

## **2006-2344: A POWER SYSTEMS ANALYSIS PROJECT**

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# A Power Systems Analysis Project

## Abstract

This paper describes a six-week laboratory project in which students analyze a small power system. The analysis is done with PowerWorld and is supplemented with MatLab calculations. The students were provided information concerning a small (seven bus, two generator) power system. They were required to assemble all of the information into tables, both in actual values as well as in per unit on a common base, before beginning the analysis. Since the system was small, students could manually calculate Ybus and Zbus values and compare them to the PowerWorld results. Any discrepancies had to be resolved in order to get the proper results. Once the system was correctly entered into PowerWorld, a power flow and fault analyses were done. Each week the students were required to produce interim products which were graded to insure they were on track. At the end, they were required to produce a formal report with all of their results. This project contributes to several of the ABET a-k outcomes, both for technology and engineering. Producing the report contributes to their communication skills, the analysis and correcting of the system contributes to their design skills, and the use of PowerWorld and MatLab, which are both commercial products, contributes to their ability to use current tools of the trade.

## Introduction

The Engineering Accreditation Commission (EAC) and the Technology Accreditation Commission (TAC) of ABET have both moved to outcomes-based accreditation of engineering and technology programs, via EC 2000 and TC2K, respectively. Both EC 2000 and TC2K require that every accredited program develop a set of program outcomes, which must insure that students have demonstrated the achievement of eleven outcomes, the so-called “a” to “k” lists. Table 1 shows a listing of the “a” to “k” outcomes for EC 2000 and TC2K.<sup>1,2</sup> With the change from previous accreditation criteria, ABET has gone away from the so-called “bean counting” that required certain numbers of credit hours in various categories, such as mathematics, sciences, social sciences, and humanities. Instead each program must evaluate and assess its curriculum on a continuous basis to show that graduates are demonstrating the required outcomes.

Although the outcomes are different for the two sets of criteria, they have a number of similarities. In particular, both the EAC and TAC require an “ability to communicate effectively,” (item g. for both). Other similarities are noted between EAC item k. and TAC item a., which require the use or mastery of techniques, skills, and modern tools; EAC item c. and TAC item d., which both deal with design; EAC item b. and TAC item c., which both include interpretation of data; and EAC item d. and TAC item f., which deal with formulating and solving technical problems. The author has found that contributions can be made to all of these criteria with a power systems analysis project. This paper provides a brief description of the

Electrical and Computer Engineering Technology ( ECET) curriculum and assessment plan at Purdue University and ECET 331, “Generation and Transmission of Electrical Power,” which is a junior/senior elective course. The paper then provides a detailed description of a six-week power systems analysis project done as part of the laboratory portion of ECET 331.

**Table 1.** Comparison of EC 2000 and TC2K Outcomes “a” to “k”

EAC Required Outcomes (Criterion 3)	TAC Required Outcomes (Criterion 2)
<ul style="list-style-type: none"> <li>(a) an ability to apply knowledge of mathematics, science, and engineering</li> <li>(b) an ability to design and conduct experiments, as well as to analyze and interpret data</li> <li>(c) an ability to design a system, component, or process to meet desired needs</li> <li>(d) an ability to function on multi-disciplinary teams</li> <li>(e) an ability to identify, formulate, and solve engineering problems</li> <li>(f) an understanding of professional and ethical responsibility</li> <li>(g) an ability to communicate effectively</li> <li>(h) the broad education necessary to understand the impact of engineering solutions in a global and societal context</li> <li>(i) a recognition of the need for, and an ability to engage in life-long learning</li> <li>(j) a knowledge of contemporary issues</li> <li>(k) an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.</li> </ul>	<ul style="list-style-type: none"> <li>a. an appropriate mastery of the knowledge, techniques, skills and modern tools of their disciplines,</li> <li>b. an ability to apply current knowledge and adapt to emerging applications of mathematics, science, engineering and technology,</li> <li>c. an ability to conduct, analyze and interpret experiments and apply experimental results to improve processes,</li> <li>d. an ability to apply creativity in the design of systems, components or processes appropriate to program objectives,</li> <li>e. an ability to function effectively on teams,</li> <li>f. an ability to identify, analyze and solve technical problems,</li> <li>g. an ability to communicate effectively,</li> <li>h. a recognition of the need for, and an ability to engage in lifelong learning,</li> <li>i. an ability to understand professional, ethical and social responsibilities,</li> <li>j. a respect for diversity and a knowledge of contemporary professional, societal and global issues,</li> <li>k. a commitment to quality, timeliness, and continuous improvement.</li> </ul>

### The Purdue ECET Curriculum

The ECET program at Purdue is a 2+2 curriculum; i.e., students spend two years working toward an Associate of Science degree in EET, followed by two more years to obtain a B.S. degree. At the Purdue West Lafayette campus, very few students stop at the A.S. point. The curriculum, like most engineering technology programs, is a very hands-on program with labs accompanying almost every ECET course. For many years, the Purdue ECET program has taken pride in being one of only a few EET programs in the United States with a strong electric power program. The plan of study includes a required electrical power and controls course that has been previously described.<sup>3</sup> Junior/senior electives include a course oriented toward electric utility generation and transmission (ECET 331), an electrical distribution course,<sup>4</sup> and two controls courses. Despite the tradition, the power courses sometimes come into question. One way of defending the power courses is to make sure they contribute to the program outcomes, which reflect the ABET a-k items in Table 1.

