

A Practical Graphical Approach for Drawing Shear Force and Bending Moment Diagrams

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Abstract:

The ability to draw shear force and bending moment diagrams on beam-like components is an important skill for mechanical engineering students. We found that some students had difficulty to draw effectively the shear force and bending moment diagrams during the course and even in their senior year. Although the method of sections can produce all for drawing the diagrams, it becomes tedious when a beam is divided into several segments. The graphical approaches by free-hand sketch are discussed in every textbook and is a good approach for drawing the diagrams. But the procedures provided in textbooks are not easy to be followed effectively by students and could be improved. Many computer-based or similar programs for drawing the diagram are available, but their effectiveness is not validated by proper assessment. We had studied this issue in the past years and had developed a practical procedure for drawing the diagrams with a free-hand sketch in 2017. In the spring and fall semesters 2017, the proposed procedure was implemented successfully in different sections of Mechanics of Materials course. The proposed procedure for drawing the diagrams was assessed by a quiz given to different sections of the course. The feedback information was collected through the class survey. Based on the assessment and class survey, most students liked the proposed procedure and believed the method was practical. The assessment based on the quiz also showed the grades with the proposed procedure were significantly improved. This paper presents the practical graphical procedure for drawing the diagrams as well as the results of the class survey and the assessment.

1. Introduction

One of the primary objectives of the mechanical engineering program is that students can conduct mechanical system designs including mechanical component design. To design beam-like components such as beams and shafts, we must analyze the loading conditions on the components, that is, the Shear force and Bending moment diagrams (the S/B diagrams). So, the ability to draw the S/B diagrams on beam-like components is an important skill for mechanical engineering students. In our mechanical engineering program, the S/B diagrams of a beam were introduced during Engineering Statics by using the method of sections in the first semester of their sophomore year. In the second semester of their sophomore year in the course Mechanics of Materials, the S/B diagrams were discussed again by using both the method of sections and the graphical approach. During teaching the Mechanics of Materials course, it had been noticed that some students had difficulty in effectively drawing the S/B diagrams. Even during senior design projects in their senior year, it was still found that some students had some difficulty in drawing the S/B diagrams.

The method of sections ^[1,2] can be used to determine fully the shear force and bending moment at any cross-section of beams and to draw the S/B diagrams. When there are several external forces on a beam, the beam must be divided into several segments. The method of sections will be used repeatedly in each segment. Since it is tedious and time-consuming to use this method, the graphical approach for drawing S/B diagrams is introduced for quickly and effectively

drawing the S/B diagrams. The basic foundations of these graphical approaches are the same and are mainly based on the relationships between the distributed load, shear force and bending moments. The relationships are explained typically with the examples of special cases such as no distributed loading, uniformly distributed loading, concentrated loading and a concentrated couple of moments are explored. Thus, the corresponding procedures for drawing S/B diagrams are presented in the textbooks [2~5]. The faculty, having taught such a course several times, believed the procedures were clear and easy to follow, and students could easily follow it to draw effectively the S/B diagrams. But some students had difficulty to follow it. After discussions with many students, we found the issues for them were: (1) they didn't know how to start and (2) the procedure was not easy to follow.

Many educators have found that some students have difficulty to draw effectively the S/B diagrams, too. Many computer-based or similar programs have been developed to facilitate students in obtaining the shear force and bending moment diagrams [6~12]. For examples, in 1996, an interactive graphics-oriented beam analysis program [6] was developed for the Strength of Materials courses, which produced the shear force and bending moment diagram through the interactive graphic interface. In 2000, a series of novel Web-based instructional units for teaching structural mechanics were developed. The instructional units were centered on computer programs written in the platform-independent object-oriented Java programming language and could be used to construct the S/B diagrams [7]. Computer-based or similar programs for constructing shear force and bending moment diagrams were especially appealing to engineering mechanics instructors and students since they used an interactive interface and automatically generated the S/B diagrams. However, the assessment data showed that such computer-based multimedia programs did not enhance learning and did not improve exam scores, so the effectiveness of these tools was not validated by the assessment data [7,8,10,11,12].

We have studied this issue in the past few years, with students having difficulty to effectively draw the S/B diagrams. For mechanical engineering students, the free-hand sketch is an important method. They need to frequently draw free-body diagrams and sketch of part models or assembly as well as the working principle of their designs. Since the computer-based programs for the S/B diagrams are dependent of computers, we focused on the graphic approach with a free-hand sketch for drawing the S/B diagrams and developed a practical and easily executable graphical procedure. In 2017, we proposed and completed a practical executable graphic procedure for drawing the S/B diagrams. According to the relationship between external forces and internal shear forces and bending moments, six rules were summarized as part of the practical executable procedure. Two rules were used to determine the types of curves, which could be a horizontal line, a linear line, and a parabolic curve. Also, two rules were used to determine the magnitude of the shear force and bending moment at the right end of the same segment and the possible maximum bending moment in this segment. The last two rules were used to determine the values of the shear force and bending moment of another side of the right end of the same segment, which was the values of the shear force and bending moment of the left end of the adjacent segment. So, a seamless practical executable graphical procedure was proposed to facilitate students to draw effectively the S/B diagrams. In spring and fall semesters 2017, the proposed procedure was successfully implemented in the Mechanics of Materials course. The feedback about the procedure was collected through the class survey. A quiz was used to assess the effectiveness of the proposed approach. The same quiz was offered to two

groups of students, one of which were taught with the proposed approach and another of which was taught by the procedure presented in the textbook [2]. Through the assessment and class survey, most students liked the proposed procedure and believed that it was practical and executable. The grades with the proposed procedure were significantly higher than the grade without the proposed procedure. The practical graphical procedure for drawing the S/B diagrams, the data analysis results of the assessment and the class survey will be presented and explained in detail in the following sections.

