

A Method for Obtaining the Transfer Function of Inverting and Non-inverting Op-amp Circuits Based on Classical Feedback Theory

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Abstract

This paper presents an alternate approach for deriving the transfer function (gain, bandwidth) for both inverting and non-inverting Op-amp circuits. The approach uses classical feedback theory, in which the Op-amp circuit is represented in terms of its corresponding closed-loop, (feedback) block diagram. The characteristics of the Op-amp (open-loop), together with the equivalent transfer function of the accompanying circuit components, are incorporated into the classical, general feedback block diagram. The equivalent transfer functions (pre-filter and feedback) are obtained by means of superposition. Then, all the blocks are reduced into a single transfer function by means of the simplification formula: $P(s)G(s)/(1+G(s)H(s))$. The resulting transfer function shows the gain for each configuration ($-R_F/R_A$ for the inverting Op-amp and $1+R_F/R_A$ for the non-inverting configuration) and bandwidth. It also shows that the Gain*Bandwidth product is constant for the non-inverting configuration, but not so for the inverting configuration. This approach is straightforward and insightful, specially for those students who have previously been exposed to feedback theory and who have backgrounds in fields other than electronics.

I. Introduction

Often non-electrical engineers and technologists find themselves using Operational amplifiers (Op-amps). This occurs because electronic instrumentation has become very pervasive, specially with the proliferation of the PC. The approach presented in this paper is helpful to someone who is attempting to understand the Op-amp's transfer characteristics. It is assumed that the individual has had a basic electrical science course and an understanding of feedback control. This approach has some advantages over the classical method used in electronic classes, aimed at electrical engineering/engineering technology students. Further, it is also insightful to students with an electronics background, even though they have the skills to understand the material as presented from an electronic presentation approach.

We will derive the equivalent transfer function for both the inverting and non-inverting amplifiers. This transfer function will convey the DC gain and the bandwidth, which allows the relationship between gain and bandwidth to be observed.

The approach assumes that the input signal source has low input impedance and that the load has relative high impedance. This approach does not produce information about input nor output impedance.

II. Op-amp's Open-loop Characteristics

The basic characteristics of an Op-amp can be described as a device that amplifies a differential input voltage (with a very high gain, G_0) and presents it at the output terminal (see Figure 1). The output equation is:

$$v_o = G_0(v_+ - v_-)$$

Notice that the Op-amp is presented in standard form, with its inverting input on top and its non-inverting one on the bottom. This is helpful for visual learners, who prefer to see the same orientation of the Op-amp throughout a given circuit.

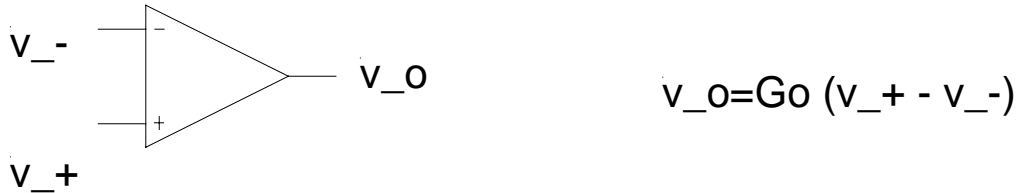


Figure 1: Conventional symbol, standard orientation for an Op-amp, and its output equation

Thus, if a +/- 15 volt supply is used, one can assume that the output is confined to the range of -14V to 14V. It is important that the student recognizes the amplifier becomes 'saturated' the moment that the output equation tries to produce an output voltage outside this range. The device also has dynamic characteristics which are presented next.

Transfer Function

It is helpful to start with the Op-amps transfer function, since engineers use this method for conveying a device's dynamic characteristic. Here, we present an experimental Bode plot, which would result if the Op-amp was to be tested in its open-loop configuration. Even though performing this test would very difficult (if not impossible), it conveys the characteristics of the device clearly. This approach ignores non-linear characteristics of the device, such as slew rate. Consider the test setup shown in Figure 2, where a very small sinusoidal signal is applied at the non-inverting input. Since the output is proportional to the input, the output is also sinusoidal. Notice that if the Op-amp becomes saturated, the output would no longer look like a sinusoidal signal. The experimental data is obtained by taking the ratio of the output to the input sinusoid, at different frequencies within for the desired frequency range. These ratios are then plotted in Bode format, to obtain something similar to what is shown at the bottom of Figure 2. From this data three things can be obtained: a) the type of transfer function, which in this case is a low-pass, first order, b) the open-loop gain G_0 , and c) the bandwidth (or corner frequency ω_o). This information can be used to represent the Op-amp, operated in open-loop mode as:

