A Novel Method of Teaching Dimensions, Units, Dimensional Homogeneity and Dimensional Analysis

By

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Abstract

When most students are asked, “what are the primary units of the Systeme Internationale unitary system” (commonly referred to as the SI system of units, or the metric system), their first response usually is, "I was never taught that." Their second response is, "what is a primary unit?" And, their third response is, "how many of these primary units are there, anyway?" Unfortunately, these first responses are just as liable to come from fourth-year engineering students, as they are to come from first-year engineering students. All too often, these are the same first responses that are given by many Engineering educators, as well. And, since the incorrect use of Dimensions and Units contributes to such a high percentage of failures in the Engineering profession, it is also an indictment of the fundamental Dimensions and Units education in this country. Dimensional homogeneity, dimensional analysis and similitude are three of the most important fundamental design tools in engineering. Many introductory college classes in Engineering, and most introductory engineering textbooks, usually start off with a discussion on Dimensions and Units. But, that cursory review of dimensions and units in most first-year engineering courses is where most of the students' Dimensions and Units education stops. The lack of a proper knowledge of Dimensions and Units is one of the greatest contributors to catastrophic failure in scientific, industrial and manufacturing endeavors. And yet, only a casual introduction to dimensions and units is given to first-year engineering students, with the expectation that they will somehow eventually teach themselves the conceptual links and intricate interrelationships between the various engineering dimensions used in their discipline. This paper will examine case studies of how incorrect dimensions resulted in catastrophic failure, and will present a Primer on Dimensions and Units that can be used by Engineering Educators as a model to effectively give engineering students at any stage of their education a thorough introduction to Dimensions and Units, Dimensional Homogeneity and Dimensional Analysis.

Introduction

In a one hour college lecture, starting with the six primary units of the SI system of units (the Amp, the Candela, the Kelvin, the meter, the second and the kilogram), most other units used by mechanical and electrical engineering students can be derived and expressed in a combination of these six primary units. For instance, a Farad is really an ($\text{Amp}^2 * \text{seconds}^4$) per ($\text{kilogram} * \text{meter}^2$). A Coulomb is an (Amp * second) and a Weber is a ($\text{kilogram} * \text{meter}^2$) per ($\text{second}^2 * \text{Amp}$). Few students, and even some Engineering Educators, do not realize that a Volt is really a ($\text{kilogram} * \text{meter}^2$) per ($\text{Amp} * \text{seconds}^3$). But, all of them should know that, in order to provide the proper factor of safety in any engineering design. And, in knowing how all of the various derived units are comprised of the primary units in any Unitary System will enhance their understanding of dimensions, and how various dimensions inter-relate to one another. In this paper, an examination of all of the various unitary systems used throughout the world could be undertaken, where the similarities and differences between the various unitary systems are examined, detailing how many of them share the same primary dimensions, and denoting the differences between the primary units in each of those unitary systems. But, that would be counterproductive in a paper that is designed to be too brief to do that topic justice. Instead, I would like to concentrate on the accepted Systeme Internationale unitary system that has been adopted as the standard unitary system throughout the world. In all engineering education programs in the United States, and throughout the entire developed world, the Systeme Internationale units are taught to all incoming engineering students, and used mainly by those students to solve both their homework problems and examination problems. The primer on dimensions and units is designed to be taught to either electrical engineering or mechanical engineering students, but can easily be modified for engineering students of other disciplines, or for science students.

Two Case Studies in Which Improper Units Caused Catastrophic Failure

On July 23, 1983, Air Canada Flight 143 was supposed to fly from Montréal, Québec to Edmonton, Alberta. Unfortunately, the fuel gauge on the Boeing 767 -- 200 was malfunctioning, so the pilots had no way of monitoring the fuel consumption in flight. This nonworking fuel gauge should have grounded the Boeing 767 -- 200 immediately. But, an Air Canada ground crew maintenance worker incorrectly informed the captain that the plane could still fly if the ground crew simply determined the required amount of fuel that is necessary to fly from Montréal to Edmonton, and then filled the tanks with that amount of fuel. The ground crew members correctly measured the fuel in each of the two tanks, determining that there were 3758 L in one tank and 3924 L in the other tank. This gave a total of 7682 L of fuel for the journey from Montréal to Edmonton. So, at this point in time, the airliner should have been grounded, but was still perfectly able to fly. And, if the series of following mistakes it not occurred, the plane, the pilot and all the passengers would have arrived in Edmonton, never having known that the fuel gauge was not working.

