AC 2007-1374: A NEW APPROACH FOR TEACHING IN-PLANE PRINCIPAL STRESSES, PRINCIPAL DIRECTIONS AND MAXIMUM SHEAR STRESS FOR PLANE STRESS

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A New Approach for Teaching In-Plane Principal Stresses, Principal Directions and Maximum Shear Stress for Plane Stress

Abstract

The topic of in-plane principal stresses, principal directions and maximum shear stress for a state of plane stress is typically taught in introductory mechanics of materials and solid mechanics courses using the following approach. First, the equations for the normal and shear stresses on an inclined plane are obtained applying the principle of static equilibrium to an infinitesimal wedge element. Then, the derivatives of the equations for the normal and shear stresses on the inclined plane with respect to the angle used to define that plane are found and set equal to zero to find the angles corresponding to the maximum and minimum algebraic values for the normal and shear stresses. Based on the expressions obtained for those angles, equations to find the in-plane principal stresses and the maximum in-plane shear stress are presented and the relative orientation of the planes corresponding to those stresses is discussed.

In this paper, a different approach is used to obtain the equations for the in-plane principal stresses, the maximum in-plane shear stress and the angles corresponding to the planes in which those stresses occur. Using the idea of a phase angle that is commonly employed in the analysis of signals or physical quantities that involve a linear combination of sine and cosine functions of the same angle, the equations for the normal and shear stress on an inclined plane are expressed in terms of trigonometric functions of twice the difference between the angle that defines the inclined plane and a phase angle. Plots of the resulting expressions are presented and used to quickly obtain formulas for the values and orientations of the planes for the stresses in question. The approach, although relatively simple, is mathematically rigorous and allows students to visualize in a convenient way the relationship between the normal and shear stress on an inclined plane. Furthermore, it can be used to show in a straightforward fashion that the two equations can be combined to represent the equation of a circle and thus serve as convenient starting point to introduce the concept of Mohr's circle.

Introduction

Exploring different ways of presenting topics covered in introductory mechanics of materials and solid mechanics courses is of particular importance. The concepts taught in those courses serve as the foundation over which students build additional knowledge and constitute an integral part of their professional expertise. Efforts to improve student learning typically focus on using new pedagogical strategies and methods to present the concepts as well as their practical application. However, in most cases the activities selected for the learning process are based on the same mathematical derivations that have been employed in the past. Sometimes the approach used to obtain certain equations is difficult for the students to understand and obscures the underlying concept. As a "solution" to this situation, the emphasis is shifted to the use the final formulas taking them as definitions. It is a good exercise for instructors to take a given derivation and explore if there are new alternatives that can be used to obtain the same formulas but that are easier for the students to comprehend.

The topic of in-plane principal stresses, principal directions and maximum shear stress for a state of plane stress is typically taught in undergraduate mechanics of materials and solid mechanics courses using the following approach. Equations for the normal and shear stresses on an inclined plane are obtained applying the principle of static equilibrium to an infinitesimal wedge element. In those equations, the normal and shear stresses on the inclined plane are expressed in terms of the components of the stress tensor with respect to a rectangular Cartesian x-y coordinate system and the angle that the outward normal to the inclined plane makes with the positive x-axis. Then, one of following two options is used. In the first and most common one, the derivatives of the equations for the normal and shear stresses on the inclined plane with respect to the angle used to define it are found and set equal to zero to find the angles corresponding to the maximum and minimum algebraic values for the normal and shear stresses at the point. Then, based on the expressions for those angles, equations to find the in-plane principal stresses and maximum shear stress are presented. In the second option, the equations for the normal and shear stresses acting on the inclined plane are combined to define Mohr's circle and this graphic tool is used to find the desired equations.

A survey of many textbooks for mechanics of materials and solid mechanics courses dating as far back as 1934 showed that, besides the two options stated above, other alternatives have not been employed. For example, Seely[1], Timoshenko and MacCullough[2], Laurson and Cox[3], Marin and Sauer[4], Higdon et al.[5], Byars and Snyder[6], Popov[7], Bickford[8], Craig[9], Bedford and Liechti[10], Gere[11], Hibbeler[12], and Riley et al.[13], follow the approach of finding the maximum and minimum algebraic values of the expressions for the normal and shear stresses on the inclined plane using derivates. Other authors such as Beer and Johnston[14] first present the concept of Mohr's circle and then proceed to use it to determine the in-plane principal stresses, maximum shear stress and the orientations of the planes corresponding to those stresses.