$$\frac{G_0}{s/\omega_o + 1}$$

Further, this information can also be used to calculate the Op-amp's Gain Bandwidth Product (GBP), by multiplying the open-loop gain G_0 times the corner frequency ω_0 . GBP is usually expressed in Hz. For this Op-amp, the GBP is:

$$GBP = G_0 \omega_0 \text{ (in rad / s)}$$

$$GBP = \frac{G_0 \omega_0}{2\pi} \text{ (in Hz)}$$

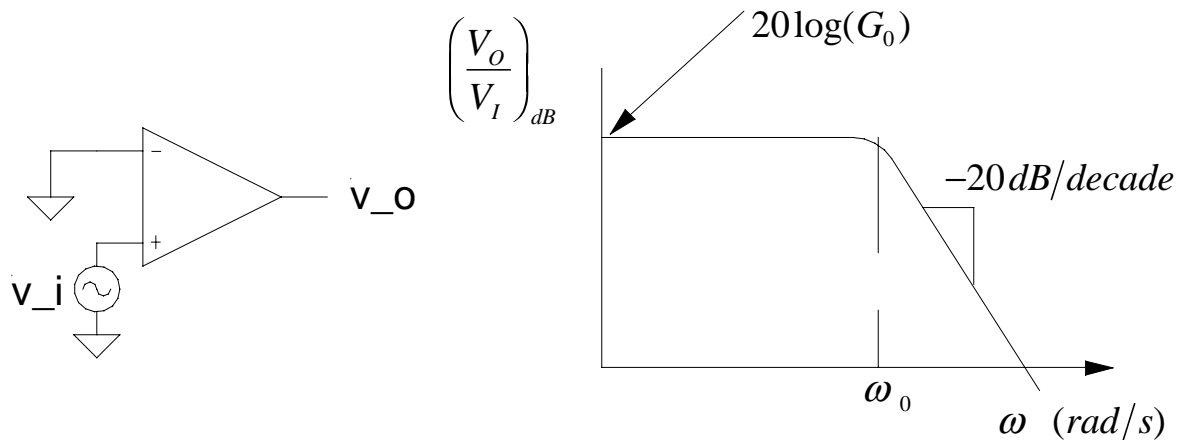
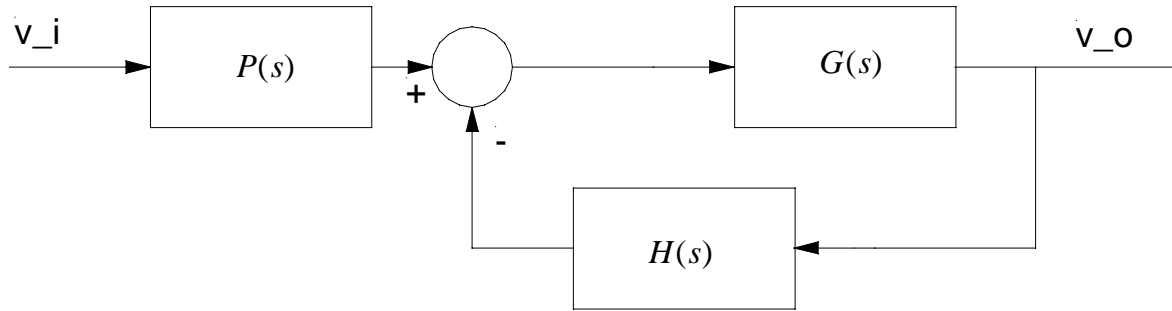


Figure 2: Open-loop testing of an Op-amp

For most Op-amps, the gain is very high (in the order of 10^6) and the corner frequency is relatively low (in the order of $2\pi \cdot 1$ rad/s or 1 Hz). As we put the Op-amp in a closed-loop circuit, we will obtain a much higher bandwidth at the expense of a lower gain. The next section deals with such a configuration, the non-inverting Op-amp.