The ground crew knew that the plane needed 22,300 kg of fuel to fly from Montréal to Edmonton. And, they knew that the plane already contained 7682 L of fuel. What they did not know was fundamental engineering education concerning dimensions and units concepts. In fact, all they had was a conversion factor with no units. They did not have the basic knowledge to determine if this was the correct conversion factor, nor any way of independently verifying that the conversion factor was correct. But, they used the conversion factor anyway. The conversion factor was 1.77. Unfortunately, this was not the conversion factor between liters of fuel on the plane and kilograms of fuel on the plane. Instead, it was the conversion factor between liters of fuel on the plane and pounds of fuel on the plane. So, even though it was calculated that the plane had 13,597 kg of fuel already on the plane, in reality the plane only had 13,597 pounds of fuel on the plane.

After the ground crew estimated that the plane had 13,597 kg of fuel already on the plane, they then deduced that the plane would need an additional 8703 kg of fuel to make it to Edmonton with the adequate safety factor. Dividing 8703 kg by 1.77, they incorrectly determined that the plane would need an additional 4916 L of fuel to make it to Edmonton. They then added an additional 5000 L of fuel to the plane, and told the captain that everything was fine and ready to go. But—everything was not fine and ready to go. In fact, everything was wrong. Now, the plane that was never supposed to leave Montréal because the fuel gauge was not working, was about to take off with only 12,598 L of fuel in its two tanks. Since the conversion factor that the ground crew had used was the conversion factor between liters and pounds, rather than between liters and kilograms, the plane only had 10,136 kg of fuel onboard. This was less than half the fuel that the Boeing 767 -- 200 needed to make it to Edmonton.

In actuality, the correct conversion factor that should have been used was .803 kg for every liter of fuel in the plane's fuel tanks. That would have meant that the plane originally had 6169 kg of fuel on board. Since 22,300 kg of fuel were needed to make the flight to Edmonton, that meant that 16,131 kg of fuel was needed to be added to the plane's fuel tanks. 16,131 kg of fuel divided by .803 kg per liter of fuel, meant that 20,163 L of fuel had to be added to the fuel tanks to allow the plane to make it to Edmonton. But, the plane never made it to Edmonton. At41,000 feet, the left forward fuel pump ceased operation. And, a few seconds later, a second fuel pump ceased operation.

Now, two critical safeguards that had been put in place for the safety of the passengers on the Air Canada Boeing 767 -- 200 had failed. The plane never should have taken off because the fuel gauge was not working. And, the ground crew, which should have known how much fuel to put in the fuel tanks, was critically under educated in the use of dimensions and units. Now, at 41,000 feet, the only remaining safeguard to the lives of the 61 passengers on board flight 143 were the pilot and the copilot sitting in the cockpit. Thankfully, they were up to the task. They made a dead stick landing of the plane on a drag strip in Gimli, Canada, and saved the lives of all of the people on that plane.

NASA did something quite similar with the loss of the Mars Climate Orbiter, a $125 million Mars orbiter that was to work in conjunction with the Mars Polar Lander in a $500 million mission to study the red planet. NASA had turned over the guidance of the Mars Climate Orbiter to a Lockheed Martin team that had helped develop the spacecraft. Unfortunately, the Lockheed Martin team had been performing all their calculations in pounds of thrust, rather than in newtons of thrust that NASA required. Of course, over the entire 286 days of the mission, the discrepancy was only approximately 60 miles (or, should I say 100 km), so the slight course correction was lost in the vast distance that the orbiter had traveled to Mars. In fact, as the orbiter reached Mars, the trajectory was so slightly off, that it was imperceptible to the tracking devices used to determine if the orbiter was on the correct course.

It was only after the orbiter had entered the Martian atmosphere, and had disappeared behind Mars that the full extent of the problem was realized. As the Mars orbiter disappeared behind Mars, its main thruster shutdown due to overheating. But, it was not that the thruster was overheating, it was that the orbiter was slowly sinking into the atmosphere of Mars, and was beginning to burn up. In actuality, the Mars orbiter did not burn up. But, after being charred and ripped to pieces as it made its way through the Martian atmosphere, it continued on into outer space, and is now in orbit around the sun. This great loss was entirely due to the fact that the Lockheed Martin team that was overseeing the orbiter's trajectory to Mars was still using nonconventional, nonstandard English units, rather than the accepted, standardized SI units that the rest of the world uses on a daily basis.