As the department prepared for its first accreditation under TC2K, we developed program

outcomes. Later, as the process continued, it was decided that every course in the curriculum should contribute to some of the outcomes. In order to strengthen the case for ECET 331, I decided to incorporate a multi-week project as part of the lab, which would help meet several of the program outcomes.

### ECET 331

ECET is titled, “Electrical Generation and Transmission.” As such it covers power systems topics including construction, operation, and steady-state analysis of the synchronous machine; calculation of resistance, inductance, and capacitance of transmission lines, analysis of transmission line performance; power flow; and fault studies. A detailed listing of the topics and the hours allocated to each topic is shown in Table 2.

**Table 2.** Topics Covered and Hours Allotted in ECET 331

Introduction, energy usage (1)	Transmission line capacitance calculation (2)
The power system (1)	Transmission line performance (3)
Power circuit fundamentals (1)	Power flow analysis (2)
Synchronous machine: construction & operation (1)	Introduction to fault current analysis, three-phase faults, circuit breakers, and fuses (4)
Sync. mach., equiv. Circuit/phasor diagram (1)	Symmetrical components/sequence networks (2)
Sync. Mach. performance and V-curves (1)	Effect of transformers on fault currents (1)
Synchronizing & transient performance (1)	Unbalanced fault calculations (2)
Load sharing between generators (1)	Relays, applications, & coordination (5)
Transformer equiv. ckts., and phase shift (1)	Over-voltage protection & surge arrestors (2)
Per-unit system & calculations (2)	Substation grounding (2 hrs)
Phase-shifting and autotransformers (1)	Summary and review (1 hr)
Transmission line construction, resistance (1)	Exams (3)
Transmission line inductance calculation (2)	

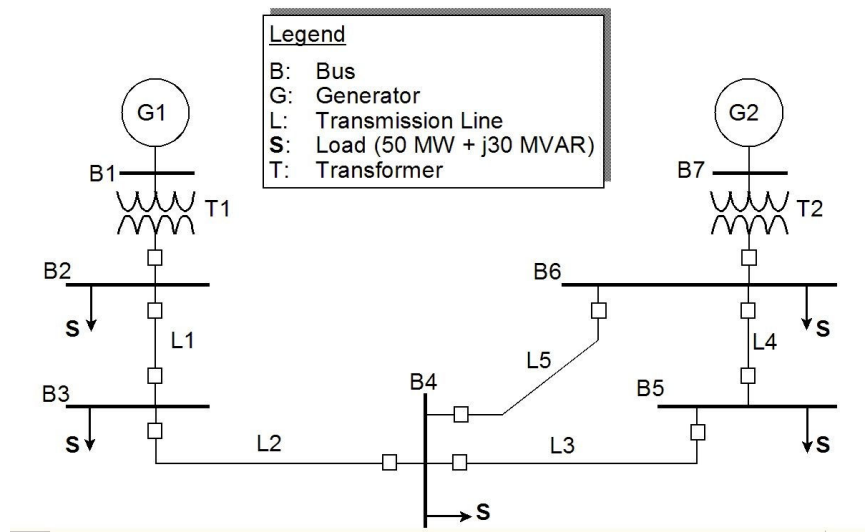
One problem, of course, with this type of material is the laboratory. At Purdue, we are fortunate to have some small synchronous machines, transmission line simulators, phase-shifting transformers, and other equipment. As a result approximately seven hardware-based lab experiments were developed for the course, which still left eight weeks in the semester. Over the last decade, these weeks were filled with computer studies utilizing various freeware packages, as well as an academic version of PowerWorld.<sup>5-7</sup>

PowerWorld began as a power flow program, but has evolved into a much more comprehensive power systems analysis package. In particular, it is also capable of conducting a variety of fault studies, both balanced and unbalanced. As a result, I decided to replace the several independent computer analyses with a more comprehensive project, using PowerWorld, that would take the student through the process of putting data into proper form, conducting power flow studies and correcting the system where required, conducting fault analysis studies, and documenting their work in a comprehensive report. The inclusion of this project makes a strong case for helping the students to meet program outcomes dealing with written communication, design, and data analysis. The project was done during the laboratory portion of the course, with outside time

for writing up the results. Regular homework assignments were still done throughout the six weeks of the project.

### The Power System Analysis Project

Figure 1 shows a one-line diagram of the system that was chosen for the project. It was taken from the course textbook.<sup>9</sup> Table 3 shows the parameters and other system modeling information for the system. This system was chosen for several reasons. Obviously it was in the textbook, but it also is complex enough to demonstrate some of the problems that occur in a power system, while being simple enough to allow some calculations by hand to verify the computer results. In addition, the free trial/academic version of PowerWorld is limited to twelve buses. This means the students can download the current version and use it at home as well as in the lab.



