

2006-44: A MATHCAD FUNCTION SET FOR SOLVING THERMODYNAMICS PROBLEMS

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A MathCAD Function Set for Solving Thermodynamics Problems

Abstract

A set of MathCAD functions was constructed to evaluate the thermodynamic properties of steam, R-134a, air, and twelve ideal gases. The functions were constructed to ease the need for time-consuming interpolation using tabularized thermodynamic data while reinforcing the functional representation of traditional thermodynamic property tables. A consistent scheme was adopted for naming the functions. Three examples are provided to demonstrate the use of the functions in an undergraduate applied thermodynamics course. The examples include the design of a reversed-Rankine refrigeration cycle, the analysis of a turbojet Brayton cycle, and the selection of a reheat pressure that maximizes the efficiency of a Rankine-reheat power cycle provided fixed boiler and condenser pressures. The examples are presented in their entirety in appendices to demonstrate the report-quality worksheets possible using MathCAD. While the examples provided focus on the use of the functions in an undergraduate applied thermodynamics course, the functions have also been used in an internal combustion engines course and in a turbomachinery course. Survey results from three courses and several student comments are discussed.

Introduction

Learning with a combination of a textbook and a software package is a contemporary engineering-thermodynamics pedagogy. Many software tools are available for evaluating thermodynamic properties of engineering fluids. Many of these software tools are proprietary packages sold by textbook publishers, such as “Interactive Thermodynamics: IT” [1]. In fact, finding a thermodynamics text that does not come with a software package is difficult. Some textbooks are now built around using a software or web-based internet package [2]. While many educational software packages are available for evaluating thermodynamic properties, evidence that shows that practicing engineers continue to use these thermodynamic-property software packages after entering the workforce is not readily available.


MathCAD, MatLab, and Engineering Equation Solver (EES) are all powerful computational and analytical packages [3,4,5]. Many schools teach and require the use of a computational tool such as MathCAD, MatLab, or EES [6]. From informal conversations with engineers who learned to use one of these computational tools, many of them continue to use these tools after graduation. Developing extensions or toolkits for software that the students will use after graduation seems more appropriate than developing complete software packages that will only be used by students in an educational environment. Because of the need for thermodynamic property functions for the widely used computational tools, functions were generated to evaluate the thermodynamic properties of water, R-134a, air, and twelve species of the CHON system in MathCAD. Equilibrium functions for eight independent reactions involving species of the CHON system are also included. The function set created provides the minimum number of thermodynamic functions required to teach a two-course sequence in undergraduate engineering thermodynamics.

Each computational analysis package has strengths and weaknesses when compared to the others. EES also has the thermodynamic functions discussed here (and functions for many other fluids). While EES is not a traditional structured programming language, the appearance of EES programs is similar to the appearance of C or FORTRAN programs, and some users find the unit conversion procedures awkward in EES. MathCAD was chosen for this project because of its mathematical report appearance, because of its ability to perform calculations with automatic unit handling and conversion, and because of its wide use in the Department of Mechanical Engineering at the University of Alabama at Birmingham.

While developed for a two-course sequence in undergraduate thermodynamics, the function sets are useful for many other engineering courses. In fact, this effort started in an ME 448/548 Internal Combustion Engines course [7,8]. Undergraduates in thermodynamics based courses often become frustrated using property tables and fail to comprehend either the material or the significance of the material. As the case with all thermodynamic property evaluation software, the intent was 1) to take some of the effort and distraction away from working problems requiring extensive interpolation in the tables and 2) to allow the students to analyze more complicated problems.

Referencing the Function Sets in MathCAD

The functions are contained in three MathCAD worksheets. Table 1 presents the files and the information contained in each file. To use the functions in a new MathCAD worksheet, the information in the property worksheets does not have to be copied into the new worksheet. The function worksheet may be referenced by using the <Insert, Reference> command, and identifying the file required. When this is done correctly, a statement similar to

 Reference:C:\ThermoII\SteamProps.mcd

will appear in the worksheet. All functions generated in referenced file will then be available for use in the new worksheet.

All of the functions generated are available to the educational public on the condition that if the functions are used for instruction, the instructor agrees to have his students complete a survey form at the end of the course. The function sets may be obtained by emailing the author at smcclain@uab.edu. Please include the subject line “MathCAD: Thermo Function Set Request” in your email.

Table 1. Property Function Files

File	Information Contained
<i>SteamProps.mcd</i>	Thermodynamic properties of water
<i>R134aProps.mcd</i>	Thermodynamic properties of R-134a
<i>IGASProps.mcd</i>	Thermodynamic ideal gas properties of CO, CO ₂ , H ₂ , H, OH, H ₂ O, N ₂ , N, NO, NO ₂ , O, O ₂ , and dry air and equilibrium constants for eight independent reactions

Water and R-134a

A consistent naming scheme was chosen for the phase change fluids. The function calls for water and R-134a begin with the variable to be determined, followed by an underscore, followed by the independent property(s), followed by a fluid identifier (H2O or R134a), followed finally by the values of the independent property(s) in parenthesis. Figure 1 presents the format of function calls for the phase change substances. The functions require that all independent properties be entered with the appropriate absolute units.

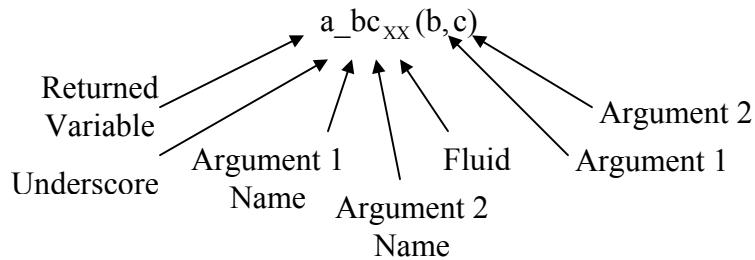


Figure 1. Scheme for Naming Property Functions

The functions were constructed to reflect and reinforce how property information is provided in traditional property tables. Functions are provided for the saturation pressure based on temperature and for the saturation temperature based on pressure. Saturated liquid and saturated vapor functions are provided for specific volume (v), specific internal energy (u), specific enthalpy (h), and specific entropy (s) based on either temperature or pressure. Functions for v , u , h , and s of saturated mixtures are also provided as functions of quality and either temperature or pressure. Functions that return quality are provided. Functions that return temperature or enthalpy as functions of pressure and entropy, often required in the analyses of Rankine and reversed-Rankine cycles, are provided. Finally, a function that determines temperature based on pressure and enthalpy, often used when analyzing reversed-Rankine cycles, is provided. Table 2 presents a summary of the functions provided for the phase-change substances.

