A Nusselt Number Correlation Classification System

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I. Introduction

A main component of any undergraduate heat transfer course is the teaching of convective heat transfer and, in particular, the use of Nusselt number correlations to calculate convective heat transfer coefficients. Most of the standard heat transfer textbooks teach convection in a very compartmentalized way. There is normally a chapter on forced convection external flows, a chapter on forced convection internal flows, and a chapter on natural convection. This approach makes very good sense from the perspective of teaching students the physics of convective heat transfer; however, this does not follow a logical decision making process required for solving actual convective heat transfer problems. In actual heat transfer problems, the challenge is often in applying the appropriate judgment to identify the type of convective processes occurring in the situation. A disservice is done to the students if they are taught only from a compartmentalized perspective. They do not completely develop the judgment needed to solve actual heat transfer problems. A difficulty in teaching from a more integrated approach is the identification of the appropriate Nusselt number correlations once the nature of convective heat transfer process has been recognized. To address this concern a Nusselt number classification system has been developed.

There are examples of heat transfer classification systems that have appeared in the literature. One of the first of these is a convection coefficient classification by Lauer [1]. In this system, convection coefficients are classified and cataloged according to twenty two different geometries (e.g., Single Flat Vertical Surface-Vertical, Inside of Tubular Surface-Horizontal Tube, and Granular Solids), followed by six different thermal conditions (e.g., Cool Wall-Warm Gas, Cool Wall-Condensing Vapors, and Warm Wall-Cool Liquid), and finally by flow condition (natural convection, streamline flow, and turbulent flow). In fact it was this classification system and catalog that prompted the development of the system presented in this paper. Another example of a classification system in heat transfer is the transient heat conduction system proposed by Beck [2]. This a system that is used to classify different transient heat conduction problems and allows for the cataloging of analytical conduction solutions. Its basis is first identification of the coordinate system, Cartesian, cylindrical or spherical, followed by the type of boundary conditions specified for the problem. Finally, the form of the initial condition is specified. Using Beck's numbering system the problem denoted by X13B01T0Y21B21 represents the physical problem of heat conduction in a two dimensional rectangular slab with a specified temperature boundary condition (1st kind boundary condition) at x = 0 and convective boundary condition (3rd kind boundary condition) at x = a. X13 tells us that we have rectangular coordinates with first and third kind boundary conditions along the x-coordinate. The boundary condition modifier B01 indicates that at x = 0 the specified temperature condition is T = 0 and

at x = a the convective condition is a constant function. The next set is T0 which indicates that the initial temperature distribution is zero. Y21 indicates the y coordinate conditions are at y = 0 a prescribed heat flux and at y = b a specified temperature condition. The boundary condition modifier (B21) on the y-coordinate at y = 0 the prescribed heat flux is a function of time q(t) =ct and the boundary condition at y = a is a non-zero specified temperature. Once a problem is categorized, one may then select the appropriate Green's Function solution. There also exists a supplement to Beck's book that categorizes the solutions in Carslaw and Jeager [3] with the same system. A Green's Function may be determined for these problems as well.

This paper continues with a description of the Nusselt number correlation classification system. Then two tools to utilize the system are presented, a PC DOS program and a web based system. Finally, to demonstrate the classification system an example is presented that involves multiple convective processes.

II. Classification System

The basis of the classification system is to lead the user through a decision-making process: identifying the flow driver, flow domain, surface geometry, and flow or surface orientation. This will lead to a sufficiently thorough description of the convective process such that a limited number of appropriate Nusselt number correlations may be identified. The first decision that must be made by the user is identifying the mechanism which is driving the flow. Three possibilities are presented:

- I. Forced Convection
- II. Natural Convection
- III. Mixed Convection

Next the flow domain must be specified. Two choices are possible:

- A. External Flow
- B. Internal Flow

The surface geometry is then specified from the list below:

- 1. Flat surface
- 2. Sphere
- 3. Cylinder
- 4. Tube Bank
- 5. Packed Bed
- 6. Annulus
- 7. Parallel Plates
- 8. Parallelepiped
- 9. Duct
- 10. Round Impinging Jet
- 11. Slot Impinging Jet

- 12. Hexagon
- 13. Ellipse
- 14. Triangle
- 15. Octagon

If the flow driving force has been identified as natural convection the surface's orientation with respect to gravity is specified as

- a. Horizontal
- b. Vertical
- c. Inclined

Finally, if the flow driving force has been identified as forced convection the flow direction is specified as

- (i) Parallel to Axis(ii) Normal to Axis
- (iii) Oblique to Axis

For example the classification I-A-3-(ii) would represent the forced convection over (and across) a cylinder. An overall view of the classification scheme is shown in Figure 1.