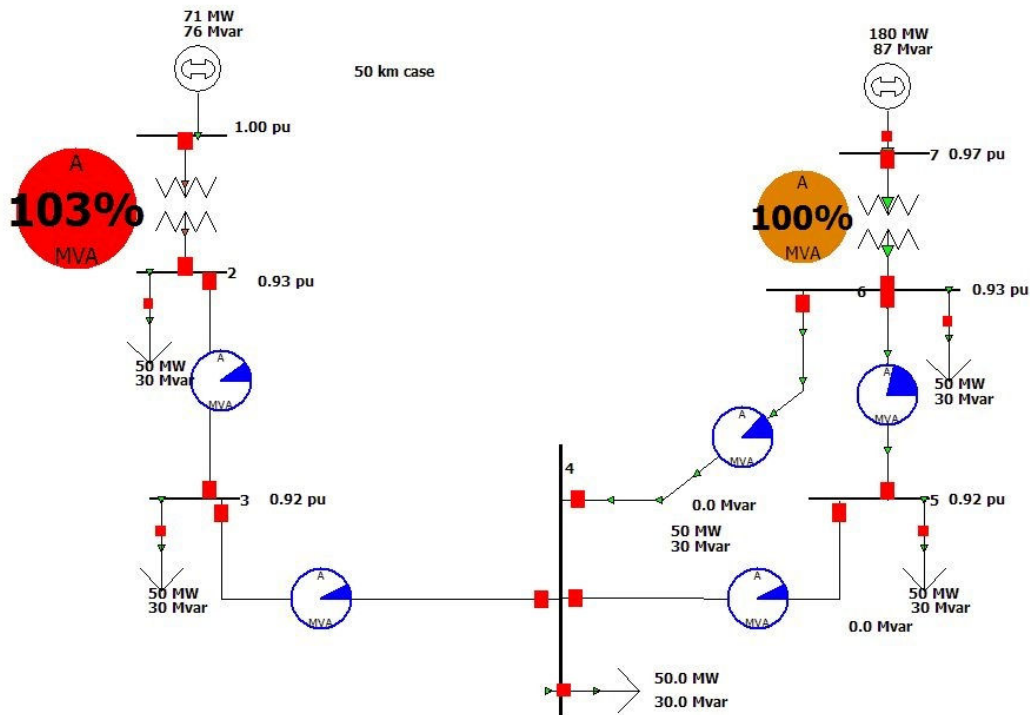
**Figure 1.** System One-Line Diagram for Power Systems Analysis Project

Observation of the system data shows that it is in a variety of forms; i.e., some of the data are in actual values, some in per-unit, and some in ohms/km. Thus, the first week's assignment for the students was to create data tables putting the data into the form required for the PowerWorld program. These tables were turned in as an interim product, so they could be verified. Errors were annotated and the students were required to fix them before continuing. Tables A1 to A7 in the Appendix show the results expected from week 1. Once the data was in the proper form, students were required to manually calculate  $Y_{bus}$ , neglecting the charging capacitance of the transmission lines, which is very straightforward.<sup>9</sup>

During the second week, students entered their system into PowerWorld. PowerWorld is a graphically-oriented program that allows the user to draw the system and enter data via dialog boxes. Figure 2 shows the solved case for the given system. Looking at the results in Figure 2, some problems are evident. First, transformer T1 is overloaded (for purposes of this study, it was assumed that it is permissible to load the transformers to 100%). Secondly, there are low voltages at all the buses except the generator buses. Finally, generator G2 is at its hard limit for providing reactive power and the bus voltage has dropped from the desired 1.0 to 0.97 pu.