A Primer on Dimensions and Units, Dimensional Homogeneity and Dimensional Analysis

The six primary units of the Systeme Internationale are the Amp, the Kelvin, the Candela, the meter, the second and the kilogram. All of the derived units are derived as a combination of these six primary units in the Systeme Internationale. In any discussion with students concerning dimensions and units, it is always essential first to differentiate between those two entities--dimensions and units. Dimensions are the physical attributes of an engineering system, such as pressure, velocity, force or energy. These dimensions are independent of the unitary systems that are created to quantify them. Perhaps, the easiest derived units to teach engineering students are the units that they already have an intuitive understanding for in daily life. And, then to extend their understanding from those intuitive units that they see in everyday life into a more meaningful understanding of the derived units that they will be encountering throughout their engineering education.

Examples of dimensions that the students encounter in everyday life, and have learned since childhood, are the dimensions of area and volume. The combination of primary units that comprise dimensions of area and volume have no nicknames like the Newton or the Joule, but are taught to students in elementary school, and are very familiar to all students entering an introductory engineering course. As an added simplicity, both the dimensions of area and volume only use one primary unit to describe themselves. The Systeme Internationale units of area are square meters, and the units of volume are cubic meters.

Another dimension that is used by engineering students in their daily life is velocity. They know and understand the concept of velocity through their interaction with driving a vehicle. From an early age, they have the experience of knowing how fast the car is going in (miles) per (hour). Therefore, after introducing the familiar derived units of area and volume, the third derived unit that is easy for engineering students to understand is velocity. However, at this point, their intuitive construct of the concept of velocity includes (miles) per (hour), which are not the Systeme Internationale units that quantify the dimension of velocity. Also, the combination of primary units that denote the units of velocity are not a singular primary unit, as in the case of the dimensions of area and volume, but is a combination of two primary units, meters and seconds. Luckily, it is not much of an extension of understanding for the students to exchange meters for miles, and seconds for hours. The engineering students can easily exchange the SI units for the old Imperial or British gravitational units that are used traditionally in America to express velocity. In this way, the engineering student has exchanged his ideas of square feet, cubic feet and (miles) per (hour) for square meters, cubic meters and (meters) per (second).

Once velocity is introduced to the engineering student, it is not a far extension to also introduce the dimension of acceleration. Although not all students have learned about the gravitational constant in high school, most of them have had some familiarity with the concept of acceleration. When it is explained that acceleration is merely the change in velocity over the change in time, most students quickly can see that the result of the complex fraction of (meters) per (second) divided by seconds is (meters) per (second^2). In addition, most of the students have an understanding of the gravitational constant, g, which equals 9.81 m/s^2. This gives them a familiarity with the gravitational constant, but not with how the constant was derived. Once the student is shown that acceleration is simply the change in velocity with the change in time, and reminded that velocity is the change in distance over the change in time, then the revelation of where the units of the gravitational acceleration constant come from are no longer a mystery.

With students having an understanding of the dimensions of area, volume, velocity and acceleration, it is easy to then introduce Newton's second law, which introduces them to the concept of the dimension of force. Force is different from the preceding derived units that have been presented to the students up to this point. Area and volume were simply combinations of the same primary unit to describe two different dimensions. Velocity and acceleration, although combining two different primary units, still were familiar derived units for the students, and only included a combination of two different primary units. But, Force is different in the fact that it uses a combination of three primary units to describe itself. It is different in another aspect, as well. The combination of primary units that make up the units for the dimension of force have a nickname that allows engineering students to never have to understand the true underlying conceptual fundamentals of the dimension of force. Newton's second law tells us that force equals mass times acceleration. Therefore, combining the unit of mass, which is a primary unit in the SI system, and represented by a kilogram, with the combination of primary units that comprise the dimension of acceleration, meters per second^2, the result is a (kilogram*meter) per (second^2). Usually, engineering students are then told that a (kilogram*meter) per (second^2) is referred to as a Newton, and to use the term Newton, instead of (kilogram*meter) per (second^2), in all their engineering calculations from that point on in their life. I believe that allowing students to substitute 1N for 1 (kilogram*meter) per (second^2) is one of the greatest deficiencies in the instruction of dimensions and units in this country today.