The property functions for water were created using the IAPWS 97 formulation [9]. The IAPWS formulation presents fundamental property equations for five regions. The functions provided are for Region 1 (compressed liquid below 623.15 K), Region 2 (superheated vapor), and Region 4 (saturated mixture below 623.15 K). Functions for Region 3 (near and above the critical point) or Region 5 (low pressure very high temperature vapor) were not created but may be added at a later time.

Table 2. Functions for the Thermodynamic Properties of Water and R-134a

Function or Constant	Description
Tsat_P _{XX} (P)	Returns the saturation temperature as a function of pressure
Psat_T _{XX} (T)	Returns the saturation pressure as a functions of temperature
zf_T _{XX} (T)	Returns the z property ($v, u, h, \text{ or } s$) for saturated liquid as a function of temperature
zf_P _{XX} (P)	Returns the z property ($v, u, h, \text{ or } s$) for saturated liquid as a function of pressure
zg_T _{XX} (T)	Returns the z property ($v, u, h, \text{ or } s$) for saturated vapor as a function of temperature
zg_P _{XX} (P)	Returns the z property ($v, u, h, \text{ or } s$) for saturated vapor as a function of pressure
z_T _{XX} (T,x)	Returns the z property ($v, u, h, \text{ or } s$) for a saturated mixture as a function of temperature and quality
z_P _{XX} (P,x)	Returns the z property ($v, u, h, \text{ or } s$) for a saturated mixture as a function of pressure and quality
z_TP _{XX} (T,P)	Returns the z property ($v, u, h, \text{ or } s$) for either a compressed liquid or a superheated vapor as a function of temperature and pressure
x_T _{ZXX} (T,z)	Returns the quality of a saturated mixture as a function of the temperature and the z property ($v, u, h, \text{ or } s$)
x_P _{ZXX} (T,z)	Returns the quality of a saturated mixture as a function of the temperature and the z property ($v, u, h, \text{ or } s$)
T_P _{sXX} (P,s)	Returns the temperature as a function of pressure and entropy
h_P _{sXX} (P,s)	Returns the enthalpy as a function of pressure and entropy
T_P _{hXX} (P,h)	Returns the temperature as a function of pressure and enthalpy

The reader should note that *all* functions presented in this paper are for educational use only, and that the user assumes all responsibility for calculations performed using the functions if used for an industrial application. For the steam functions in particular, the function results have been compared to steam table values; however, the functions have not been verified to the level required by the IAPWS 97 formulation for industrial use. If verified steam functions for MathCAD are required, Mathsoft no longer supports the steam functions that were once available in a MathCAD extension pack. Fortunately, steam functions from third-part software companies are available [10].

The property functions for R-134a are based on the equation of state proposed by Tillner-Roth and Baehr [11]. The functions are valid for temperatures from 170 K to 455 K at pressures up to 70 MPa. Because the functions are based on the formulation provided by Tillner-Roth and Baehr, energy and entropy values reported by the functions differ from the tabulated values reported in some undergraduate engineering thermodynamics texts that are based on the ASHRAE formulation [12].

Air and Ideal Gases

The data used to create the functions for ideal gases and dry air came from “The Chemkin Thermodynamic Data Base” as reported by Turns [12]. Turns reports fourteen constants used to determine thermodynamic data for twelve species (CO, CO₂, H₂, H, OH, H₂O, N₂, N, NO, NO₂, O, O₂) of the carbon-hydrogen-oxygen-nitrogen (CHON) system as a function of temperature. The fourteen constants for each species are used to determine thermodynamic properties that are valid for the temperature range of 300 K to 5000 K. Table 3 summarizes the constants and functions available for the twelve ideal gases.

Table 3. Functions for the Thermodynamic Properties of Ideal Gases

Function or Constant	Description
M_{XX}	The molecular weight of the XX species.
R_u	The universal gas constant, 8.314 kJ/(kmol·K)
R_{XX}	The gas constant of the XX species.
$hm_{XX}(T)$	Returns the molar specific enthalpy of the XX species as a function of temperature.
$sm_{XX}(T)$	Returns the molar specific entropy of the XX species as a function of temperature.
$um_{XX}(T)$	Returns the molar specific internal energy of the XX species as a function of temperature.
$\mu m_{XX}(T)$	Returns the molar specific Gibbs free energy of the XX species as a function of temperature.
$cpm_{XX}(T)$	Returns the molar specific heat at constant pressure of the XX species as a function of temperature.
$cvm_{XX}(T)$	Returns the molar specific heat at constant volume of the XX species as a function of temperature.

The functions for ideal gases follow a slightly different naming scheme than used for the phase change substances. Since properties of ideal gases are functions only of temperature, the underscore and independent property identifiers found in the function names of the phase change substances are not present. An “m” follows the returned property variable in the ideal gas function names to remind the user that the functions return molar specific properties.

The ideal gas functions were used to generate the thermodynamic functions for air using a composition of 79% N₂ and 21% O₂. The specific internal energy, the specific enthalpy, and the standardized specific entropy were generated as functions of temperature. The reference values of the properties at 298 K were added so that the values reported by the functions equaled the values found in traditional ideal-gas air tables [13]. The functions were created on a gravimetric basis for the same reason. Along with the functions for internal energy, enthalpy, and entropy, functions were also created for the relative pressure and relative volume. The inverse functions, which find temperature from the other air properties, are also available in the worksheet. Table 4 presents the available ideal gas functions for air.