For this classification system to be useful in engineering calculations there must be a catalog. Such a catalog has been developed for three of the standard heat transfer textbooks: Incropera and DeWitt [4], Çengel [5], and Bejan[6]. The results of applying this catalog are a list of equations from these texts and are presented in Table 1. Students are strongly urged to read the text associated with the equation, so that they can be certain of using the right equation for the specified problem.

III. Software Access Tools

Two software tools have been developed to enhance the students utilization of the Nusselt Number classification system. A PC/DOS program is available that provides the appropriate correlations in symbolic form. An example of a run of the program is shown in Figure 2. Note that only numbered menus are used for each judgment in the classification system rather then the letter/number combination introduced above. This is for computational convenience and has not been found a hindrance to students using either the PC/DOS program or the hard copy versions of the classification system. For the physical situation classified, II-B-8-b (natural convection inside a parallelepiped oriented vertically with respect to gravity), there are multiple correlations available. These are presented one at a time, interrupted by the Pause command, with any

Figure 1. Nusselt Number Classification System

Flow Driving Force

- I. Forced Convection
- II. Natural Convection
- III. Mixed Convection

Fluid Domain

- A. External Flow
- B. Internal Flow

Geometry

- 1. Flat surface
- 2. Sphere
- 3. Cylinder
- 4. Tube Bank
- 5. Packed Bed
- 6. Annulus
- 7. Parallel Plates
- 8. Parallelepiped
- 9. Duct
- 10. Round Impinging Jet
- 11. Slot Impinging Jet
- 12. Hexagon
- 13. Ellipse
- 14. Triangle
- 15. Octagon

Surface Orientation with respect to Gravity (for natural convection)

- a. Horizontal
- b. Vertical
- c. Inclined

Flow direction (for forced convection)

- (i) Parallel to Axis
- (ii) Normal to Axis
- (iii) Oblique to Axis

Nusselt Number Classification	Çengel [5]	Incropera and DeWitt [4]	Bejan [6]
I-A-1-(i) [laminar, average, isothermal]	Eq. (6-18)	Eq. (7.25)	Eqs. (5.82a), (5.82b), (5.83)
I-A-1-(i) [laminar, average, uniform heat flux]		Eq. (7.43b)	
I-A-1-(i) [laminar, local, isothermal]	Eq. (6-15)	Eqs. (7.21), (7.27)	Eqs. (5.79a), (5.79b)
I-A-1-(i) [laminar, local, isothermal, liquid metal]		Eq. (7.26)	
I-A-1-(i) [laminar, local, unheated starting length]		Eq. (7.37)	Table 5.4
I-A-i-(i) [laminar, local, non-uniform temperature]			Table 5.4
I-A-1-(i) [laminar, local, uniform heat flux]	Eq. (6-30)	Eq. (7.39)	Table 5.4
I-A-1-(i) [laminar, local, non-uniform heat flux]			Table 5.4
I-A-1-(i) [mixed, average, isothermal]	Eq. (6-27)	Eq. (7.33)	Eq. (5.134)
I-A-1-(i) [turbulent, average, isothermal]	Eq. (6-23)	Eq. (7.35)	
I-A-1-(i) [turbulent, local, isothermal]	Eq. (6-21)	Eq. (7.30)	Eq. (5.131')
I-A-1-(i) [turbulent, local, uniform heat flux]	Eq. (6-31)	Eq. (7.40)	
I-A-1-(i) [turbulent, local, unheated starting length]		Eq. (7.38)	
I-A-1-(ii)	Table 6-3	Eq. (7.45b) with Table 7.3	
I-A-2-(i,ii,iii)	Eq. (6-33)	Eq. (7.49)	Eq. (5.139)
I-A-2-(ii)[liquid droplet, average]		Eq. (7.50)	
I-A-3-(ii)	Table 6-3, Eq. (6-32)	Eqs. (7.45b) with Table 7.2, Eqs. (7.46), (7.47)	Eqs. (5.135), (5.136)
I-A-3-(ii)[local]		Eqs. (7.45a), Figure 7.9	
I-A-4-(ii)[average]		Eqs. (7.53), (7.57)	Eqs. (5.144), (5.147) with Figure 5.26
I-A-5-(ii)[average]		Eq. (7.80)	
I-A-9-(ii)	Table 6-3	Eq. (7.45b) with Table 7.3	
I-A-10-(ii) [array]		Eq. (7.73)	
I-A-10-(ii) [single]		Eq. (7.68)	
I-A-11-(ii) [array]		Eq. (7.76)	
I-A-11-(ii) [single]		Eq. (7.71)	
I-A-12-(ii)	Table 6-3	Eq. (7-45b) with Table 7.3	Eq. (5.142) w/ Table 7.1, Figure 7.16
I-A-13-(ii)	Table 6-3		Eq. (5.142) w/ Table 7.1, Figure 7.16