Classical Feedback Block Diagram

The reader should verify that the classical block diagram, shown in Figure 3, can be simplified to the transfer function shown in the same figure. $P(s)$ is called the pre-filter, $H(s)$ the plant transfer function, and $H(s)$ the feedback transfer function, often referred to as β in electronic texts [1].



$$\frac{V_o}{V_i} = P(s) \frac{G(s)}{1 + G(s)H(s)}$$

Figure 3: Simplification of classical feedback block diagram into a single transfer function

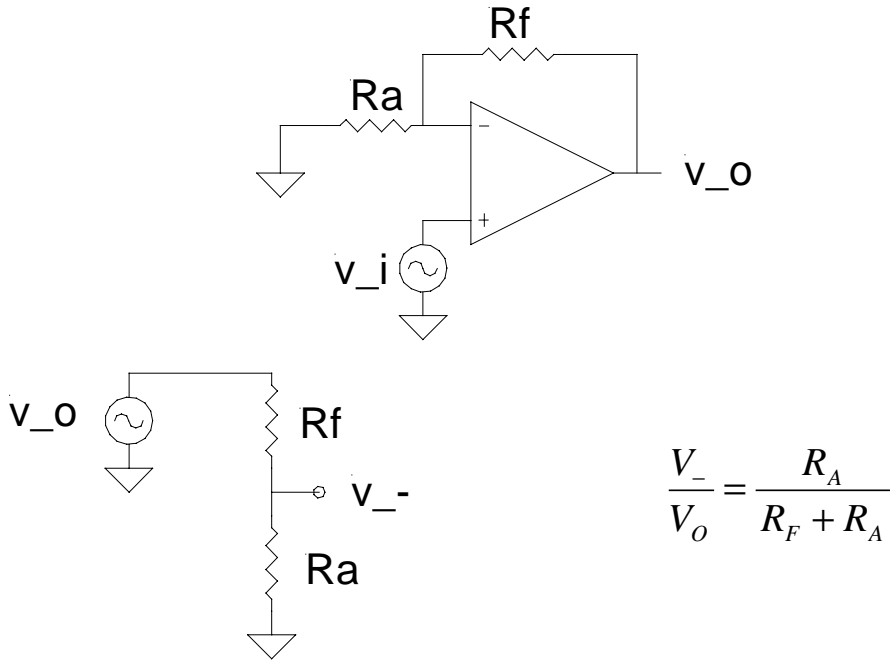
III. Non-Inverting Op-amp

Derivation of Transfer Function

Consider placing the basic Op-amp, discussed in the previous section, in the non-inverting circuit shown in Figure 4. To analyze this circuit, we want to use the classical, feedback control reduction formula (Figure 3) to obtain an overall transfer function for the non-inverting Op-amp circuit. We need to know how to translate a resistor bridge circuit into a transfer function. For instance, consider how the output is ‘feedback’ to input via the resistor bridge, composed of R_F and R_A . To obtain this transfer function, we need to use the superposition theorem, which allows us to investigate the contribution of individual signals, one at a time. It is helpful to redraw the resistors in the form of the bridge circuit shown in Figure 4. Here the input is V_o and the input is V_- . We assume that the input impedance of the Op-amp is infinite and the input signal source's impedance is zero. The resulting transfer function is:

$$\frac{V_-}{V_o} = \frac{R_A}{R_F + R_A}$$

This feedback effect corresponds to $H(s)$ in Figure 3.



$$\frac{V_-}{V_o} = \frac{R_A}{R_F + R_A}$$

Figure 4: Method for representing feedback from output to input.