Table 4. Functions for the Thermodynamic Properties of Air

Function	Description
$h_{T_{\text{air}}}(T)$	Returns the enthalpy of dry air as a function of temperature.
$u_{T_{\text{air}}}(T)$	Returns the internal energy of dry air as a function of temperature.
$s_{T_{\text{air}}}(T)$	Returns the standardized specific entropy of dry air as a function of temperature with $P = 1$ atm.
$pr_{T_{\text{air}}}(T)$	Returns the relative pressure of dry air as a function of temperature.
$vr_{T_{\text{air}}}(T)$	Returns the relative volume of dry air as a function of temperature.
$T_{h_{\text{air}}}(h)$	Returns the temperature of dry air as a function of enthalpy.
$T_{u_{\text{air}}}(u)$	Returns the temperature of dry air as a function of internal energy.
$T_{s_{\text{air}}}(s)$	Returns the temperature of dry air as a function of standardized specific entropy.
$T_{pr_{\text{air}}}(pr)$	Returns the temperature of dry air as a function of relative pressure.
$T_{vr_{\text{air}}}(vr)$	Returns the temperature of dry air as a function of relative volume.
$h_{pr_{\text{air}}}(pr)$	Returns the enthalpy of dry air as a function of relative pressure.
$h_{vr_{\text{air}}}(vr)$	Returns the enthalpy of dry air as a function of relative volume.
$u_{pr_{\text{air}}}(pr)$	Returns the internal energy of dry air as a function of relative pressure.
$u_{vr_{\text{air}}}(vr)$	Returns the internal energy of dry air as a function of relative volume.

Equilibrium Constants

Chemical equilibrium functions are also included in the function set. Equilibrium constant functions were generated for eight independent reactions of the CHON system. Those eight reactions are:

- I. $H_2 \Leftrightarrow 2H$
- II. $O_2 \Leftrightarrow 2O$
- III. $N_2 \Leftrightarrow 2N$
- IV. $H_2 + \frac{1}{2}O_2 \Leftrightarrow H_2O$
- V. $2H_2O \Leftrightarrow H_2 + 2OH$
- VI. $N_2 + O_2 \Leftrightarrow 2NO$
- VII. $CO_2 \Leftrightarrow CO + \frac{1}{2}O_2$
- VIII. $CO_2 + H_2 \Leftrightarrow CO + H_2O$

The functions for the equilibrium constant are called using “Kp_{YY}(T)”, where “YY” represents the roman numeral listed for each reaction. For example, the equilibrium constant for $CO_2 + H_2 \Leftrightarrow CO + H_2O$ at 4500 K is found in MathCAD using the statement:

$$K_{p_{\text{VII}}}(4500\text{K}) = 8.932$$

The equilibrium constant functions were validated using data from the *JANAF Thermochemical Tables* as reported by Russell and Adebisi [12].

Examples

Three example problems are provided demonstrating the use of the function sets in the analysis of thermodynamic cycles. The examples provided include 1) the design of a reversed-Rankine cycle for refrigeration, 2) the optimization of a Rankine power cycle with reheat, and 3) the analysis of a Brayton turbojet cycle. The examples demonstrate how the functions relieve the need for tedious or repetitive interpolation in the thermodynamic property tables. The full solutions of the example problems are provided in the appendix to demonstrate how the functions are not just property evaluators. Because of the format of MathCAD, the functions are used to create report-quality worksheets that may be submitted by the students as homework solutions.

Design of a Reversed-Rankine Cycle

Problem Statement: A residential refrigeration system must maintain a room temperature of 65 F when the outside temperature is a maximum of 110 F. Design a reversed-Rankine cycle, refrigeration system using R-134a as the refrigerant to cool the space. If the system is to be a 4 ton unit, what is the mass flow rate of refrigerant? What is the second law efficiency of the system?

The complete MathCAD solution for this example is presented in Appendix A. The solution demonstrates the purpose of all thermodynamic property evaluation software: to ease the need for multiple or double interpolation using the tables. In refrigeration system design, setting the phase-change temperature of the refrigerant in the condenser based on the temperature of the high temperature sink is common practice. In this case the minimum temperature in the condenser is set to 10 F above the maximum outside temperature. Setting the minimum condenser temperature fixes the condenser pressure for an ideal refrigeration cycle.

When the minimum condenser temperature is an even or rounded number, the resulting condenser pressure is not usually a pressure for which superheated vapor tables are provided in most textbooks. Thus, evaluating the properties of the refrigerant exiting the compressor (using the condenser pressure and the entropy of the refrigerant entering the compressor) requires double interpolation. As demonstrated in Appendix A, the R-134a functions are easily used to evaluate the states of the refrigeration cycle with un-rounded condenser and evaporator pressures.

The reversed-Rankine cycle also demonstrates MathCAD's ability to handle user-defined units. The unit of heat transfer "ton_c" which equals 12,000 BTU/hr is not an automatically recognized unit in the version of MathCAD used to create the example. However, the any new unit may be defined in MathCAD in terms of units that are recognized by MathCAD. Appendix A demonstrates how a new unit may be defined and used in any MathCAD worksheet.

Optimization of a Rankine Power Cycle with Reheat

Problem Statement: A Rankine power cycle with reheat operates between a boiler pressure of 1000 psia, a maximum cycle temperature of 1260 °R, and a condenser pressure of 2 psia. Steam exits the reheat boiler at 1260 °R. By varying the reheat boiler pressure between 5 psia and 900 psia, determine the reheat boiler pressure that maximizes the cycle efficiency and the resulting maximum efficiency.

The complete MathCAD solution for this example is presented in Appendix B. The solution to this example demonstrates the use of the property functions for steam and demonstrates how MathCAD is used to increment or “sweep” variables.

A reheat pressure exists between the main boiler pressure and condenser pressure that maximizes the cycle efficiency. The problem statement indicates the optimum reheat pressure is to be found by “sweeping” the reheat pressure between 5 psia and 900 psia. In MathCAD a variable with a range of values may be created using a counting variable and creating an array using the counting variable as the index. In the MathCAD solution, the variable “i” is defined as a variable ranging from 0 to N_d , where N_d is the number of increments desired for the sweep. The range of reheat pressure values is then constructed by assigning the reheat pressure, P_{rh} , a counting subscript and defining each pressure as a function of the counting variable. The range variable construction statement appears in the MathCAD solution as:

$$i := 0..N_d$$
$$P_{rh_i} := 5\text{-psi} + \frac{i}{N_d} \cdot (900\text{-psi} - 5\text{-psi})$$

The variable P_{rh} is a vector with $N_d + 1$ elements ranging from 5 psi to 900 psi. All of the state properties following the reheat boiler and to entering the pump, must also be evaluated as arrays using the counting variable as the index. The construction is used to easily determine the reheat boiler pressure that maximizes the cycle efficiency and to plot the cycle efficiency versus the reheat boiler pressure, shown in Figure 2.