**Table 3.** System Parameters and Modeling Information

<p><u>Generator Information</u>                  G1: 100 MVA, 13.8 kV, <math>X'' = 0.12</math> pu,  <math>X_2 = 0.14</math> pu, <math>X_0 = 0.05</math> pu                  G2: 200 MVA, 15.0 kV, <math>X'' = 0.12</math> pu,  <math>X_2 = 0.14</math> pu, <math>X_0 = 0.05</math> pu                  Generator neutrals are solidly grounded</p> <p><u>Transformer Information</u>                  T1: 100 MVA, 13.8–230 kV, <math>\Delta</math>-Y,  <math>X = 0.1</math> pu                  T2: 200 MVA, 15.0–230 kV, <math>\Delta</math>-Y,  <math>X = 0.1</math> pu                  Transformer neutrals are solidly grounded</p> <p><u>System Base Quantities</u>  <math>S_{base} = 100</math> MVA (three-phase)  <math>V_{base} = 13.8</math> kV (line-to-line) at Bus 1</p>	<p><u>Transmission Line Ratings</u>  <math>z_1 = (0.08 + j 0.5) \Omega/\text{km}</math>;  <math>z_0 = (0.2 + j 1.5) \Omega/\text{km}</math>;  <math>y_1 = 3.3(10)^{-6}</math> S/km                  Maximum MVA = 400 for all lines                  L1 = 15 km;                  L2 = 35 km;                  L3 = 40 km;                  L4 = 15 km;                  L5 = 50 km</p> <p><u>Power Flow Data</u>                  Bus 1: Slack bus                  Bus 2, 3, 4, 5, 6: Load buses                  Bus 7: Generator (PV) bus; <math>V = 15</math> kV;  <math>P = 180</math> MW; <math>-87</math> MVAR <math>&lt; Q &lt; 87</math> MVAR</p>
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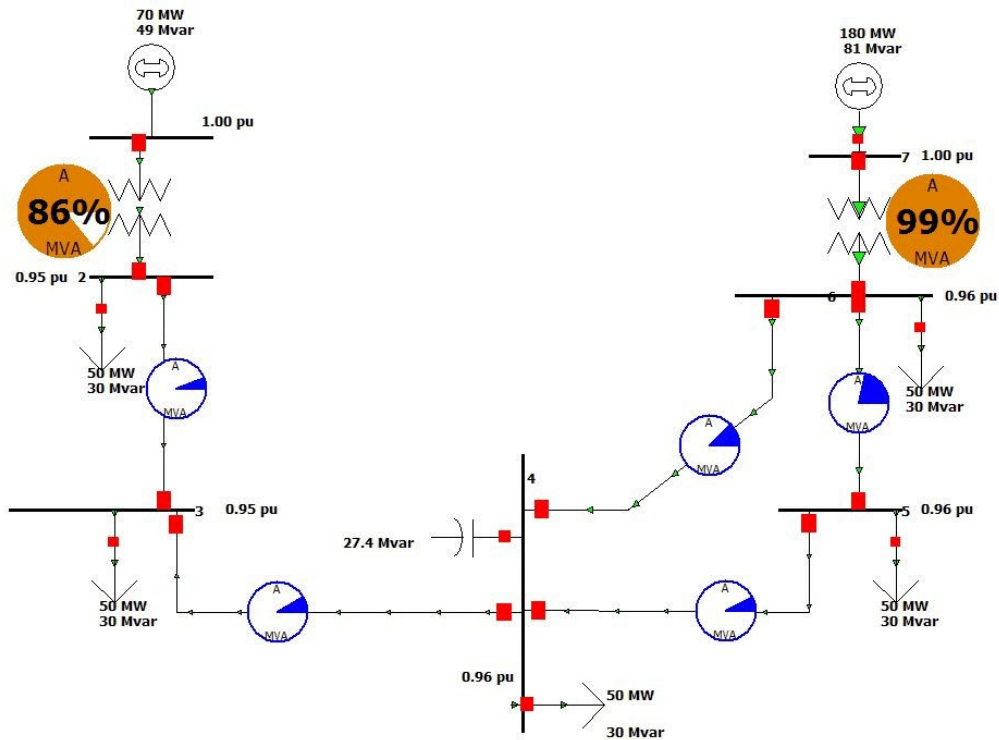


**Figure 2.** Power Flow Results for Base Case of System Shown in Figure 1

Once they had their power flow simulation running, students were required to verify that they

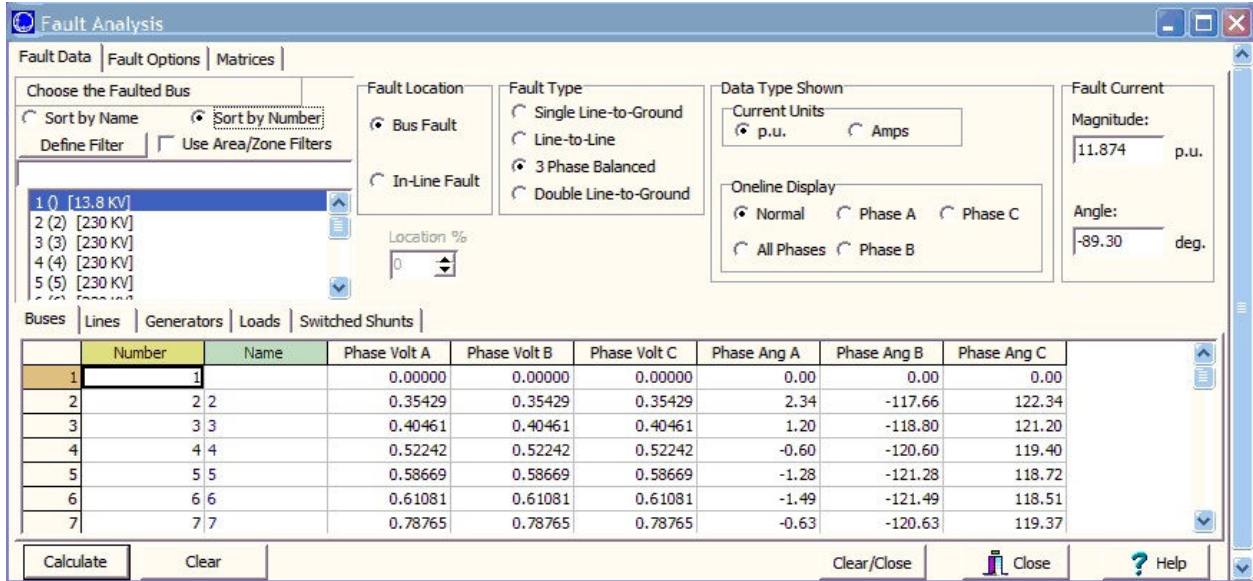
had entered their data correctly. PowerWorld provides a method to view Ybus, with and without charging capacitances, so students could compare their manually-calculated Ybus to the values shown in the program. Table A8 shows the positive-sequence Ybus values. Once their results were verified, students were assigned the task of correcting the system deficiencies. In particular, no device was to be loaded at greater than 100% of its rating and all bus voltages were to be greater than 0.95 pu.