The rest of the corresponding transfer functions are shown in Figure 5: P(s) is simple: it is equal to 1 since the complete input signal reaches the input port. G(s) is the Op-amps transfer function obtained in the previous section. Namely:

$$\frac{G_0}{s/\omega_o + 1}$$

Now we can use the classical feedback control equation to simplify this collection of blocks into a single transfer function, or:

$$\frac{V_o}{V_i} = 1 \frac{\frac{G_0}{s/\omega_o + 1}}{1 + \frac{G_0}{s/\omega_o + 1} \frac{R_A}{R_F + R_A}}$$

Substituting and collecting terms the result in:

$$\frac{V_o}{V_i} = \frac{1 + \frac{R_F}{R_A}}{\frac{s}{G_0 \omega_0} + 1}$$

This resulting transfer function reveals that for this configuration, the GBP is constant and equal to:

$$G_0 \omega_0$$

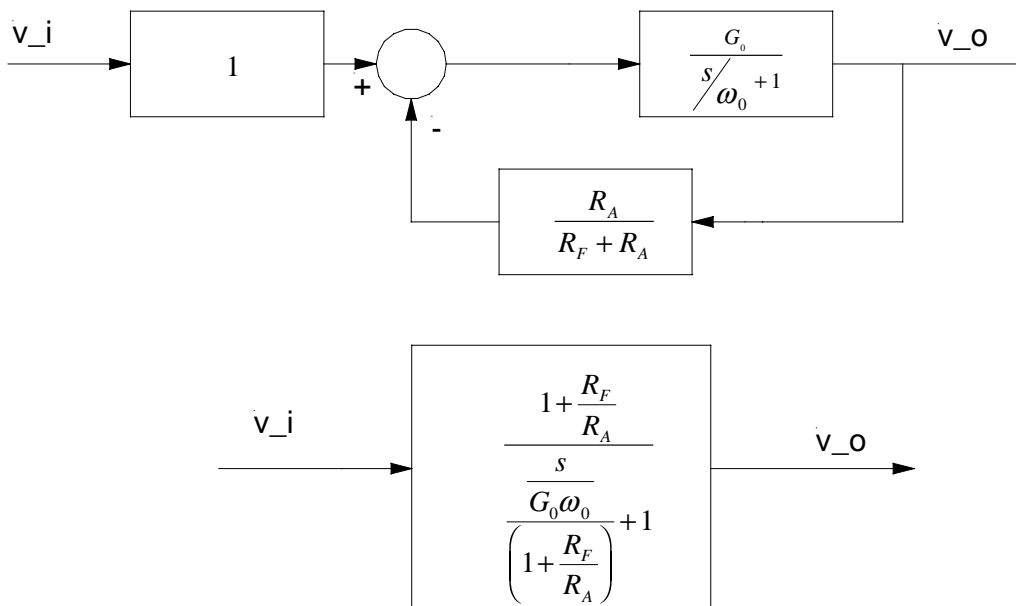
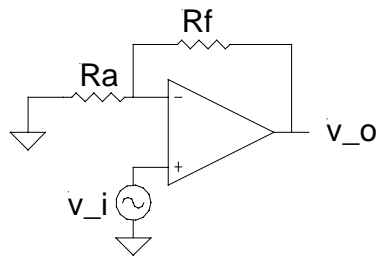


Figure 5: Equivalent block diagram for a non-inverting Op-amp, and its resulting transfer function

IV. Inverting Op-amp

Derivation of Transfer Function

The inverting configuration, shown in Figure 6, is more commonly used because one can add additional input signals without changing the gains of the existing input signals. The procedure for obtaining the individual transfer function follows.

In this case, the input signal goes first into a resistor bridge. It is also assumed that the input impedance of the Op-amp is infinite and its output impedance is zero. Also notice that the input has an effect on the inverting side. Thus the corresponding transfer function for P(s) is:

$$-\frac{R_F}{R_F + R_A}$$

The rest of the corresponding transfer functions are shown in Figure 6.

When the classical feedback formula is used, the following result is obtained:

$$\frac{V_O}{V_I} = \frac{-\frac{R_F}{R_F + R_A} G_0}{\frac{s}{\omega_0} + 1 + G_0 \frac{R_A}{R_F + R_A}}$$

Notice that 1 is small compared with $G_0 \frac{R_A}{R_F + R_A}$, so it can be neglected. After combining terms and putting the transfer function in standard form, the result is:

$$\frac{V_O}{V_I} = \frac{-\frac{R_F}{R_A}}{\frac{s}{G_0 \omega_0} + 1 + \frac{R_F}{R_A}}$$