Analysis of a Brayton Turbojet Cycle

Problem Statement: Air enters the diffuser of a turbojet engine with a velocity of 750 ft/s, a temperature of 420 °R, a pressure of 9 psia. The frontal area of the turbojet engine is 5 ft². The pressure ratio for the compressor is 12, and its isentropic efficiency is 88%. Air enters the turbine at 2400 °R. The turbine efficiency is 92%. Using the ideal-gas analysis, determine (a) the rate of heat addition in BTU/hr, (b) the pressure at the turbine exit in psi, (c) the compressor power requirements in BTU/hr, (d) the velocity at the nozzle exit in ft/s, and (e) the thrust of the engine in lb_f.

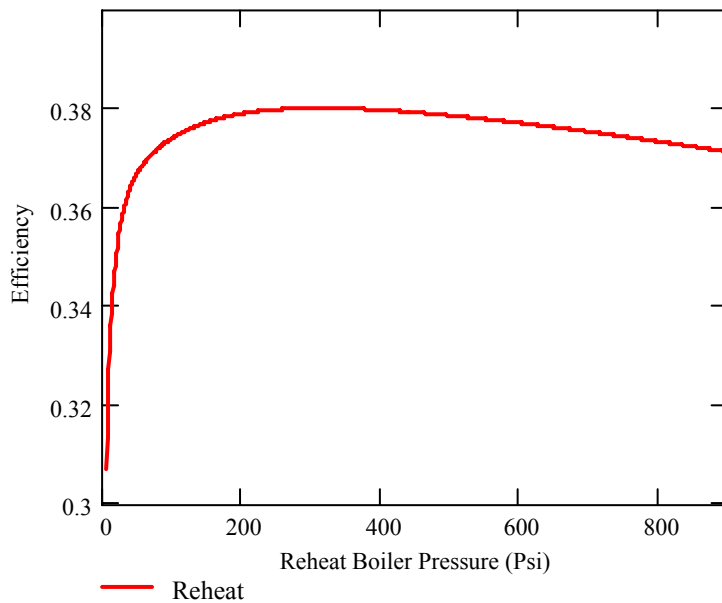


Figure 2. The Variation in Cycle Efficiency Versus Reheat Boiler Pressure

The complete MathCAD solution for this example is presented in Appendix C. The solution to this example demonstrates the use of the ideal-gas property functions for air. An interesting aspect of the Brayton-Turbojet cycle solution is the determination of the pressure exiting the non-isentropic turbine, a calculation often missed by students in an undergraduate applied thermodynamics course. Appendix C demonstrates how the relative pressure functions are used in MathCAD to determine the pressure of the air exiting the turbine and entering the nozzle.

Pedagogy and Delivery

The MathCAD functions were presented to students in three applied thermodynamics courses during the Spring 2005 semester. The three courses were: ME 242 – Thermodynamics II (the University of Alabama at Birmingham), ME 448 – Internal Combustion Engines (the University of Alabama at Birmingham), and MECH 4703 – Aerospace Propulsion (at John Brown University).

In ME 242 and ME 448, the students were given access to the function files and documentation. The students were also provided a one-hour, in-class MathCAD introduction. For each major topic in ME 242 and ME 448, a lecture would be provided on the theory, and examples would be provided using traditional methods of problem solution. Solutions to the same examples would then be provided using the MathCAD functions. In some cases in ME 242, more examples were worked in class using a laptop and computer projector system. The students were required to use the functions in selected homework problems and on their projects. The students were not allowed to use the MathCAD functions to take their tests. The students in MECH 4703 were provided the function files and documentation, but were not given an in-class MathCAD introduction.

Survey Results and Student Comments

A survey was developed to explore the effectiveness of teaching the courses using the MathCAD functions. The survey consisted of twelve questions to which the students could reply: strongly agree (5), agree (4), neutral (3), disagree (2), and strongly disagree (1). The survey questions and the student responses, by course and overall, are presented in Table 5.

While rigorous statistics have not been performed on the survey results, the results in Table 5 provide several interesting observations. Most importantly, the overall student response to using the MathCAD functions was highly positive. The overall student response to Statements #8 and #10 demonstrate that the functions and MathCAD made completing their homework easier and that the students preferred to use MathCAD to complete their homework assignments. The responses to Statement #3, especially from ME 242, in which most students had no MathCAD experience, demonstrates that students can be learn MathCAD and learn the use of the thermodynamic functions in a very short period of time.

Two interesting observations may be made regarding the effectiveness of the functions related to course instructions. First, the student responses were “neutral” to “agree” regarding the helpfulness of the MathCAD functions in learning the course material (Statement #9). Second, a disparity seems to exist between the student responses to Statements #8 and #9 compared to the response to Statement #10, especially for ME 242, Thermodynamics. These two observations seem to imply that while the functions made solving their homework problems easier, many students preferred to solve the homework problems in a more difficult and time consuming manner and that the functions were only marginally effective in improving learning. However, after post course interviews with the students and after reflection by the ME 242 professor, the two observations are most likely due to the facts that 1) some students became dependent on the MathCAD functions to solve their thermodynamics problems, 2) the students were not allowed to use their computers and the MathCAD functions during the course tests, and 3) the professor gave tests requiring manual solution techniques and failed to recognize that the pedagogical changes required changes in testing and assessment. Some of the students in the ME 242 course took advantage of the MathCAD resources in completing almost all of their homework assignments, yet the same students performed poorly on the tests which were created to evaluate the students ability to use the tabular property data. The student responses reflect the need to adapt testing methods and expectations as the course content and delivery changes.

Conclusions

A set of functions for evaluating the thermodynamic properties of many working fluids has been created for MathCAD. An example problem using the functions is presented in its entirety in the appendices to demonstrate the ability of MathCAD to create report quality solutions to applied thermodynamics problems.

The function sets have been used recently in three undergraduate, applied thermodynamics courses. The functions were highly successful in allowing the students to complete complicated homework and project assignments. The function sets eased the effort of completing the homework assignments that otherwise require the arduous use of tabularized thermodynamic

property data. Student response to using the functions and MathCAD was also highly positive. While student response was positive, the survey and other feedback from the students identified the need to adjust testing methods when MathCAD and the thermodynamic functions are used heavily during instruction.