Textbooks intended for machine design courses usually include a brief review of how to find the in-plane principal stresses, maximum shear stress and the orientation of the planes corresponding to those stresses. Those books typically provide a summary of key formulas and only mention an approach that can be used to obtain them. Once again, the same two approaches discussed before have been employed. For example, Shigley[15], Edwards and McKee[16], and Hamrock et al.[17], refer to finding the maximum and minimum of the expressions for the normal and shear stresses using derivates. Other authors like Juvinall and Marshek[18], Spotts and Shoup[19], Mott[20], and Norton[21] use the Mohr's circle concept.

In this paper, an alternative approach to obtain the equations for finding the in-plane principal stresses, maximum shear stress and angles corresponding to the planes in which those stresses act is presented. Applying the idea of a phase angle that is frequently used in the analysis of signals or physical quantities involving linear combinations of sine and cosine functions of the same angle, the equations for the normal and shear stresses on an inclined plane are expressed in terms of a single trigonometric function involving the double of the difference between the angle that defines the orientation of the plane and a phase angle. Plots of the resulting expressions in terms the difference of those two angles allow finding in a straightforward fashion equations for the values and orientations of the planes for the stresses in question.

In the following sections a brief overview of the most common approach that has been used to teach this topic is presented. Next, the proposed approach is explained including the information that an instructor would present to the students during a class session. Finally, a brief preliminary assessment of the proposed approach is given and conclusions are presented.

Traditional Approach

For plane stress conditions, the typical starting point to derive equations to find the in-plane principal stresses, maximum shear stress and the orientation of the planes where those stresses act are the formulas for the normal and the shear stresses on an inclined plane. Those formulas are obtained by applying the principle of static equilibrium to an infinitesimal wedge element like the one shown in Fig. 1 and can be written as

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \tag{1}$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \tag{2}$$

where σ_x, σ_y, and τ_{xy} ($\tau_{yx} = \tau_{xy}$) are the components of the stress tensor with respect to the x-y coordinate system, θ is the angle (positive measured counterclockwise) that the unit outward normal to the inclined plane makes with the positive x-axis, and σ_n and τ_{nt} are the normal and shear stresses acting on the inclined plane. The angle θ completely defines the inclined plane under consideration as well as the right-handed n-t rectangular coordinate system associated with it.
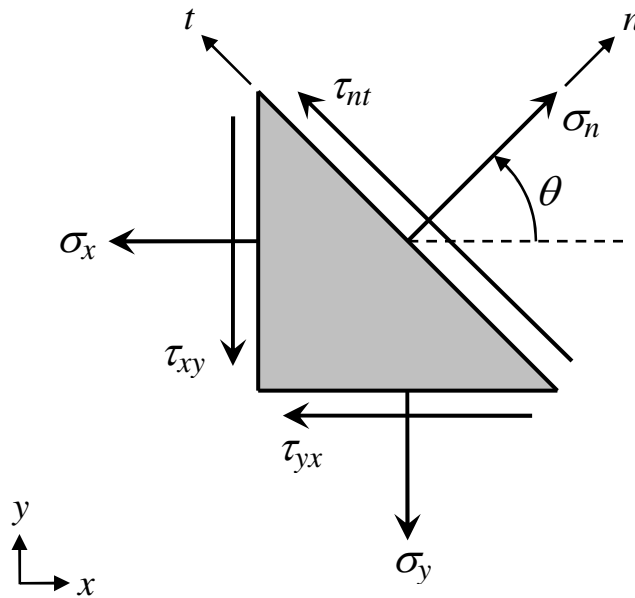

Figure 1. Normal and shear stresses on an inclined plane

The orientation of the planes in which the maximum and minimum algebraic values of the normal stress σ_n occur is found first. Using basic ideas from calculus regarding the relative maximum and minimum of a function of a single variable, the first derivative of the normal stress σ_n with respect to θ is set equal to zero to obtain:

$$\frac{d\sigma_n}{d\theta} = -(\sigma_x - \sigma_y)\sin 2\theta + 2\tau_{xy}\cos 2\theta = 0 \tag{3}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{4}$$

where θ_p is used to represent those angles for which the normal stress σ_n has a maximum or minimum algebraic value.