Students should understand that the units of force are really (kilogram*meters) per (second^2). Because the understanding of force being (kilogram*meters) per (second^2) allows students to readily remember Newton's second law, it can also assist in understanding higher order derived dimensions. For instance, one equation that describes energy is that force times distance equals energy. So, if the students know that force times distance equals energy, then they can also understand that a (kilogram*meter) per (second^2) times a meter equals a (kilogram*meter^2) per (second^2). And, they can quickly be told that a (kilogram*meter^2) per (second^2) also has a nickname, and that is the Joule. This way, they are not lulled into a sense of using the nickname to represent the true underlying characteristics of the derived unit. They see and understand the derived unit as a combination of characteristics of the six primary units, rather than as a new separate unit.

To extend from the dimension of energy to the dimension of power is relatively easy for the engineering students to understand. That is not to say that most first-year engineering students understand the relationship between power and energy, but once it is explained to them, they easily understand the relationship. In my classes, I like to point out that I would never buy a 200 GigaJoule power plant, and then ask the students why. Eventually, after several students offer an incorrect explanation, it is explained to them that if 200 GigaJoule's of energy were being delivered by the power plant over a 100-year period, then the generator on the front wheel of my bicycle would easily be able to deliver that type of power.

I also ask the students if any of them had ever paid an electricity bill. There are always a few students that actually have paid an electricity bill prior to coming to college, and they explain that they pay for their electricity in kilowatt-hours. The students are then asked if kilowatt-hours are a measure of power or a measure of energy. Are people paying for the amount of energy that they use, or are they paying for the rate at which they use that energy? Surprisingly, most of the students do not know the answer to that question. But, when it is pointed out to the students that power is simply energy per unit time, then it all becomes clear that they are paying for energy when their electricity bill comes. But, when they buy a power plant, it would be better to know the number of watts generated by the power plant, rather than the amount of Joules of energy produced.

At that point, the students can also see that the (kilogram*meters^2) per (seconds^2) that comprise the Joule, divided by a second, equals a (kilogram*meters^2) per (seconds^3). And, just like the Newton and the Joule, a (kilogram*meter^2) per (seconds^3) has a nickname, and that nickname is the Watt. Other dimensions that only use three primary units in their combination of primary units can also be introduced to the students at this time. For instance, although most first-year engineering students do not know that pressure is force divided by cross-sectional area, once that relationship is described to them, they can easily figure out the combination of primary units that describe the dimension of pressure. A (kilogram*meter) per (second^2), divided by square meters, equals (kilograms) per (meter*second^2). The students are then told that a (kilogram) per (meter*second^2) is usually referred to as a Pascal, which is the nickname for the combination of SI units that describe the dimension of pressure. Other similar dimensions, which are comprised of the kilogram, meter and second can also be introduced to the students at this time.

Generally, the students have a much better understanding of the conceptual underpinnings of the meter, the kilogram and the second than they do of the other three primary units of the Kelvin, the Candela and the Ampere. But, an easy way to bridge the gap between the three primary units that they are familiar with, and the three primary units that they are not as familiar with, is by using the dimension of power. Using the concept of power, and the Ampere, the combination of primary units that comprise the derived dimensions of current, voltage, resistance and impedance can quickly be ascertained. From the preceding discussion, the combination of primary units that make up the dimension of power is a (kilogram*meter^2) per (seconds^3). But, perhaps the most easily recognized equation of an electrical engineering student is that power equals current times voltage. Well, if power equals current times voltage, then voltage must equal power divided by current. Fortunately, the unit that describes the dimension of current in the SI system is also a primary unit, the Ampere. Therefore, the combination of SI primary units that describe the dimension voltage can quickly be determined to be a (kilogram*meter^2) per (seconds^3 *Amp). Students are then asked what the nickname might be for the combination of primary units that describe a unit of voltage in the SI system. Almost universally, the students realize that the nickname that is given to a (kilogram*meter^2) per (seconds^3 *Amp) is a volt.

Now that the students know that a volt is actually a (kilogram*meters^2) per (seconds^3 *Amp), they can use the second most widely familiar equation in electrical engineering to determine what the combination of primary units are for the dimension of resistance. Ohms law is very well known amongst electrical engineering students, and is routinely known amongst engineering students in general. Ohms law states that the voltage drop across a resistor is equal to the current passing through the resistor times the resistance of the resistor. But, if voltage equals current times resistance, then resistance must equal voltage divided by current. Since the combination of primary units that describes the volt is a (kilogram*meters^2) per (seconds^3 *Amp), then the combination of primary units that must describe the dimension of resistance must be a (kilogram*meters^2) per (seconds^3 *Amp^2).