In looking at the system as originally simulated, it is evident that the low voltages are due to the large lagging loads at buses 2 through 6. Thus, one possible method to correct the system deficiencies would be to add one or more capacitor banks. Figure 3 shows the power flow results for the system, with the addition of a large capacitor bank at bus 4. With this addition, both transformers are below 100% load and all of the system voltages are greater than 0.96 pu. Some students put in more than one bank and some used larger amounts.

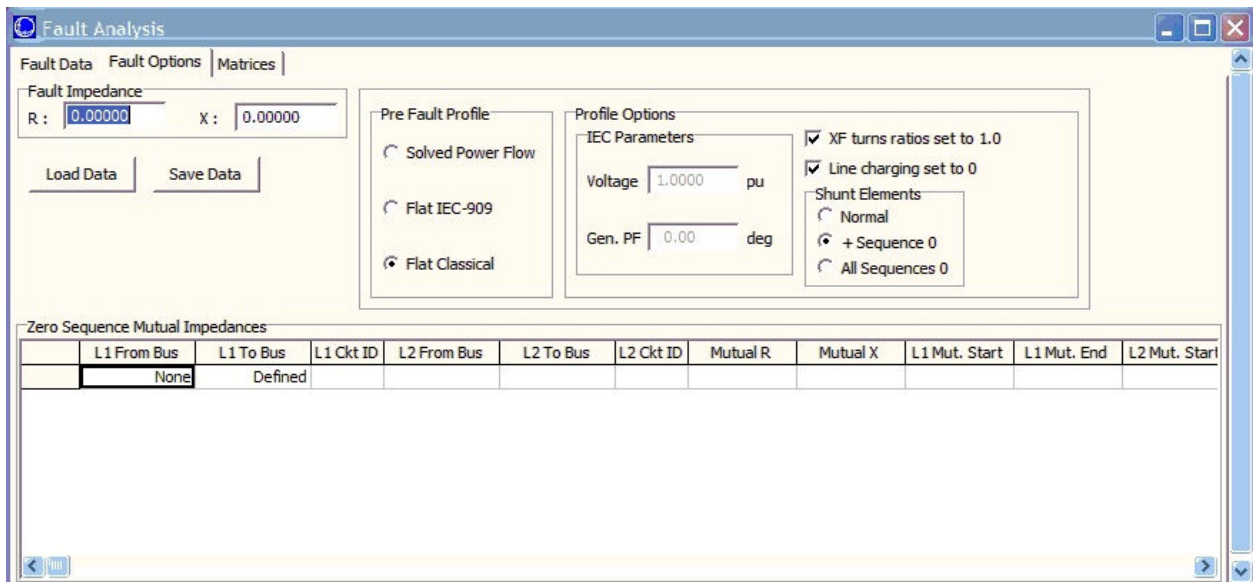


**Figure 3.** Power Flow Results for Corrected System

The third week of the project was devoted to balanced, three-phase fault analysis. PowerWorld provides a variety of options for fault analysis, as shown in Figures 4 and 5. Figure 4 shows the main setup tab for fault analysis. As shown, a balanced, three-phase fault has been selected at bus 1. Figure 5 shows the “Fault Options” tab. This tab allows the insertion of fault impedance (zero was used here). A “Flat Classical” pre-fault profile was chosen, along with line charging set to zero. These choices make it relatively simple for the student to verify the calculations. As is well known, the three-phase fault currents are obtained by dividing the pre-fault voltage (1.0 pu in this case) by the diagonal elements of Zbus, which are themselves the Thevenin



**Figure 4.** PowerWorld Fault Analysis Dialog Box



**Figure 5.** PowerWorld Fault Options Dialog Box

equivalents at each bus. Students were required to calculate the Thevenin equivalent at several buses and compare them to the Zbus values. PowerWorld does not display the values of Zbus, but it does allow the export of Ybus values as a MatLab m-file. The Ybus m-file was entered into MatLab and the inverse was found to yield the Zbus values. Table A12 in the appendix shows the positive-sequence Zbus. Once they had agreement between their manual calculations and Zbus, students were required to calculate the fault currents using the diagonal elements of Zbus and compare them to the values produced by PowerWorld. When done correctly, the values agree almost exactly. This process helps the student to understand how the program calculates fault currents, and provides methods for them to check their results. Note, for



example,  $Z_{bus}(1,1)$  in Table A12 has a value of  $(0.001 + j0.0842)$ . Putting that into polar form yields a magnitude of 0.0842, and taking the inverse yields a fault current of  $(1/0.0842) = 11.876$ , which compares almost exactly to the value of 11.874 shown in the upper right corner of Figure 4.