The result of Equation (3) is used together with Equation (2) to show that the shear stress is zero in a plane where the normal stress has an algebraic maximum or minimum value. In addition, the planes in which maximum or minimum algebraic values of the normal stress occur are identified as principal planes, their corresponding angles are used to identify the principal directions and the maximum and minimum algebraic values of the normal stress are defined as the principal stresses.

Since for any angle α the tangent of $2(\alpha + 90^\circ)$ is equal to the tangent of 2α, students are told that the possible solutions for Equation (4) correspond to two directions that are perpendicular to each other.

Finally, using arguments such as the right-triangle presented in Fig. 2, Equation (1) is used to obtain the two in-plane principal stresses as

$$\sigma_{p1,p2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{5}$$

and the angles corresponding to σ_{p1} and σ_{p2} are identified as θ_{p1} and θ_{p2}, respectively.

A similar approach is followed to find the orientation of the planes in which the magnitude of the in-plane shear stress is a maximum as well as the value of that stress. Taking the first derivative of τ_{nt} with respect to θ and setting it equal to zero, the following expressions are obtained:

$$\frac{d\tau_{nt}}{d\theta} = -(\sigma_x - \sigma_y)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0 \tag{6}$$

$$\tan 2\theta_\tau = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}} \tag{7}$$

where θ_τ is used to represent those angles for which the shear stress τ_{nt} has a maximum or minimum algebraic value.

$$\cos 2\theta_p = \frac{\dfrac{\sigma_x - \sigma_y}{2}}{\sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\sin 2\theta_p = \frac{\tau_{xy}}{\sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\dfrac{\sigma_x - \sigma_y}{2}} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$
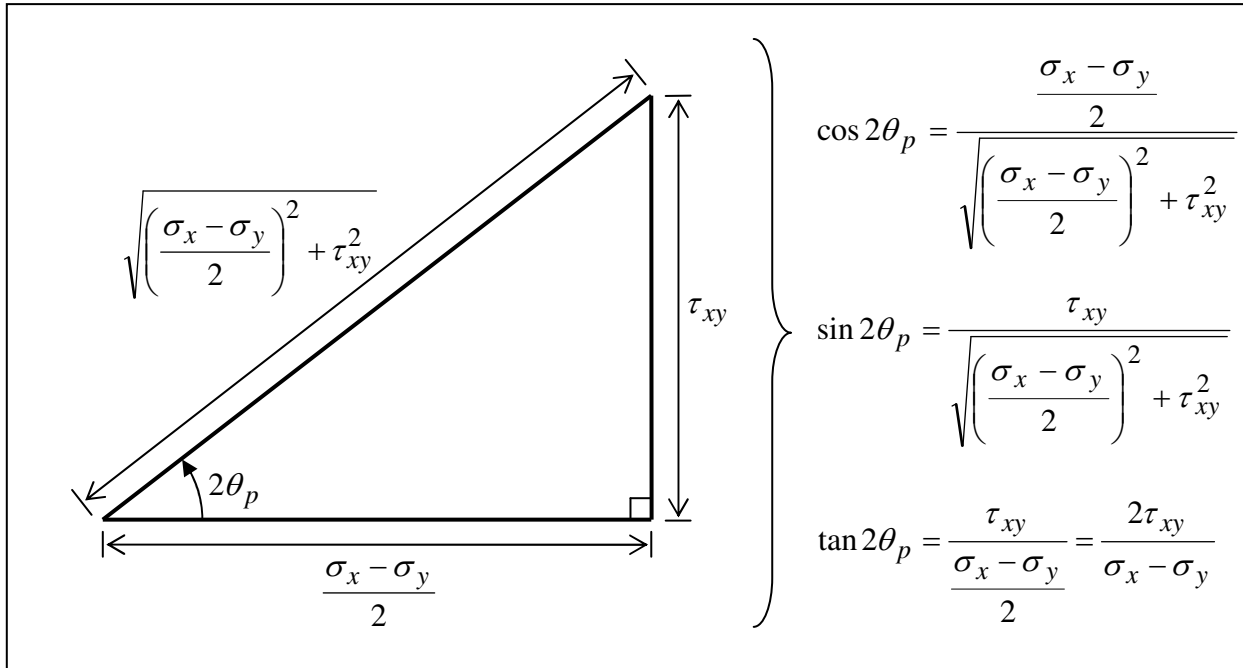
Figure 2. Right-triangle used to find the expressions for the principal stresses

Equation (6) is used together with Equation (1) to show that in the planes where the shear stress is an algebraic maximum or minimum, the normal stress is different than zero and equal to the average of the normal stresses σ_x and σ_y.