Table 5. MathCAD Survey Results

Statement about the use of MathCAD in this course.	Course			Overall
	ME 242	ME 455	MECH 4703	
1. I had significant programming experience (2 hours credit or equivalent) in C or FORTRAN before using MathCAD in this course.	4.118	2.778	4.571	3.848
2. I have explored or used <i>IT: Interactive Thermodynamics</i> or <i>EES</i> software in my basic thermodynamics course.	2.235	1.778	3.000	2.273
3. A MathCAD introduction was provided in this course, and the introduction was sufficient to get me started using MathCAD.	4.647	4.556	2.286	4.121
4. I completed a numerical analysis or programming course in my engineering curriculum prior to my enrollment in this course.	3.188	2.667	3.857	3.188
5. The documentation and examples provided regarding the MathCAD functions were understandable and helpful.	4.412	4.444	4.286	4.394
6. The automatic unit handling and conversion features of MathCAD were easy to understand and control.	4.176	4.111	4.714	4.273
7. I used the MathCAD functions for multiple homework solutions in this course.	4.294	4.667	5.000	4.545
8. I preferred to use the MathCAD functions for my homework solutions as opposed to using thermodynamic property tables.	3.882	4.000	4.857	4.121
9. The MathCAD functions helped me learn the material in this course.	3.588	3.556	3.571	3.576
10. The thermodynamic property functions in MathCAD made solving problems easier.	4.824	4.333	4.714	4.667
11. Because of my experience using MathCAD in this course, I will use MathCAD in subsequent engineering courses.	4.647	4.556	4.714	4.636
12. Because of my experience using MathCAD in this course, I will be more likely to use MathCAD in my engineering practice.	4.588	4.333	5.000	4.606

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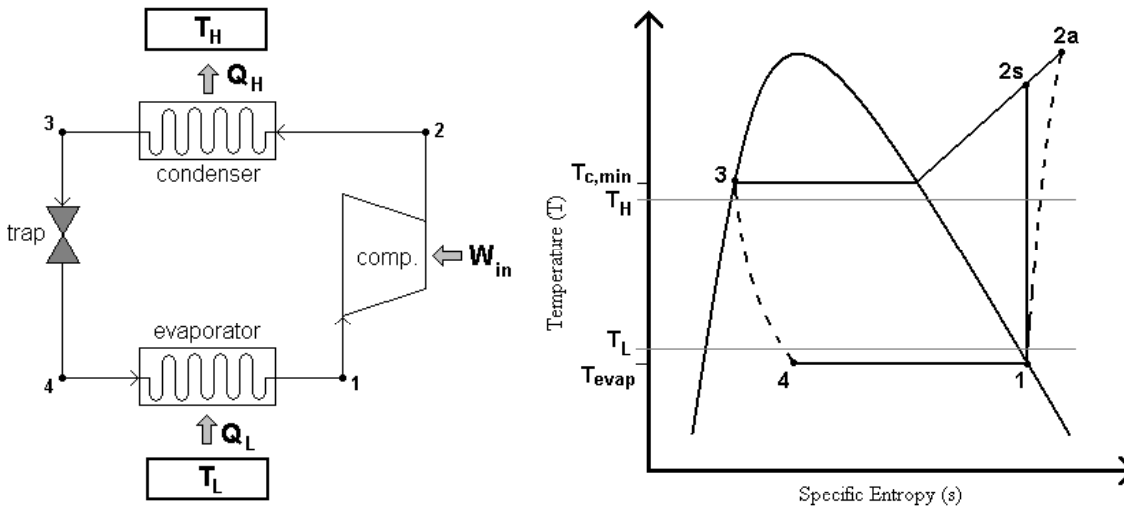
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Appendix A - Reversed-Rankine (Refrigeration) Cycle Design

Reference: C:\ThermoII\R134aProps.mcd

Problem Statement: A residential refrigeration system must maintain a room temperature of 65 F when the outside temperature is a maximum of 110 F. Design a reversed-Rankine cycle, refrigeration system using R-134a as a refrigerant to cool the space. If the system is to be a 4 ton unit, what is the mass flow rate of refrigerant? What is the second law efficiency of the system?

Sketches: A system schematic and T-s diagram are:



Solution: The design parameters from the problem statement are entered to begin the solution.

$$Q_L := 4 \text{ ton}_c \quad T_L := (65 + 460) \cdot R \quad T_H := (110 + 460) \cdot R \quad \Delta T := 10 \cdot R$$

The minimum condenser temperature and evaporator temperature are then evaluated using the a reasonable temperature difference to achieve the required heat transfer.

$$T_{cmin} := T_H + \Delta T \quad T_{cmin} = 580R \quad \frac{T_{cmin} - 460R}{R} = 120$$

$$T_{evap} := T_L - \Delta T \quad T_{evap} = 515R \quad \frac{T_{evap} - 460R}{R} = 55$$

The condenser pressure and evaporator pressure are then set as the saturation pressures at the minimum condenser temperature and the evaporator temperature, respectively.

$$P_{cond} := P_{sat_T_R134a}(T_{cmin}) \quad P_{cond} = 186.777 \text{ psi}$$

$$P_{evap} := P_{sat_T_R134a}(T_{evap}) \quad P_{evap} = 66.304 \text{ psi}$$

With the important temperatures and pressures specified, the cycle analysis follows. The R-134a is assumed to be a saturated vapor at the evaporator pressure at state 1.

$$h_1 := h_{g_P_R134a}(P_{evap}) \quad h_1 = 110.836 \frac{\text{BTU}}{\text{lb}} \quad s_1 := s_{g_P_R134a}(P_{evap}) \quad s_1 = 0.221 \frac{\text{BTU}}{\text{lb} \cdot R}$$

No efficiency is provided for the compressor, so it is assumed to be isentropic. The temperature at state 2 (the maximum cycle temperature) and the enthalpy at state 2 may be determined since the pressure is known and the entropy at state 2 equals the entropy at state 1.

$$T_{max} := T_{Ps_R134a}(P_{cond}, s_1) \quad T_{max} = 586.358R \quad \frac{T_{max} - 460R}{R} = 126.358$$

Appendix A - Reversed-Rankine (Refrigeration) Cycle Design (continued...)

$$h_2 := h_{Ps_{R134a}}(P_{cond}, s_1) \quad h_2 = 120.045 \frac{\text{BTU}}{\text{lb}}$$