Notice that the resulting gain is $-R_F/R_A$ and the GBP is almost constant (for $R_F \gg R_A$)

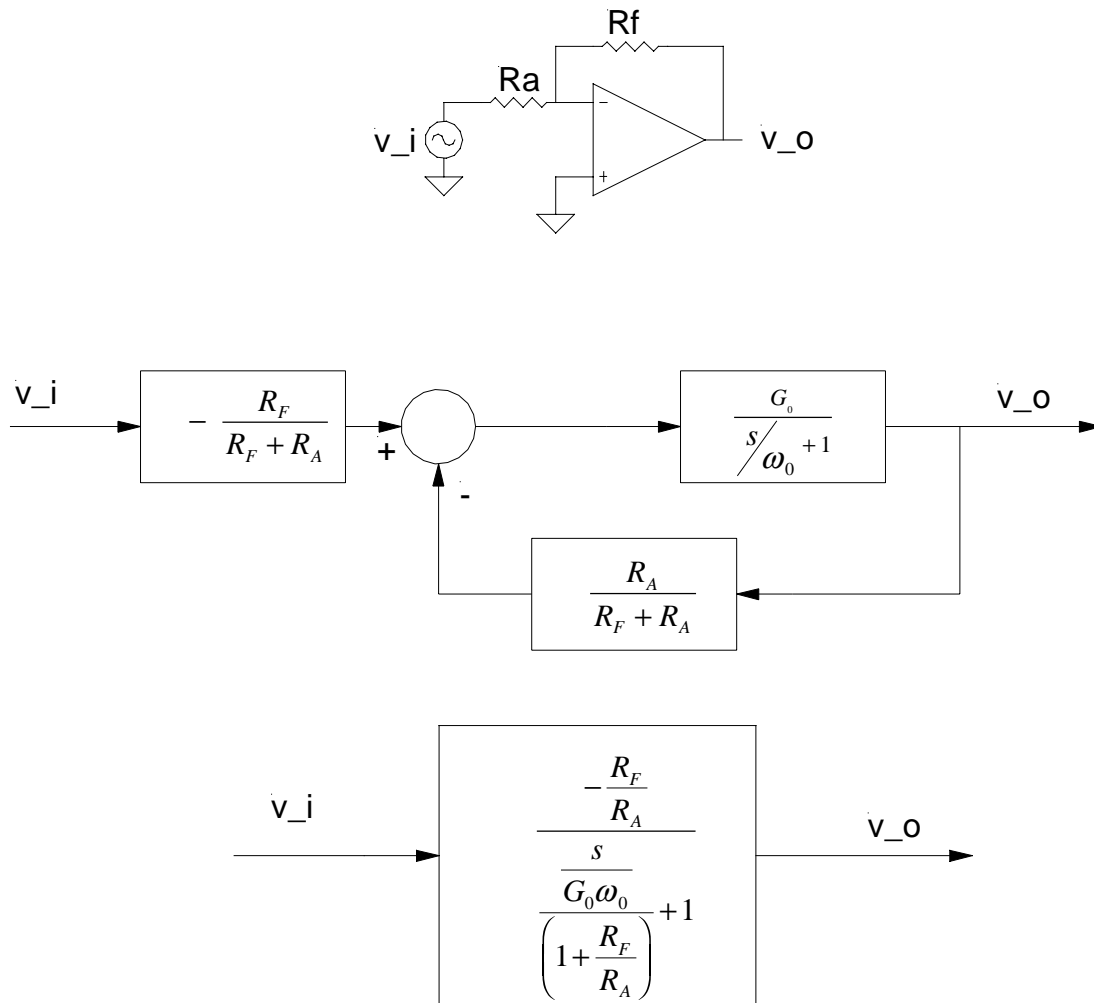


Figure 6: Equivalent block diagram for an inverting Op-amp, and its resulting transfer function

V. Conclusions

Both non-inverting and inverting Op-amp configurations have been represented in terms of a classical feedback block diagram. These block diagrams have been simplified to obtain a single transfer function that describes dynamic characteristics for the inverting and non-inverting Op-amp circuits.

Bibliography

1. Stanley, W.S., *Operational Amplifiers with Linear Integrated Circuits, 2nd. Ed.*, Merrill, 1989

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