The fact that for any angle α the tangent of 2α and the tangent of $2(\alpha + 90°)$ have the same value is used again to explain that the possible solutions for Equation (7) correspond to two directions that are orthogonal to each other. In addition, Equations (4) and (7) are compared to show that the angles θ_τ and θ_p are $45°$ apart since the tangent of $2\theta_p$ and the tangent of $2\theta_\tau$ are negative reciprocals.

Finally, a right-triangle similar to the one employed to find the in-plane principal stress is used to show that the maximum magnitude of the in-plane shear stress is given by:

$$\tau_p = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{8}$$

Typically, most textbooks proceed to present graphical representations using stress elements to show the orientation of the planes corresponding to the maximum and minimum algebraic values of the normal and shear stresses and the relative location of those planes with respect to each other and the *x-y* coordinate system.

Proposed Approach

In the proposed approach to teach this topic, Equations (1) and (2) are used as the starting point. However, the derivations are done in a different way making use of the concept of a phase angle that is commonly employed in the analysis of signals or physical quantities that involve a linear combination of sine and cosine functions of the same angle.

For the case of σ_n we propose that

$$\left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta = C\cos(2\theta - 2\beta) \tag{9}$$

where the magnitude C is greater or equal to zero and β is a phase angle. Expanding the right hand side of the above equation we obtain:

$$\left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta = C(\cos 2\theta \cos 2\beta + \sin 2\theta \sin 2\beta) \tag{10}$$

$$\left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta = (C\cos 2\beta)\cos 2\theta + (C\sin 2\beta)\sin 2\theta \tag{11}$$

In order for both sides of the above equation to be equal, the following relationships need to be satisfied:

$$C\cos 2\beta = \left(\frac{\sigma_x - \sigma_y}{2}\right) \tag{12}$$

$$C\sin 2\beta = \tau_{xy} \tag{13}$$

Using these expressions, we can find the value of C as follows:

$$C^2 \cos^2 2\beta + C^2 \sin^2 2\beta = C^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \tag{14}$$

$$C = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (15)$$

The value of the phase angle β can be obtained dividing Equation (13) by Equation (12):

$$\frac{C\sin 2\beta}{C\cos 2\beta} = \tan 2\beta = \frac{\tau_{xy}}{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)} \qquad (16)$$

$$\beta = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \qquad (17)$$

At this point, students are told that they need to be very careful while finding the inverse tangent using a calculator or the digital computer. The conventional inverse tangent function available in a calculator or a programming language will usually return an angle between 0° and 90° when the input argument is positive and an angle between -90° and 0° when the argument is negative. This corresponds to the correct answer when the denominator of Equation (17) is positive. However, when the denominator is negative, it is necessary to add 180° to the value of the inverse tangent provided by the calculator or the digital computer to obtain the correct answer. Figure 3 shows a simple visual aid that the instructor can use in order to explain this point and that students can use to make sure that they find the correct angle.
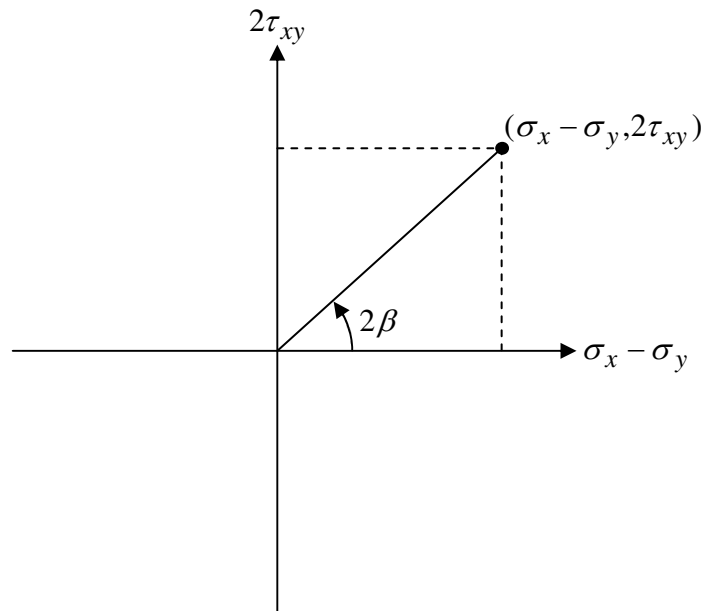

