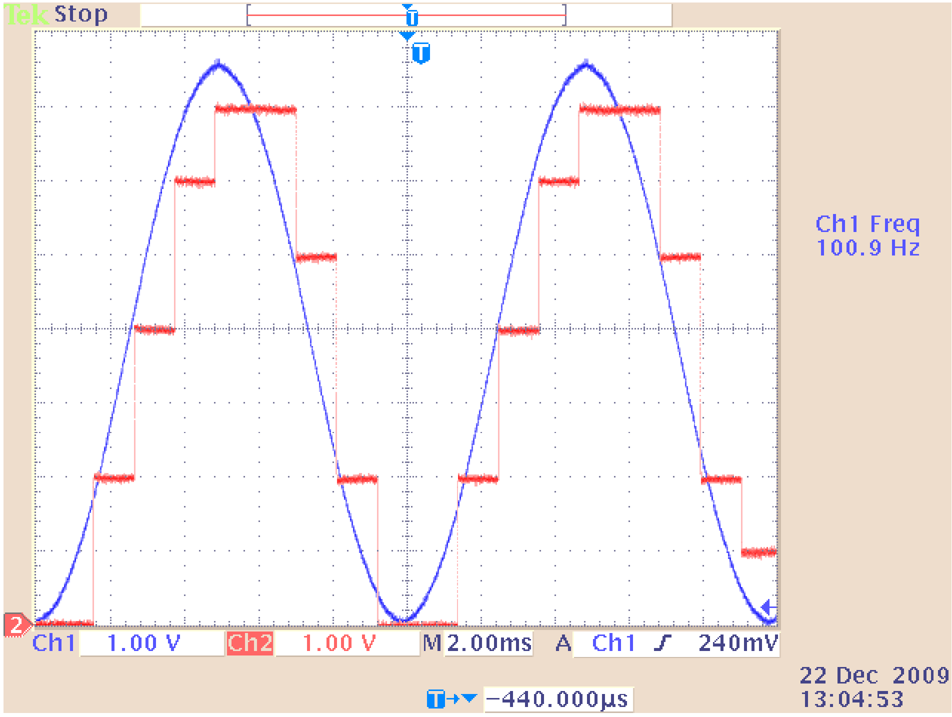

Figure 8: No Aliasing Occurs if the Input Frequency is Low Enough

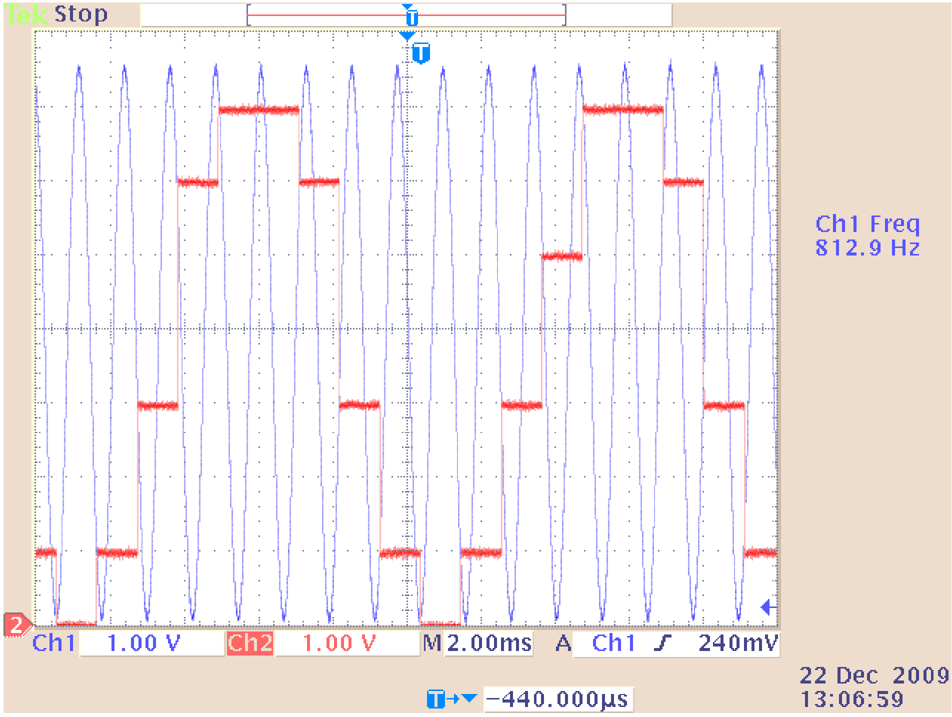

Figure 9: Aliasing Occurs if the Input Frequency is Too High

### Audio Demonstration

It is very interesting to hear the results of the ADC and DAC on audio signals. Since CD quality audio uses 16 bits per sample, one might expect very poor audio quality with only a 3-bit ADC and DAC. However, the circuit in Figure 10 can be used to pass speech or music through the 3-bit ADC and DAC to allow students to hear for themselves how quantization and aliasing affect the quality of a signal.