The maximum cycle temperature is 126.4 F. For an ideal Rankine cycle, state 3 is assumed to be a saturated liquid at the condenser temperature. Thus the enthalpy at state 3 is:

$$h_3 := hf_{T_{R134a}}(T_{cmin}) \quad h_3 = 52.467 \frac{\text{BTU}}{\text{lb}}$$

Since the trap produces no work and is assumed to be adiabatic, the enthalpy at state 4 equals the enthalpy at state 3.

$$h_4 := h_3 \quad h_4 = 52.467 \frac{\text{BTU}}{\text{lb}}$$

With all of the states evaluated, the cycle analysis proceeds very quickly. The COP_c is then the enthalpy difference across the evaporator divided by the enthalpy difference across the compressor.

$$\text{COP}_C := \frac{h_1 - h_4}{h_2 - h_1} \quad \boxed{\text{COP}_C = 6.339}$$

The mass flow rate of refrigerant is the total cooling load divided by the evaporator enthalpy difference.

$$m_{\text{dot}} := \frac{Q_L}{h_1 - h_4} \quad \boxed{m_{\text{dot}} = 0.228 \frac{\text{lb}}{\text{s}}}$$

The required compressor power is then the mass flow rate time the compressor enthalpy difference.

$$W_c := m_{\text{dot}} \cdot (h_1 - h_2) \quad W_c = -2.219 \text{ kW} \quad \boxed{W_c = -2.976 \text{ hp}}$$

A 3 hp motor is required to drive the compressor. The second law efficiency is then evaluated by determining the exergy of the heat entering the evaporator. The dead state temperature is assumed to be the ambient (outside) temperature. The boundary temperature is evaluated as the temperature of the space being cooled.

$$\chi_{QL} := Q_L \cdot \left(1 - \frac{T_H}{T_L} \right) \quad \eta_{II} := \frac{\chi_{QL}}{W_c} \quad \eta_{II} = 54.331\%$$

The system has a second law efficiency of 54.33%

Problem extension: If the system is expected to operate 60% of the time during the month of August, what is the cost of cooling the space if electricity costs 11 cents per kW-hr?

Solution: The rate of electricity cost and operating time are entered.

$$c := 0.11 \frac{\$}{\text{kW} \cdot \text{hr}} \quad \Delta t := 0.6 \cdot (31 \cdot 24) \cdot \text{hr}$$

The cost is then the rate times the operating time and the compressor power.

$$C := c \cdot W_c \cdot \Delta t$$

$$\boxed{C = -108.98}$$

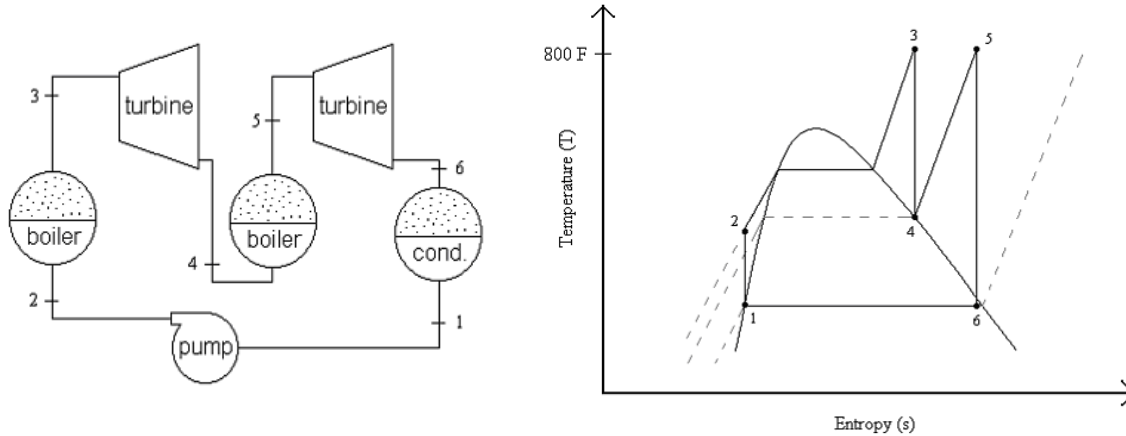
This system will cost almost \$109 to operate during the month of August provided the stated electricity cost rate and demand time.

Appendix B - Rankine-Reheat Power Cycle Optimization

Reference: C:\ThermoII\SteamProps.mcd

Problem Statement: A Rankine power cycle with reheat operates between a boiler pressure of 1000 psia and a condenser pressure of 2 psia. The maximum boiler temperature and reheat boiler temperature are both 1260 R. By varying the reheat boiler pressure between 5 psia and 900 psia, determine the reheat boiler pressure that maximizes the cycle efficiency and the resulting maximum efficiency.

Sketches: The schematic and T-s diagrams for a Rankine cycle with reheat follow.



Solution: The solution begins by entering the cycle parameters identified in the problem statement.

$$P_{\text{boiler}} := 1000 \frac{\text{lbf}}{\text{in}^2} \quad P_{\text{cond}} := 2 \frac{\text{lbf}}{\text{in}^2} \quad T_{\text{bmax}} := (800 + 460) \cdot \text{R} \quad T_{\text{bmax}} = 700 \text{K}$$

To vary the reheat boiler pressure, a counting variable is identified ranging from 0 to 50. A vector of reheat pressures between 5 and 900 psi is then created using the counting variable.

$$N_d := 50 \quad i := 0..N_d \quad P_{\text{rh}_i} := 5 \cdot \text{psi} + \frac{i}{N_d} \cdot (900 \text{psi} - 5 \cdot \text{psi})$$

The properties at each state are then evaluated starting at **State 1**: which is assumed to be a saturated liquid entering the pump. For State 1, the temperature, enthalpy, and specific volume are required.

$$T_1 := \text{Tsat}_{\text{P}_{\text{H}_2\text{O}}}(P_{\text{cond}}) \quad T_1 = 585.745 \text{R} \quad h_1 := \text{hf}_{\text{P}_{\text{H}_2\text{O}}}(P_{\text{cond}}) \quad h_1 = 94.034 \frac{\text{BTU}}{\text{lb}}$$

$$v_1 := \text{vf}_{\text{P}_{\text{H}_2\text{O}}}(P_{\text{cond}}) \quad v_1 = 0.016 \frac{\text{ft}^3}{\text{lb}}$$