Figure 3. Visual aid for the determination of the correct value of the inverse tangent

In the case of the expression for τ_{nt}, we propose the following:

$$-\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta = -D\sin(2\theta - 2\phi) \tag{18}$$

where the magnitude D is greater or equal to zero and ϕ is a phase angle. Expanding the terms on the left hand side of the above equation we get:

$$-\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta = -D(\sin 2\theta \cos 2\phi - \cos 2\theta \sin 2\phi) \tag{19}$$

$$-\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta = -(D\cos 2\phi)\sin 2\theta + (D\sin 2\phi)\cos 2\theta \tag{20}$$

In order for both sides of the above equation to be equal, the following relationships need to be satisfied:

$$D\cos 2\phi = \left(\frac{\sigma_x - \sigma_y}{2}\right) \tag{21}$$

$$D\sin 2\phi = \tau_{xy} \tag{22}$$

Comparing the last two expressions with equations (12) and (13), we immediately conclude that $D = C$ and $\phi = \beta$.

Based on the above results, the equations for the normal and shear stresses on an inclined plane can be expressed as follows:

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left[\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\right]\cos(2\theta - 2\beta) \tag{23}$$

$$\tau_{nt} = -\left[\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\right]\sin(2\theta - 2\beta) \tag{24}$$

where the phase angle β is given by Equation (17).