The final, and probably most complex, in-lab part of the project was to conduct unbalanced fault analyses at every bus in the system for each of the three unbalanced fault types shown in Figure 4. Calculation of unbalanced faults by hand requires the use of symmetrical components; however, the fault is relatively easy to calculate by hand, once the Thevenin equivalent impedances are known for the positive, negative, and zero sequence circuits. For this part of the project, the students were required to draw the sequence circuits and then to calculate the Thevenin impedance at bus 4. As shown in Tables A10 and A11, PowerWorld provides  $Y_{bus}$  matrices for the negative and zero sequence circuits. The students exported those to MatLab m-files and calculated  $Z_{bus}$  for each sequence. Table A13 and A14 show the negative and zero sequence  $Z_{buses}$ . With these, they were able to compare their manually-calculated values with the computer-derived values. They also were able to calculate the line-to-neutral unbalanced fault current at bus 4 and compare it to the PowerWorld results. As with the balanced fault, very good comparisons resulted when the proper procedures were followed. Once verified, the students ran PowerWorld to calculate all three unbalanced faults at each bus. As shown in Figure 4, PowerWorld shows the voltages at each bus in the box at the bottom. In addition, there are tabs for generators and lines, which show the currents in each phase for each device. The students were directed to export all of the data to Excel for use in their project report. For the final portion of the project, students were required to select oil circuit breakers from a table in the textbook to protect the transmission lines.

From the work done in lab, students should learn how to analyze their data and to verify it, which contributes directly to ABET outcomes. In correcting the power system, they made design decisions, also contributing to ABET outcomes. To complete the project, the students were required to write a formal report, which accounted for about 40% of their lab grade. Since the lab counts 25% of the course grade, the report essentially counted for 10% of the final grade.

### **The Final Report**

Table 4 shows the requirements given to the students for their final report. The reports were graded for both technical content and grammar/format. Due to the constant checks made along the way, virtually every student (about 50 in the last two years) was able to obtain data that was reasonably close to my values. The format was specified in some detail to insure that the students knew what was expected of them. Nevertheless, there was significant variation in the level of completeness and adherence to the specified format. Having the students do a detailed, lengthy report contributes directly to the ABET requirement for communication skills, and samples of the reports were used in the displays for our ABET visit.

**Table 4.** Requirements for Final Report

Prepare a **formal**, final report for the power system analysis project (labs 8 through 12). It should incorporate all of the work done in those labs. All figures should have figure numbers and captions and tables should have table numbers and captions. The report should be organized as follows:

**Introduction:**

Describe the system being studied (note a jpg image of the system can be downloaded from the 331 webpage) and the studies that were to be performed using PowerWorld.

**Procedures:**

Describe what you did for the power flow study (labs 8 and 9) and for the fault analysis study (labs 10 - 12). Describe how you prepared your data for entry into PowerWorld, and include your data tables.

**Results:**

Provide results from the power flow study of the base system with figures and tables.

Provide results of the fault studies. Include a table of fault currents showing the fault current for each type of fault at each bus, and include the all of the short-circuit analysis results as an appendix to your report.

**Analysis and Recommendations:**

Describe deficiencies noted from the original power flow and your suggestions to correct them. Show the results of your corrected system. Include appropriate figures and tables.

Indicate circuit breaker sizes and ratings as requested in lab 11. Explain your reasoning in selecting the size and ratings that you chose.

**Calculations and Verification:**

Include your sequence diagrams for the system and show the comparison of your manually-calculated Ybus values to those obtained from Power World. Using your sequence diagrams, calculate the Thevenin equivalent impedances in all three sequences from bus 4 to ground. Compare your values to those you obtained from MatLab. Calculate the three-phase fault current at bus 4 using the positive-sequence Thevenin impedance and calculate the line to ground fault current at bus 4 using the Thevenin impedances of all three sequences. Compare your results to the data from table 1.

**Summary and Conclusions**

Describe your confidence in your simulation of the power system; e.g., would you go to your boss and present a case for installing capacitor banks or upgrading the circuit breakers because of the results you obtained.