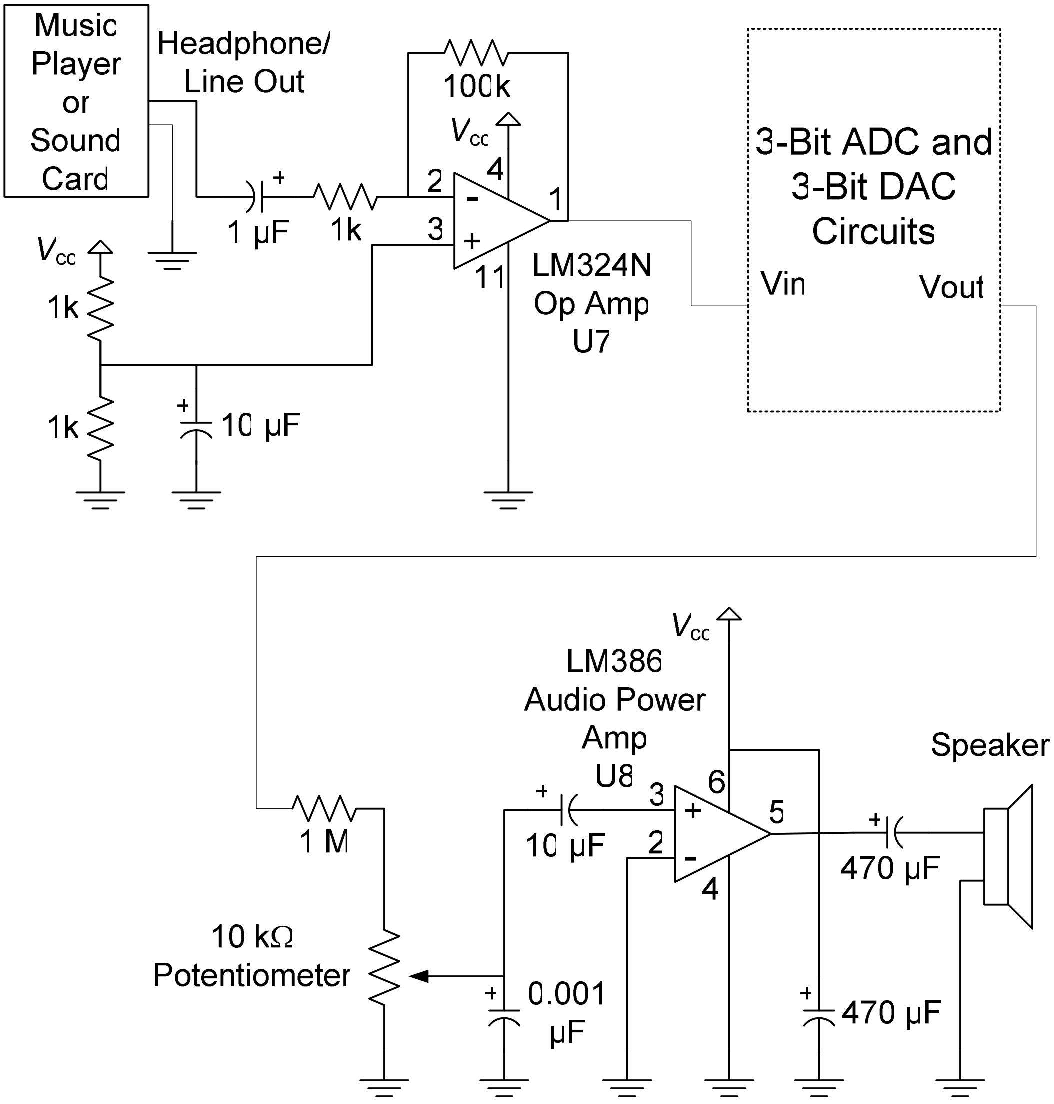

Figure 10: Circuit for Audio Demonstration

In order to avoid clipping the negative part of an audio signal, the headphone output of a music player or a computer's sound card needs to be shifted to be centered in the center of the ADC's input range. The signal also needs to be amplified to cover most of the ADC's range to get reasonable quality. The op amp in Figure 10 amplifies the line level output and shifts it to be centered around 4 Volts. Then the ADC and DAC circuits convert the signal to digital and back to analog. The potentiometer in Figure 10 is used to adjust the volume of the output and the audio amplifier is used to amplify the signal to provide enough current to drive an 8 Ω speaker.

If the circuit in Figure 1 is used with the circuit in Figure 10, then only quantization, and not aliasing, will result. If the circuit in Figure 7 is used with the circuit in Figure 10, then both quantization and aliasing will result, and the aliasing distortion will be quite severe if a low sampling frequency such as 874.5 Hz is used.

If the volume of the music player is adjusted so that the signal $V_{in}$ covers the whole range of the ADC (0 to 7.5 Volts), and if the sampling rate is high enough (or the circuit in Figure 1 is used), then the quality of the resulting quantized signal is reasonably good. Music and speech are definitely recognizable, although the quantization noise is audible.

Another interesting way to demonstrate aliasing is to disconnect the op amp circuit from $V_{in}$ in Figure 10, and instead connect a function generator which is set to generate sinusoids. Then using the ADC with D flip flops in Figure 7 with the sampling rate set to a frequency around 874.5 Hz, listen to the output while the input frequency is slowly increased. The perceived frequency of the output increases until it gets to about half of the sampling rate, and then it decreases because of the effects of aliasing. If a spectrum analyzer is available that covers the audio band, it is also very instructive to watch the spectrum of the signal while performing this experiment to see the aliasing components of the output signal $V_{out}$ while changing the input frequency and listening to the output.

## Conclusion

The circuits presented in this paper are designed to demonstrate ADC and DAC circuits. They allow the students to directly measure the error caused by quantization with a digital multi-meter, and they allow the students to see the distortion caused by quantization and aliasing on an oscilloscope and hear the distortion through speakers. Working with these circuits helps reinforce the theory of ADC's and DAC's with hands-on experience.

## References

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