Plots of σ_n and τ_{nt} versus the difference in angles $\theta - \beta$ (see Figs. 4 and 5) allow to determine in a very straight forward fashion the maximum and minimum algebraic values for σ_n and τ_{nt} as well as the orientation of the planes in which those stresses occur.
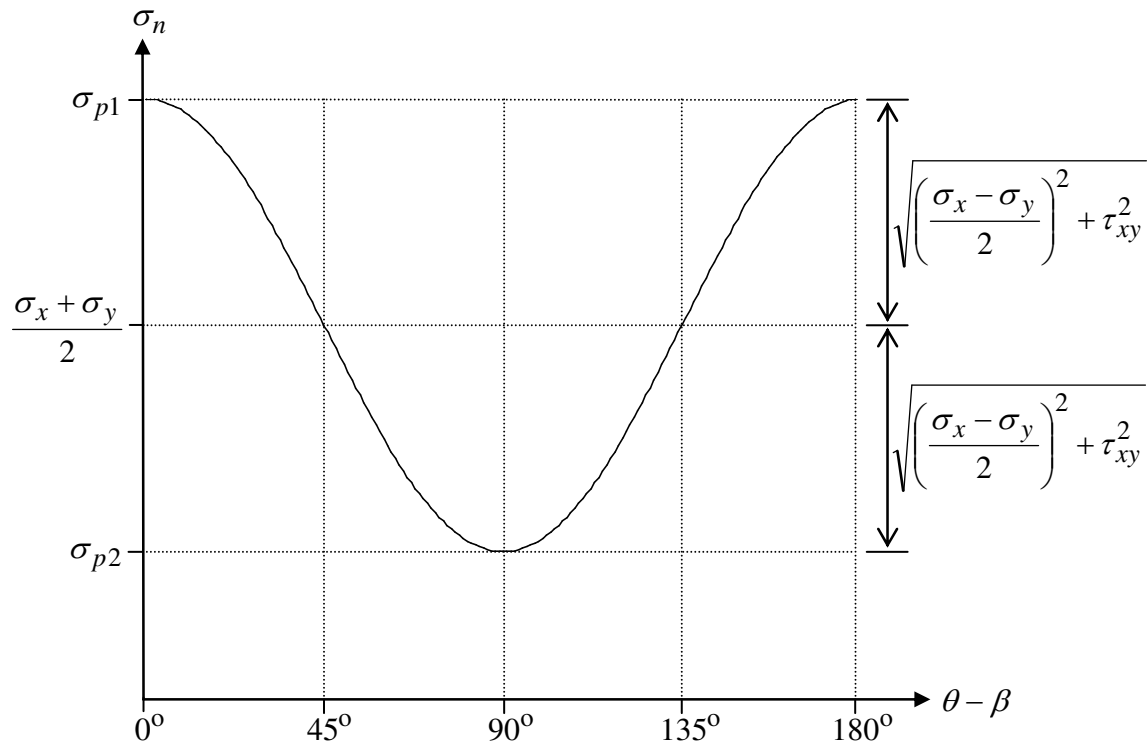
Figure 4. Plot of the normal stress σ_n vs. the difference in angles $\theta - \beta$
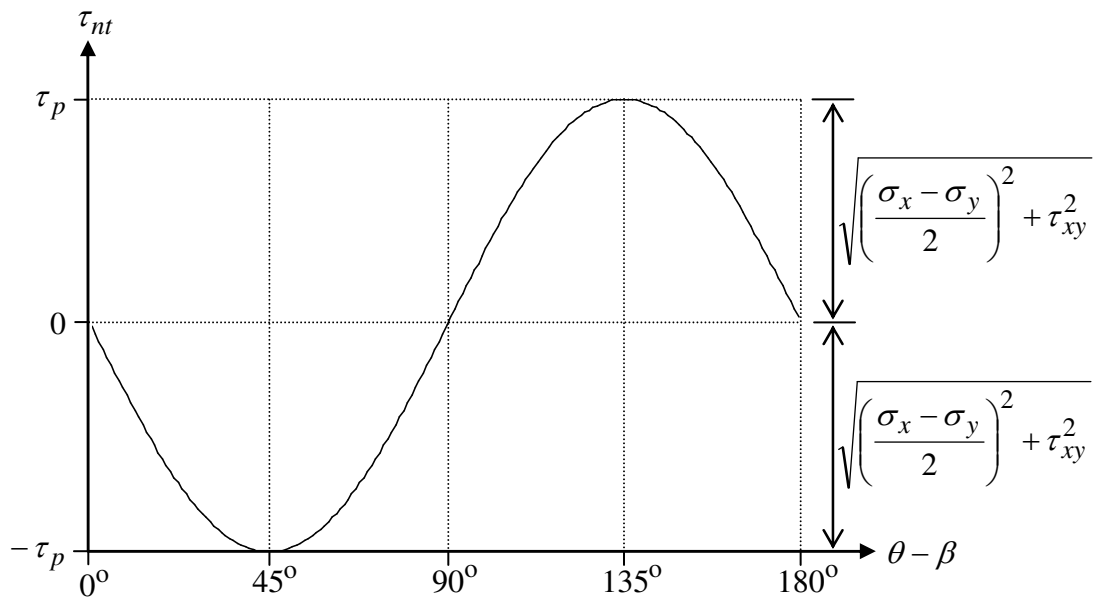

Figure 5. Plot of the shear stress τ_{nt} vs. the difference in angles $\theta - \beta$

Denoting the maximum and minimum algebraic values of the normal stress σ_n by σ_{p1} and σ_{p2}, respectively, one can easily see from Equation (23) and Fig. 4 that those quantities, which are the in-plane principal stresses, are given by:

$$\sigma_{p1,p2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{25}$$

Similarly, from Equation (24) and Fig. 5 it is easy to see that the maximum magnitude of the in-plane shear stress τ_{nt} is given by:

$$\tau_p = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{26}$$

From Fig. 4 it is evident that the planes in which the maximum and minimum algebraic values of σ_n occur correspond to $\theta - \beta = 0^{\rm o}$ and $\theta - \beta = 90^{\rm o}$, respectively. Thus, denoting by θ_{p1} the angle corresponding to $\sigma_n = \sigma_{p1}$ and by θ_{p2} the angle corresponding to $\sigma_n = \sigma_{p2}$, we have that $\theta_{p1} = \beta$ and $\theta_{p2} = \theta_{p1} + 90^{\rm o}$.