**Appendices**

## Conclusion

With the availability of a powerful power systems analysis package in a free demonstration/education version, it has become practical to implement a detailed power systems analysis project in an introductory power systems course. The project reported in this paper required students to correctly assemble their data, enter it into PowerWorld, conduct power-flow studies, correct the power system to overcome observed deficiencies, conduct balanced and unbalanced fault analyses, and to write a detailed final report on the project. Throughout the project students were required to verify the information coming from the simulation by comparing it to manual calculations. This helped to insure they were obtaining the correct results and taught them to verify computer results as much as possible. This multi-week project and final report contribute directly to several ABET outcomes including system design, analysis of data, and communication skills. For future offerings, it might be desirable to have the students do economic evaluations of alternatives to correct the power system, such as comparison of the cost of a new line versus capacitor banks.

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## Appendix

Table A1. Bus Data, Actual Values

Bus	Type	kV	delta	Gen P (MW)	Gen Q (MVAR)	Load P MW	Load Q MVAR	Gen Min MVAR	Gen Max MVAR
1	Slack	13.8							
2	Load	230				50	30		
3	Load	230				50	30		
4	Load	230				50	30		
5	Load	230				50	30		
6	Load	230				50	30		
7	Gen	15		180					

Table A2. Bus Data, Per-Unit Values

Bus	Type	V PU	delta	Gen P (PU)	Gen Q (PU)	Load P PU	Load Q PU	Gen Min Q PU	Gen Max Q PU
1	Slack	1	0						
2	Load					0.5	0.3		
3	Load					0.5	0.3		
4	Load					0.5	0.3		
5	Load					0.5	0.3		
6	Load					0.5	0.3		
7	Gen	1		1.8					

Table A3. Bus Knowns and Unknowns

Bus	Input Data	Unknowns
1	V & delta	Pgen & Qgen
2	P & Q	V & delta
3	P & Q	V & delta
4	P & Q	V & delta
5	P & Q	V & delta
6	P & Q	V & delta
7	V & Pgen	Q & delta

Table A4. Transmission Line Parameters, Actual Values

Bus-Bus	length	Series Impedance		Shunt Admittance		Max MVA
		R ohms	X ohms	G (S)	B (S)	
2-3	15	1.2	7.5		0.0000495	400
3-4	35	2.8	17.5		0.0001155	400
4-5	40	3.2	20		0.000132	400
4-6	15	1.2	7.5		0.0000495	400
5-6	50	4	25		0.000165	400

Table A5. Transmission Line Parameters, Per-Unit Values

Bus-Bus	length	Series Impedance		Shunt Admittance		Max MVA
		R ohms	X ohms	G (S)	B (S)	
2-3	15	0.002268	0.01418		0.02619	4
3-4	35	0.005293	0.03308		0.06110	4
4-5	40	0.006049	0.03781		0.06983	4
4-6	15	0.002268	0.01418		0.02619	4
5-6	50	0.007561	0.04726		0.08729	4

Table A6. Transformer Parameters, Actual Values

Bus-Bus	Equiv. Impedance		Shunt Admittance		Max MVA	Max tap
	R ohms	X ohms	G (S)	B (S)		
1-2	0	52.9			100	
6-7	0	52.9			200	

Table A7. Transformer Parameters, Per\_unit Values

Bus-Bus	Equiv. Impedance		Shunt Admittance		Max MVA	Max tap
	R ohms	X ohms	G (S)	B (S)		
1-2	0	0.1			1	
6-7	0	0.1			2	

Table A8. Positive Sequence Ybus							
Number	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6	Bus 7
1	0.00 - j18.33	0.00 + j10.00					
2	0.00 + j10.00	11.01 - j78.76	-11.01 + j68.76				
3		-11.01 + j68.76	15.72 - j98.24	-4.71 + j29.48			
4			-4.71 + j29.48	12.14 - j75.90	-4.13 + j25.79	-3.3 + j20.63	
5				-4.13 + j25.79	15.13 - j94.55	-11.01 + j68.76	
6				-3.3 + j20.63	-11.01 + j68.76	14.31 - j109.39	0.00 + j20.00
7						0.00 + j20.00	0.00 - j36.67
Table A9. Negative Sequence Ybus							
Number	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6	Bus 7
1	0.00 - j17.14	0.00 + j10.00					
2	0.00 + j10.00	11.01 - j78.76	-11.01 + j68.76				
3		-11.01 + j68.76	15.72 - j98.24	-4.71 + j29.48			
4			-4.71 + j29.48	12.14 - j75.90	-4.13 + j25.79	-3.3 + j20.63	
5				-4.13 + j25.79	15.13 - j94.55	-11.01 + j68.76	
6				-3.3 + j20.63	-11.01 + j68.76	14.31 - j109.39	0.00 + j20.00
7						0.00 + j20.00	0.00 - j34.29
Table A10. Zero Sequence Ybus							
Number	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6	Bus 7
1	0.00 - j20.00	0.00 + j0.00					
2	0.00 + j0.00	3.08 - j33.10	-3.08 + j23.10				
3		-3.08 + j23.10	4.40 - j33.00	-1.32 + j9.90			
4			-1.32 + j9.90	3.40 - j25.49	-1.16 + j8.66	-0.92 + j6.93	
5				-1.16 + j8.66	4.23 - j31.76	-3.08 + j23.10	
6				-0.92 + j6.93	-3.08 + j23.10	4.00 - j50.03	0.00 + j0.00
7						0.00 + j0.00	0.00 - j40.00