The enthalpy at **State 2**, exiting the pump, is then found using the expression for isentropic pumping of a liquid.

$$h_2 := h_1 + v_1 \cdot (P_{\text{boiler}} - P_{\text{cond}}) \quad h_2 = 97.031 \frac{\text{BTU}}{\text{lb}}$$

The enthalpy and entropy at **State 3**, exiting the boiler, are then found using the boiler pressure and maximum cycle temperature.

$$h_3 := \text{h}_{\text{TP}_{\text{H}_2\text{O}}}(T_{\text{bmax}}, P_{\text{boiler}}) \quad h_3 = 1389.833 \frac{\text{BTU}}{\text{lb}}$$

$$s_3 := \text{s}_{\text{TP}_{\text{H}_2\text{O}}}(T_{\text{bmax}}, P_{\text{boiler}}) \quad s_3 = 1.568 \frac{\text{BTU}}{\text{lb} \cdot \text{R}}$$

Appendix B - Rankine-Reheat Power Cycle Optimization (continued...)

The enthalpy at **State 4**, exiting the first turbine, depends on the reheat pressure and the isentropic efficiency of the turbine. Since no turbine efficiency is stated, a value of 100% is assumed. A vector of enthalpies at State 4 is evaluated based each reheat pressure and the entropy at State 3.

$$h_{4_i} := h_{\text{Ps H2O}}(P_{\text{rh}_i}, s_3)$$

The enthalpy and entropy at **State 5**, exiting the reheat boiler, is determined for each reheat pressure and using the maximum reheat temperature.

$$h_{5_i} := h_{\text{TP H2O}}(T_{\text{bmax}}, P_{\text{rh}_i}) \quad s_{5_i} := s_{\text{TP H2O}}(T_{\text{bmax}}, P_{\text{rh}_i})$$

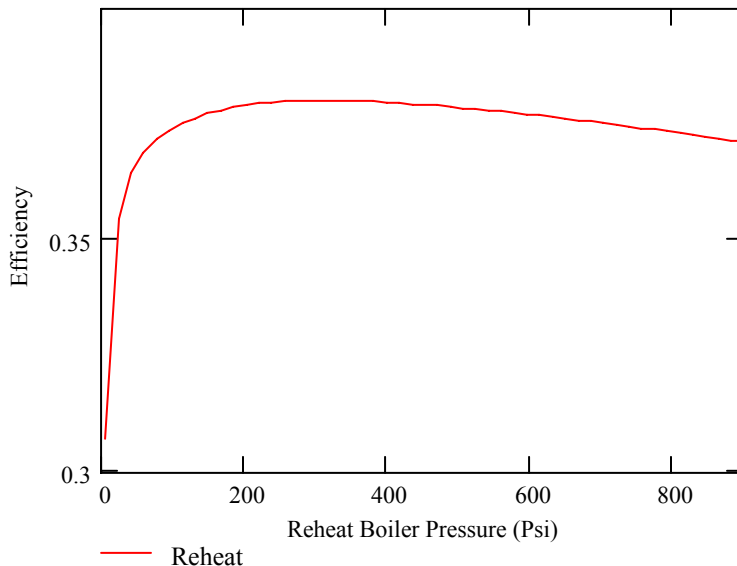
Like the first turbine, the second turbine is also assumed to be isentropic. **State 6**, exiting the second turbine, is then evaluated using the condenser pressure and the entropy at State 5.

$$h_{6_i} := h_{\text{Ps H2O}}(P_{\text{cond}}, s_{5_i})$$

With all of the states evaluated, the thermal efficiency is evaluated as the net work of the cycle divided by the net heat input into the cycle. The net work is the sum of the work of the first turbine, the second turbine and the pump. The net heat input is the sum of the heat delivered to the water in the main boiler and the reheat boiler.

$$\eta_{1_i} := \frac{(h_{5_i} - h_{6_i}) + (h_3 - h_{4_i}) + (h_1 - h_2)}{(h_3 - h_2) + (h_{5_i} - h_{4_i})}$$

The array of efficiencies is then plotted versus the reheat boiler pressure to identify the maximum thermal efficiency.



The maximum thermal efficiency of the reheat cycle is evaluated using the "max" function in MathCAD and is found to be 38% at a reheat boiler pressure of 309 psi.

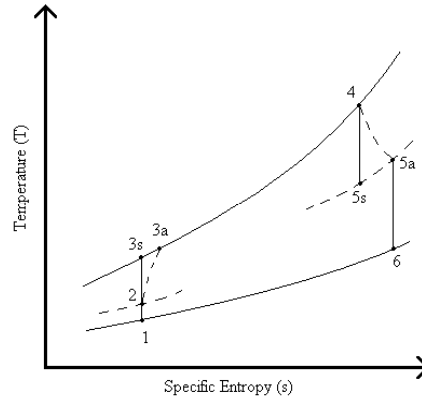
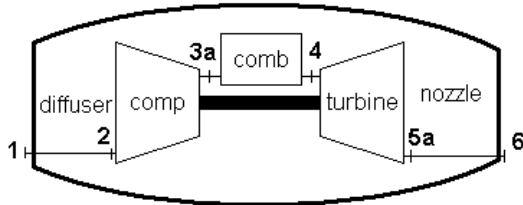
$$\max(\eta_1) = 38.002\%$$

Appendix C - Turbojet Analysis

Reference: C:\Thermo\IMGASProps.mcd

Problem Statement: Air enters the diffuser of a turbojet engine with a velocity of 750 ft/s, a temperature of 420 R, a pressure of 9 psi. The frontal area of the turbojet engine is 5 ft². The pressure ratio for the compressor is 12, and its isentropic efficiency is 88%. Air enters the turbine at 2400 R. The turbine efficiency is 92%. Using the ideal-gas analysis, determine (a) the rate of heat addition in BTU/hr, (b) the pressure at the turbine exit in psi, (c) the compressor power requirements in BTU/hr, (d) the velocity at the nozzle exit in ft/s, and (e) the thrust of the engine in lbf.