At this point one can mention to the students that the planes in which the principal stresses occur are known as the principal planes and the directions corresponding to those planes are known as the principal directions. Also, Fig. 5 can be used to show that for the planes corresponding to $\theta = \theta_{p1}$ and $\theta = \theta_{p2}$ the shear stress τ_{nt} is equal to zero.

Regarding the shear stress τ_{nt}, Fig. 5 clearly shows that the minimum and maximum algebraic values for that quantity occur when $\theta - \beta = 45^{\rm o}$ and $\theta - \beta = 135^{\rm o}$, respectively. Denoting by $\theta_{\tau 1}$ and the angle corresponding to $\tau_{nt} = -\tau_p$ and by $\theta_{\tau 2}$ the angle corresponding to $\tau_{nt} = \tau_p$, we have that $\theta_{\tau 1} = \theta_{p1} + 45^{\rm o}$ and $\theta_{\tau 2} = \theta_{p1} + 135^{\rm o}$. Based on these results, one can quickly see that $\theta_{\tau 2} - \theta_{\tau 1} = 90^{\rm o}$. In addition, Fig. 4 shows that for the planes corresponding to $\theta = \theta_{\tau 1}$ and $\theta = \theta_{\tau 2}$ the normal stress σ_n is equal to

$$\sigma_{pa} = \frac{\sigma_x + \sigma_y}{2} \tag{27}$$

which is the average of the normal stresses in the *x-y* plane.

The results obtained in the previous paragraphs are summarized in Table 1. The information contained in the table can easily be used to draw differential elements corresponding to the principal stresses and to the maximum in-plane shear stress.

Table 1. Planes corresponding to the maximum and minimum algebraic values of σ_n and τ_{nt}

Plane	$\theta - \beta$	σ_n	τ_{nt}
$\theta_{p1} = \beta$	0°	σ_{p1}	0
$\theta_{p2} = \theta_{p1} + 90^o$	90°	σ_{p2}	0
$\theta_{\tau1} = \theta_{p1} + 45^o$	45°	σ_{pa}	$-\tau_p$
$\theta_{\tau2} = \theta_{p1} + 135^o$	135°	σ_{pa}	τ_p

It is interesting to note that Equations (23) and (24) can be very easily combined to obtain the equation of a circle. Thus they serve as a convenient starting point to introduce the concept of Mohr's circle. For that purpose, Equation (23) is rewritten in the following form:

$$\sigma_n - \left(\frac{\sigma_x + \sigma_y}{2}\right) = \left[\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\right] \cos(2\theta - 2\beta) \tag{28}$$

Squaring both sides of Equations (28) and (24) and adding the resulting expressions gives:

$$\left[\sigma_n - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{nt}^2 = \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right] \tag{29}$$

which is the equation of a circle in the σ_n - τ_{nt} plane.

Preliminary Assessment of the Proposed Approach

At the university were the authors teach, the mandatory solid mechanics course sequence for the Mechanical Engineering undergraduate program is comprised of the following 3-credit courses: Introduction to Solid Mechanics (second semester of the sophomore year), Solid Mechanics (first semester of the junior year), and Machine Design I (second semester of the junior year). Usually only one section of each of those courses is offered every semester. The concepts and definitions corresponding to the topics of analysis of stress and analysis of strain are covered in the first course of the sequence.

The proposed approach was used for the first time during the fall of 2006 in a class with 23 students (all of them majoring in Mechanical Engineering). The information presented in the previous section was taught during a 50-minute lecture in which the students also learned how to draw properly oriented stress elements showing the in-plane principal stresses and the maximum

in-plane shear stress. Two examples were solved the following lecture and homework problems about the topic were given.

In general, students did not have difficulties understanding the mathematical derivation. The trigonometric identities corresponding to the cosine and the sine of the difference of two angles were written on the board before the derivation was started. The graphs shown in Figs. 4 and 5 together with Table 1 helped students to visualize the relative orientation between the planes in which the in-plane principal stresses and the maximum and minimum algebraic values of the in-plane shear stress occur. Also, they were a useful aid for students to determine the correct sense for the maximum magnitude of the in-plane shear while drawing stress elements.

Since figures similar to Figs. 4 and 5 were previously presented in the course while covering the topic of stress transformations for the case of axial loading, they provided a convenient way to link that special case with the more general one of a component subjected to a general state of stresses in two dimensions.