Since impedance also shares the same combination of primary units, but is a separate dimension, this is a good time in the dimensions and units instruction to denote that several dimensions can have the same units that represent them, while still being absolutely different dimensions. Not only is resistance and impedance a good example of this, but also the vastly different dimensions of pressure and stress. Both pressure and stress are described by (kilograms) per (meter*second^2), which is commonly referred to as a Pascal. However, the dimensions of pressure and stress are vastly different, only sharing the same combination of primary units that describe them in the SI system of units. In other unitary systems that could be devised, the combination of primary units that describe the dimensions of pressure and stress might be vastly different from one another, depending on the bases of which the primary units for that unitary system are chosen.

The last of the purely electrical dimensions that is derived is that of capacitance. In order to deduce the combination of primary units that describes the dimension of capacitance, the electrical dimension and concept of charge has to be taught to the students. The concept of charge is easy to teach to the students when it is pointed out that one Ampere of current flowing through a wire for one second transports exactly one Coulomb of charge. I use an analogy of a gallon of water flowing through a pipe in one second to give them some understanding of current flow through copper wire. The wire is the pipe, a gallon of water is the Coulomb, and the (gallons) per (second) of water flow is the Ampere. In this way, the students can see that an Amp is simply a (Coulomb) per (second), or more correctly, a Coulomb is an (Amp*second). Thus, they have another electrical dimension, that of charge, described to them as a combination of two primary units.

The dimension of capacitance equals charge divided by voltage. The students now know that the combination of primary units that describe charge is an Amp* second, and that the combination of primary units that describe voltage is a (kilogram*meters^2) per (seconds^3 *Amp). Therefore, charge divided by voltage gives the combination of primary units for the dimension and capacitance as an (Amp^2 *seconds^4) per (kilogram*meter^2). Students are told that the common name for an (Amp^2 *seconds ^{4}) per (kilogram*meter^2) is a Farad.

Introducing new electrical dimensions, in addition to the dimension of capacitance, requires the students to understand dimensions that pertain to magnetism, and how electricity and magnetism interrelate to one another. The easiest magnetically related dimension to introduce to the students of electrical engineering is the Weber. If there is a change in magnetic flux of one Weber in the time period of 1 second, then a one Volt electromotive force will be induced in a wire. That relationship is usually referred to as Faraday's law. But, if a (Weber) per (second) equals a volt, then a volt times a second must equal a Weber. The students know from previous instruction that the combination of primary units that comprises a volt is a (kilogram*meter^2) per (seconds^3 *Amp). So, the combination of primary units that describes a Weber must be a (kilogram*meter^2) per (second^2 *Amp). After determining what the combination of primary units are that describe a Weber, it is much easier to describe the other two prominent dimensions used by engineers in the study of magnetism, those being the dimensions of inductance and of magnetic flux density.

It is a simple extension for the students to understand the concept of magnetic flux density, once they have been able to understand the concept of magnetic flux. Magnetic flux density is simply the amount of magnetic flux that is fluxing through a specified cross-sectional area. So, if the combination of primary units that describes magnetic flux is a (kilogram*meter^2) per (second^2 *Amp), then the combination of primary units that describes magnetic flux density is simply the units of magnetic flux divided by square meters. That quick division results in the combination of primary units for magnetic flux density being equal to a (kilogram) per (second^2 *Amp). This combination of primary units that describe the dimension of magnetic flux density is usually referred to by the nickname Tesla. It is interesting to note that in the combination of primary units that describe a Tesla, the primary unit of the meter is nonexistent. Only the three primary units of the kilogram, the second in the Ampere are involved in the combination of primary units that describes the Tesla.

The last dimension that has its combination of primary units derived for the students is the dimension of inductance. Wikipedia describes the SI unit that quantifies the dimension of inductance as:

> If the rate of change of current in a circuit is one ampere per second and the resulting electromotive force is one volt, then the inductance of the circuit is one Henry

Or, to put it more succinctly, if a volt differential is created by increasing the current in a circuit at one (Amp) per (second), then the inductance of the circuit will be one Henry. So, a unit of inductance in the SI system is a volt, divided by an (Amp) per (second). The combination of primary units that describe a volt are (kilogram*meter^2) per (seconds^3 *Amp), whereas the (Amp) per (second) is already comprised of primary units. Thus, a (volt *second) per (Amp) is in fact a (kilogram*meter^2) per (second^2 *Amp^2). And that derived unit is commonly called the Henry.

All of this instruction in dimensions and units can entirely be completed in one lecture class at the beginning of the semester.

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