**Table A12. Positive-Sequence Zbus Values**

$$z_{bus_1} = \begin{pmatrix} 0.001 + 0.0842j & 0.0019 + 0.0544j & 0.0013 + 0.0501j & 0 + 0.0402j & -0.0007 + 0.0348j & -0.001 + 0.0328j & -0.0005 + 0.0179j \\ 0.0019 + 0.0544j & 0.0034 + 0.0997j & 0.0024 + 0.0919j & 0 + 0.0737j & -0.0013 + 0.0639j & -0.0018 + 0.0601j & -0.001 + 0.0328j \\ 0.0013 + 0.0501j & 0.0024 + 0.0919j & 0.0035 + 0.0978j & 0.0008 + 0.0785j & -0.0007 + 0.068j & -0.0013 + 0.064j & -0.0007 + 0.0349j \\ 0 + 0.0402j & 0 + 0.0737j & 0.0008 + 0.0785j & 0.0025 + 0.0896j & 0.0006 + 0.0776j & -0.0001 + 0.0731j & -0 + 0.0399j \\ -0.0007 + 0.0348j & -0.0013 + 0.0639j & -0.0007 + 0.068j & 0.0006 + 0.0776j & 0.0022 + 0.0883j & 0.0005 + 0.0781j & 0.0003 + 0.0426j \\ -0.001 + 0.0328j & -0.0018 + 0.0601j & -0.0013 + 0.064j & -0.0001 + 0.0731j & 0.0005 + 0.0781j & 0.0008 + 0.0799j & 0.0004 + 0.0436j \\ -0.0005 + 0.0179j & -0.001 + 0.0328j & -0.0007 + 0.0349j & -0 + 0.0399j & 0.0003 + 0.0426j & 0.0004 + 0.0436j & 0.0002 + 0.051j \end{pmatrix}$$

**Table A13. Negative-Sequence Zbus Values**

$$z_{bus_2} = \begin{pmatrix} 0.00119 + 0.09469i & 0.00204 + 0.06229i & 0.00143 + 0.05768i & -0 + 0.04692i & -0.00078 + 0.04106i & -0.00107 + 0.03887i & -0.00062 + 0.02267i \\ 0.00204 + 0.06229i & 0.0035 + 0.10677i & 0.00245 + 0.09886i & -0 + 0.08042i & -0.00134 + 0.07038i & -0.00184 + 0.06662i & -0.00107 + 0.03885i \\ 0.00143 + 0.05768i & 0.00245 + 0.09886i & 0.00353 + 0.10468i & 0.00076 + 0.08517i & -0.00076 + 0.07455i & -0.00131 + 0.07057i & -0.00077 + 0.04116i \\ -0 + 0.04692i & -0 + 0.08042i & 0.00076 + 0.08517i & 0.00253 + 0.09625i & 0.00061 + 0.08428i & -0.0001 + 0.07979i & -0.00006 + 0.04654i \\ -0.00078 + 0.04106i & -0.00134 + 0.07038i & -0.00076 + 0.07455i & 0.00061 + 0.08428i & 0.00221 + 0.09498i & 0.00056 + 0.08481i & 0.00032 + 0.04946i \\ -0.00107 + 0.03887i & -0.00184 + 0.06662i & -0.00131 + 0.07057i & -0.0001 + 0.07979i & 0.00056 + 0.08481i & 0.00082 + 0.08669i & 0.00048 + 0.05056i \\ -0.00062 + 0.02267i & -0.00107 + 0.03885i & -0.00077 + 0.04116i & -0.00006 + 0.04654i & 0.00032 + 0.04946i & 0.00048 + 0.05056i & 0.00028 + 0.05865i \end{pmatrix}$$

**Table A14. Zero-Sequence Zbus Values**

$$z_{bus_0} = \begin{pmatrix} 0.05i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.00213 + 0.07285i & 0.0015 + 0.06118i & 0.00002 + 0.03395i & -0.00078 + 0.01913i & -0.00108 + 0.01357i & 0 \\ 0 & 0.0015 + 0.06118i & 0.00561 + 0.08712i & 0.00196 + 0.04839i & -0.00003 + 0.02731i & -0.00076 + 0.01941i & 0 \\ 0 & 0.00002 + 0.03395i & 0.00196 + 0.04839i & 0.00647 + 0.08209i & 0.00172 + 0.04641i & -0.00003 + 0.03302i & 0 \\ 0 & -0.00078 + 0.01913i & -0.00003 + 0.02731i & 0.00172 + 0.04641i & 0.00483 + 0.073i & 0.00036 + 0.04044i & 0 \\ 0 & -0.00108 + 0.01357i & -0.00076 + 0.01941i & -0.00003 + 0.03302i & 0.00036 + 0.04044i & 0.00052 + 0.04321i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.025i \end{pmatrix}$$