Some students had difficulties finding the correct value of the inverse tangent in Equation (17) during homework assignments and in-class exams when the denominator was negative. They directly used the value of the inverse tangent given by the calculator without adding 180°. This particular aspect will need to be emphasized more in the future.

Conclusions

In this paper a new alternative to obtain the equations to find the values of the in-plane principal stresses, the maximum magnitude of the in-plane shear stress and the angles corresponding to the planes in which those stresses occur was presented. Although some of the ideas employed in the derivation are frequently used in other mechanics courses like mechanical vibrations, textbooks for mechanics of materials and solid mechanics undergraduate courses have not included the proposed approach despite its possible advantages from a pedagogical point of view.

The first time the approach was used in class it was well received by the students. Additional assessment is needed to see if improvements can be made in the way the material is presented and to determine if it can help students to better retain the basic concepts covered for use in the next courses of the solid mechanics course sequence. Of particular importance is to have instructors at different universities use the proposed approach and provide feedback.

Besides the topic under consideration, the proposed approach can also be used when the subject of strains analysis in two dimensions is presented in class. Once the equations for strain transformations have been obtained, identical steps to the ones presented here can be followed to derive the expressions for the in-plane principal strains, the maximum magnitude of the in-plane shear strain and their corresponding orientations.

References

1. Seely, F.B., <u>Resistance of Materials</u>, Second Edition, John Wiley & Sons, 1934.
2. Timoshenko, S. and MacCullough, G.H., <u>Elements of Strength of Materials</u>, Second Edition, D. VanNostrand, 1934.
3. Laurson, P.G. and Cox, W.J., <u>Mechanics of Materials</u>, John Wiley & Sons, 1938.
4. Marin, J. and Sauer, J.A., <u>Strength of Materials</u>, Second Edition, John Wiley & Sons, 1954.
5. Higdon, A., Ohlsen, E.D., Stiles, W.D., and Weese, J.A., <u>Mechanics of Materials</u>, John Wiley & Sons, 1968.
6. Byars, E.F. and Snyder, R.D., <u>Engineering Mechanics of Deformable Bodies</u>, IEP, 1969.
7. Popov, E.P., <u>Engineering Mechanics of Solids</u>, Prentice-Hall, 1975.
8. Bickford, W.B., <u>Mechanics of Solids</u>, Irwin, 1993.
9. Craig, R.R., <u>Mechanics of Materials</u>, Second Edition, John Wiley & Sons, 2000.
10. Bedford, A. and Liechti, K.M., <u>Mechanics of Materials</u>, Prentice-Hall, 2000.
11. Gere, J.M., <u>Mechanics of Materials</u>, Fifth Edition, Brooks/Cole, 2001.
12. Hibbeler, R.C., <u>Mechanics of Materials</u>, Second Edition, Prentice-Hall, 2005.
13. Riley, W.F., Sturges, L.D., and Morris, D.H., <u>Mechanics of Materials</u>, Sixth Edition, John Wiley & Sons, 2007.
14. Beer, F.P. and Johnston, E.R. Jr., <u>Mechanics of Materials</u>, Second Edition, SI Units, McGraw-Hill, 1992.
15. Shigley, J.E., <u>Mechanical Engineering Design</u>, Second Edition, McGraw-Hill, 1972.
16. Edwards, K.S. Jr. and McKee, R.B., <u>Fundamentals of Mechanical Component Design</u>, McGraw-Hill, 1991.
17. Hamrock, B.J., Jacobson, B., and Schmid, S.R, <u>Fundamentals of Machine Elements</u>, McGraw-Hill, 1999.
18. Juvinall, R.C. and Marshek, K.M., <u>Fundamentals of Machine Component Design</u>, Third Edition, John Wiley & Sons, 2000.
19. Spotts, M.F. and Shoup, T.E., <u>Design of Machine Elements</u>, Seventh Edition, Prentice-Hall, 1998.
20. Mott, R.L., <u>Machine Elements in Mechanical Design</u>, Fourth Edition, Prentice-Hall, 2004.
21. Norton, R.L., <u>Machine Design: An Integrated Approach</u>, Third Edition, Prentice-Hall, 2006.