Nusselt Number Classification	Çengel [5]	Incropera and DeWitt [4]	Bejan [6]
I-B-3-(i) [laminar, fully developed]	Table 6-4		Table 6.1, Eqs. (6.56), (6.51), Figure 6.8
I-B-3-(i) [laminar, fully developed, uniform wall heat flux]		Eq. (8.53)	
I-B-3-(i) [laminar, fully developed, isothermal]		Eq. (8.55)	
I-B-3-(i) [laminar, developing flow]	Eq. (6-59)		
I-B-3-(i) [laminar, developing, isothermal]			Eq. (6.57), Figure 6.10
I-B-3-(i) [laminar, combined devloping, isothermal]		Eq. (8.57)	
I-B-3-(i) [laminar, thermally devloping, isothermal]		Eq. (8.56)	
I-B-3-(i) [laminar, developing, uniform heat flux]			Eq. (6.52), Figure 6.9
I-B-3-(i) [turbulent, fully developed]	Eqs. (6-62), (6-63), (6-64)	Eqs. (8.60),(8.61), (8.62), (8.63)	Eqs. (6.90), (6.91), (6.92), (6.93), (6.93'), (6.93'')
I-B-3-(i) [turbulent, developing]		Eq. (8.64)	
I-B-3-(i) [turbulent, fully developed, liquid metal, uniform		Eq. (8.65)	Eqs. (6.94), (6.95)
heat flux]			
I-B-3-(i) [turbulent, fully developed, liquid metal, isothermal]		Eq. (8.66)	
I-B-6-(i) [laminar, fully developed]		Table 8.2, Eqs. (8.73), (8.74) with Table 8.3	
I-B-7-(i) [laminar, fully developed]		Table 8.1	Table 6.1
I-B-7-(i) [turbulent]		Eq. (8.67) with Eqs. (8.60)-(8.66)	
I-B-9-(i) [laminar, fully developed]	Table 6-4	Table 8.1	Table 6.1, Table 6.2
I-B-9-(i) [turbulent]		Eq. (8.67) with Eqs. (8.60)-(8.66)	
I-B-12-(i) [laminar, fully developed]	Table 6-4		Table 6.2
I-B-13-(i) [laminar, fully developed]	Table 6-4		
I-B-14-(i) [laminar, fully developed]	Table 6-4	Table 8.1	Table 6.1
I-B-14-(i) [turbulent]		Eq. (8.67) with Eqs. (8.60)-(8.66)	
I-B-15-(i) [laminar, fully devloped]			Table 6.2
II-A-1-(a)	Table 7-1, Eqs. (7-12), (7-13), (7-14)	Eqs.(9.30), (9.31), (9.32)	Eqs. (7.77a), (7.77b), (7.78)
II-A-1-(b)	Table 7-1, Eqs. (7-9), (7-10), (7-11)		

Table 1. Catalog of Nusselt Number Correlations (continued)