Sketches:



Solution: The parameters listed in the problem statement are entered to begin the solution.

$$V_{\text{jet}} := 750 \frac{\text{ft}}{\text{s}} \quad V_{\text{jet}} = 511.364 \frac{\text{mi}}{\text{hr}} \quad T_1 := 420 \text{ R} \quad T_{\text{max}} := 2400 \text{ R} \quad P_1 := 9 \text{ psi} \quad A_f := 5 \cdot \text{ft}^2$$

$$\eta_c := 88\% \quad r_p := 12 \quad \eta_t := 92\%$$

The mass flow rate is then found using the ambient conditions and the velocity of the engine.

$$\rho_1 := \frac{P_1}{\frac{R_u}{M_{\text{air}}} \cdot T_1} \quad \rho_1 = 0.058 \frac{\text{lb}}{\text{ft}^3} \quad m_{\text{air}} := \rho_1 \cdot V_{\text{jet}} \cdot A_f \quad m_{\text{air}} = 216.937 \frac{\text{lb}}{\text{s}}$$

The cycle analysis begins by evaluating the properties at the inlet conditions. **State 1:**

$$h_1 := h_{\text{air}}(T_1) \quad h_1 = 100.367 \frac{\text{BTU}}{\text{lb}} \quad pr_1 := pr_{\text{air}}(T_1) \quad pr_1 = 0.577$$

State 2: Exiting the diffuser, the velocity of the air is assumed to be negligible.

$$h_2 := h_1 + \frac{V_{\text{jet}}^2}{2} \quad h_2 = 111.6 \frac{\text{BTU}}{\text{lb}}$$

$$T_2 := T_{\text{air}}(h_2) \quad T_2 = 467.111 \text{ R} \quad pr_2 := pr_{\text{air}}(T_2) \quad pr_2 = 0.835$$

The pressure at state 2 is found using the relative pressure ratios between states 1 and 2.

$$P_2 := P_1 \cdot \frac{pr_2}{pr_1} \quad P_2 = 13.027 \text{ psi}$$

State 3: The compressor ratio is used to determine the isentropic properties after compression.

$$pr_3 := pr_2 \cdot r_p \quad pr_3 = 10.021 \quad T_3 := T_{\text{air}}(pr_3) \quad T_3 = 942.594 \text{ R}$$

$$P_3 := P_2 \cdot r_p \quad P_3 = 156.326 \text{ psi} \quad h_{3s} := h_{\text{air}}(pr_3) \quad h_{3s} = 227.199 \frac{\text{BTU}}{\text{lb}}$$

Appendix C - Turbojet Analysis (continued...)

The compressor efficiency is then used to evaluate the actual properties exiting the compressor.

$$h_{3a} := h_2 + \frac{(h_{3s} - h_2)}{\eta_c} \quad h_{3a} = 242.962 \frac{\text{BTU}}{\text{lb}}$$

The compressor power is then the mass flowrate times the actual enthalpy difference across the compressor.

$$W_{\text{comp}} := m_{\text{air}}(h_{3a} - h_2) \quad \boxed{W_{\text{comp}} = 1.026 \times 10^8 \frac{\text{BTU}}{\text{hr}}} \quad W_{\text{comp}} = 4.032 \times 10^4 \text{ hp}$$

State 4: State 4 is evaluated using the stated maximum temperature of the cycle. The pressure at state four equals the pressure at state 3.

$$h_4 := h_{\text{T}_{\text{air}}}(T_{\text{max}}) \quad h_4 = 618.672 \frac{\text{BTU}}{\text{lb}} \quad pr_4 := pr_{\text{T}_{\text{air}}}(T_{\text{max}}) \quad pr_4 = 374.295$$

$$P_4 := P_3 \quad P_3 = 156.326 \text{ psi}$$

The heat input is then the mass flowrate times the enthalpy difference across the combustor.

$$Q_{\text{in}} := m_{\text{air}}(h_4 - h_{3a}) \quad \boxed{Q_{\text{in}} = 2.934 \times 10^8 \frac{\text{BTU}}{\text{hr}}}$$

State 5: The *actual* enthalpy exiting the turbine is determined using the premise that the turbine produces just enough power to drive the compressor.

$$h_{5a} := h_4 - (h_{3a} - h_2) \quad h_{5a} = 487.31 \frac{\text{BTU}}{\text{lb}} \quad T_{5a} := T_{\text{h}_{\text{air}}}(h_{5a}) \quad T_{5a} = 1932.8 \text{ R}$$

The relative pressure ratio from state 4 to state 5 must be used to evaluate the pressure exiting the turbine. However, the relative pressure ratio analysis *is only valid for isentropic processes*. Thus, the properties exiting an isentropic turbine with the same pressure ratio must be found using the definition of the isentropic efficiency. The relative pressure at state 5s is then used to determine the actual pressure exiting the turbine.

$$h_{5s} := h_4 - \frac{(h_4 - h_{5a})}{\eta_t} \quad h_{5s} = 475.888 \frac{\text{BTU}}{\text{lb}} \quad T_{5s} := T_{\text{h}_{\text{air}}}(h_{5s}) \quad T_{5s} = 1891.5 \text{ R}$$

$$pr_{5s} := pr_{\text{T}_{\text{air}}}(T_{5s}) \quad pr_{5s} = 141.221 \quad P_5 := P_4 \cdot \frac{pr_{5s}}{pr_4} \quad \boxed{P_5 = 58.982 \text{ psi}}$$

State 6: The relative pressure at the actual turbine exit state must be used to determine the relative pressure and properties at state 6.

$$pr_{5a} := pr_{\text{T}_{\text{air}}}(T_{5a}) \quad pr_{5a} = 154.081 \quad pr_6 := pr_{5a} \cdot \frac{P_1}{P_5} \quad pr_6 = 23.511$$

$$h_6 := h_{\text{pr}_{\text{air}}}(pr_6) \quad h_6 = 289.276 \frac{\text{BTU}}{\text{lb}} \quad T_6 := T_{\text{pr}_{\text{air}}}(pr_6) \quad T_6 = 1189.8 \text{ R}$$

The exit velocity and thrust are then:

$$V_{\text{exit}} := [(h_{5a} - h_6) \cdot 2]^{0.5} \quad \boxed{V_{\text{exit}} = 3.149 \times 10^3 \frac{\text{ft}}{\text{s}}} \quad T_{\text{jet}} := m_{\text{air}}(V_{\text{exit}} - V_{\text{jet}}) \quad \boxed{T_{\text{jet}} = 1.618 \times 10^4 \text{ lbf}}$$

At the stated conditions, the turbojet is producing over 16,000 lbf of thrust.