Nusselt Number Classification	Çengel [5]	Incropera and DeWitt [4]	Bejan [6]
II-A-1-(b) [local]			Eqs. (7.51), (7.52a), (7.52b)
II-A-1-(b) [average]		Eqs. (9.24) with Figure 9.6, Eqs. (9.26), (9.27)	Eqs. (7.55), (7.55'), (7.61), (7.61'), (7.62), (7.62'), (7.66), (7.67), (7.69), (7.69a),
II-A-1-(c)	Table 7-1	Section 9.6.2	Eqs. (7.74), (7.75) with Eq. (7.62)
II-A-2-(a,b,c)	Table 7-1, Eq. (7-17)	Eq. (9.35)	Eq. (7.81), Eq. (7.86) w/ Table 7.1
II-A-3-(a)	Table 7-1, Eq. (7-16)	Eqs (9.33),(9.34)	Eq.(7.79), Eq. (7.86) with Table 7.1
II-A-3-(b)	Table 7-1, Eq. (7-15)		Eq. (7.83), Eq. (7.86) with Table 7.1
II-A-3-(c)			Eq. (7.86) with Table 7.1
II-A-8-(a)			Eq. (7.86) with Table 7.1
II-A-13-(a,b,c)			Eq. (7.86) with Table 7.1
II-B-2-(a,b,c)	Table 7-2, Eq. (7-38)	Eq. (9.62)	Eq. (7.109)
II-B-3-(b)	Table 7-2, Eqs. (7-22), (7-23), (7-24), (7-25), (7-26)		Table 7.2
II-B-6-(a)	Table 7-2, Eqs. (7-36), (7-37)	Eq. (9.59)	Eq. (7.107)
II-B-7-(a)		Eq. (9.49)	Eq. (7.96), Table 7.2
II-B-7-(b)		Eqs.(9.36), (9.39), (9.40), (9.45), Eq. (9.43) with Eq. (9.42), Eq.(.9.44) with Eq. (9.42), Eq.(9.46)	
II-B-7-(c)		Eq. (9.47)	
II-B-8-(a)	Table 7-2, Eqs. (7-27), (7-28), (7-29), (7-30), (7-31), (7-32), (7-33), (7-34), (7-35)	Eq. (9.49)	Eq. (7.105)
II-B-8-(b)	Table 7-2, Eqs. (7-22), (7-23), (7-24), (7-25), (7-26)	Eqs. (9.50),(9.51),(9.52),(9.53)	Eqs. (7.99), (7.100), (7.101), (7.103a), (7.103b)
II-B-8-(c)	Table 7-2	Eqs. (9.54),(9.55),(9.56),(9.57)	Eqs. (7.106a), (7.106b), (7.106c), (7.106d)
II-B-9-(b) [narrow channel]			Table 7.2
II-B-14-(b) [narrow channel]			Table 7.2
III	Eq. (7-44)	Eq. (9.64)	

Table 1. Catalog of Nusselt Number Correlations (continued)

Figure 2. Computer Run of Nu_Correlation.exe

* ***** ***** ***** ***** Nu_Correlations: A Nusselt Number Classification * and Catalog System * * Version 1.1 Copyright 1989 Todd Jammer * * * Craig W. Somerton * * Which option do you wish to pursue? 1. Find an appropriate Nusselt number correlation 2. Create a new correlation Input Choice Number: 1 Choose Flow Type 1. Forced 2. Natural 3. Mixed Input Type Number: 2 Choose Fluid Domain 1. External Flow 2. Internal Flow Input Domain Number: 2 Choose a Geometry 1. Flat Plate 2. Sphere 3. Cylinder 4. Tube Bank 5. Packed Bed 6. Annulus 7. Parallel Plates 8. Parallelepiped 9. Duct Input Geometry Number: 8

Figure 2. Computer Run of Nu_Correlation.exe (continued)

```
Choose an Orientation

1. Horizontal

2. Vertical

3. Inclined

Input Orientation Number: 2

Average, 2 < H/L < 10, Pr <10^5, Ra < 10^10

Nu = { .220 * [(Ra * Pr)/( .200 + Pr)]^ .280} * (H/L)^ -

.250

Pause - Please enter a blank line (to continue) or a DOS command.

Average, 1 < H/L < 40, 1 < Pr < 20, 10^6 < Ra < 10^9

Nu = .069 * Ra^ .333 * Pr^ .074

Would you like to run the program again? (Y or N)N
```

comments indicating additional restrictions or situations. The correlations are represented symbolically, but it seems pretty clear that a result such as

represents a correlation of the form

$$Nu = (0.220) \left[\frac{Ra \cdot Pr}{0.2 + Pr} \right]^{0.28} \left(\frac{H}{L} \right)^{-0.25}$$

When all of the appropriate correlations have been displayed the program prompts the user whether or not another convective situation will be specified. If a physical situation is specified for which a correlation does not exist, an appropriate message is displayed and the user is once again prompted for another case. Since this program was developed several years ago, not all geometries in the current classification system are included. The program is being revised to include these geometries. It should be noted that the program can build additional correlations into its catalog through choosing item 2 from the initial menu.

A web based version of the classification system has also been developed. By following hyperlinks on menu items associated with the classification scheme, the user eventually reaches a web page where the appropriate correlations are presented for the physical situation specified. Both the web based version and the PC/DOS program are available for access and download at the Thermal Engineering Computer Aided Design (TECAD) homepage in the Department of Mechanical Engineering at Michigan State University. The URL address is

http://www.egr.msu.edu/~somerton/TECAD.

IV. Example Problem

To demonstrate the classification system an example problem is presented which involves multiple convective processes.

<u>Problem:</u> We wish to calculate the heat transfer through a building wall of height 3 meters and width 7 meters. The wall is composed as follows:

0.05 m face brick with k = 1.3 W/m·K 0.30 m air gap 2" x 4" wood supports (k=0.15 W/m·K) within the air gap spaced 1 m apart in both the width and height direction 0.015 m dry wall board with k = 0.17 W/m·K

The outside air temperature is 275 K and there is wind blowing parallel to the width direction of the wall at 4 m/s. The inside air temperature is to be maintained at 293 K.

Solution: We begin by recognizing that our mathematical model for this problem will be the thermal circuit analogy. The appropriate thermal circuit is shown below:



The heat transfer through the wall will then be given by

$$\dot{q} = \frac{T_i - T_o}{R_{tot}}$$

where the total thermal resistance is given by

$$R_{tot} = R_o + R_{FB} + [1/R_w + 1/R_{AG}]^{-1} + R_{DW} + R_i$$

Hence the problem reduces to appropriately specifying the individual resistances. [Normally at this point of the solution each resistance would be specified according to the physical situation. However, since the focus of this example is the demonstration of the Nusselt number classification system only the specification of the convective resistances will be provided.]

 R_o : Outside convective resistance, $\frac{1}{h_o A_o}$, where the outside convective heat transfer coefficient will be specified by using the classification system. We recognize that we have

Forced Convection, External Flow, Flat Surface, Flow Parallel to the Axis or I-A-1-(i)

Using the hard copy catalog (Table 1) we find that we must also decide if the flow is turbulent, laminar, or mixed (laminar boundary layer at the leading edge then developing into a turbulent boundary layer). A good argument can be made that for a wind blowing around a building the flow is generally turbulent. The next decision that must be made is whether a local or average correlation should be used. Since we want the total heat transfer, the average correlation is required. There is no unheated starting length in the physical situation, so those correlations are eliminated. Finally, a decision needs to be made on whether the surface boundary condition is isothermal or uniform wall heat flux. Unfortunately, neither of these conditions are rigorously adhered to and there are no correlations for the third kind boundary condition that actually

exists. Hence, some compromise must be reached to proceed with the solution. But then real engineering is all about compromise. In an arbitrary way we chose the isothermal condition, which yields

From Incropera and DeWitt: Eq. (7.33) From Cengel: Eq. (6-23) From Bejan: Eq. (5.131)

 R_{AG} : Air gap convective resistance, $\frac{1}{h_{AG}A_{AG}}$, where the air gap convective heat transfer coefficient will be specified by using the classification system. We recognize that we have

Natural Convection, Internal Flow, Parallelepiped, Vertical Orientation or II-B-8-b

From the catalog there are no other decisions to be made which gives

From Incropera and DeWitt: Eqs. (9.50),(9.51),(9.52),(9.53) From Cengel: Table 7-2, Eq. (7-22), (7-23), (7-24), (7-25), (7-26) From Bejan: Eq. (7.99), (7.100), (7.101), (7.103a), (7.103b)

Since there are several possibilities, the student must be directed to read the text around the equation to identify the most appropriate correlation.

 R_i : Inside convective resistance, $\frac{1}{h_iA_i}$, where the inside convective heat transfer coefficient will be specified by using the classification system. We recognize that we have

Natural Convection, External Flow, Flat Surface, Vertical Orientation or II-A-1-b

From the catalog there are no other decisions to be made which gives

From Incropera and DeWitt: Eqs. (9.24),(9.26),(9.27) From Cengel: Table 7-1, Eq. (7-9), (7-10), (7-11) From Bejan: Eq. (7.55), (7.55 '), (7.61), (7.61 '), (7.62), (7.62 '), (7.66), (7.67), (7.69), (7.69a), (7.69b), (7.70), (7.70 '), (7.70 ''), (7.70 '' ')

Once again since there are several possibilities, the student must be directed to read the text around the equation to identify the most appropriate correlation.

V. Conclusions and Recommendations

A classification system for identifying the physical situation for a convective heat transfer problem has been developed and when coupled with several catalog systems allows for the easy access of appropriate Nusselt number correlations. Students are strongly encouraged to read the text associated with these correlations to confirm their proper use. Catalogs have been assembled for three of the more popular heat transfer texts. A PC/DOS based catalog system and a web based catalog system have also been developed. Utilizing this classification approach in teaching students convective heat transfer enhances the modeling capabilities for the students and can lead them to realize appropriate approximations for physical situations.

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