2. The practical graphical approach for drawing S/B diagrams

2.1 What are the actual steps for drawing the S/B diagrams?

The proposed graphical approach in this paper will deal with typical loadings on the beam-like components such as uniformly distributed loading, concentrated lateral force and concentrated couples of moments because the main purpose of the graphical approach is to draw quickly and effectively the S/B diagrams on a beam. For more complicated loading conditions such as linear distributed loading, it is strongly recommended that the method of sections be used for drawing the S/B diagrams. This is because the graphical approach could become complicated and inconvenient and could not provide accurate diagrams for complicated loadings.

Some students claimed the procedure provided in the textbook [2] was not easy to follow, which is copied and attached as Appendix A at the end of this paper. To understand this, we scrutinized each step for drawing the S/B diagrams with a free-hand sketch. After discussions with students, we identified several key steps which should be used to explain how to draw the S/B diagrams with the free-hand sketch.

- (1) The beam is divided into several segments according to the applied loadings.
- (2) The S/B diagrams are first drawn in the most left segment and then continuously drawn in the next adjacent segment until the S/B diagrams are completed.
- (3) The shear force and bending moment at the left segment in the S/B diagrams are equal to the lateral force and a couple of moment at the left end of the beam.
- (4) The values of the shear force and bending moment of the left end of the same segment in the S/B diagrams are already known. So, the starting points of the S/B diagrams in this segment has been plotted. When the type of curves and the values of the shear force and bending moment at the right end are known, the S/B diagrams in this segment can be easily drawn by the free-hand sketch.
- (5) There is a joint (or dividing point) between current segment and the next adjacent segment. The values of the shear force and bending moment at the right side of the joint must be determined so that continuous diagrams in the adjacent segment can be drawn.

Based on our observation and conversations with students, they typically didn't have any issues with items (1) ~ (3), but they did have some problems with the items (4) and (5). This can be solved by the relationship between the distributed loading, shear force and bending moment and will be discussed in next section.

2.2 Six rules for the graphical approach

The Items (4) and (5) mentioned in section 2.1 lead to three questions: a) how to determine the type of curves of the S/B diagrams, b) how to calculate the values of the shear force and bending moment of the right end of the same segment, and c) how to calculate the values of the shear force and bending moment of the right side of the joint according to the loading conditions on the joint.

A free-body diagram of a very small segment Δx from a beam subjected to an arbitrary loading is shown in Figure 1, where $\omega(x)$ is a distributed loading; V and ΔV refer the shear force and its increment respectively; M and ΔM refer the bending moment and its increment respectively; $k(\Delta x)$ is the location of the equivalent concentrated force $\omega(x)\Delta x$ of the distributed loading over the segment Δx . The point O in Figure 1 is the geometrical center of the cross-section of the right side of the segment Δx .

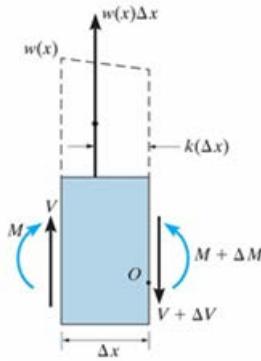


Figure 1 Free-body diagram of segment Δx ^[2]

When $\Delta x \rightarrow 0$, the equations of equilibrium on the segment will produce the following two equations:

$$\frac{dV}{dx} = w(x), \text{ or } \Delta V = \int w(x)dx \quad (1)$$

$$\frac{dM}{dx} = V(x), \text{ or } \Delta M = \int V(x)dx \quad (2)$$

The physical meaning of Equation (1) is the slope of shear force diagram is equal to the corresponding distributed loading or the difference of shear forces at two different points will equal to the area under the distributed loading between these two points. The physical meaning of Equation (2) is the slope of bending moment diagram is equal to the corresponding shear force, or the difference of the bending moment at two different points is equal to the area under the shear force between the two points. Equations (1) and (2) can be summarized in the following six rules, which can visually guide students to draw the S/B diagrams.

The shape of the diagrams in the same segment

Rule #1: For no load in the segment, that is, $\omega(x) = 0$

- The shear force diagram is a horizontal line.

- The bending moment is an oblique straight line. For positive V , it is an upwards oblique straight line; for negative V , it is a downwards oblique straight line; and for a zero-shear force, it is a horizontal line.

Rule #2: For a constantly distributed load $\omega(x) = \text{Constant}$

- The shear force diagram is an oblique straight line. For positive $\omega(x)$, it is an upwards oblique straight line, for negative $\omega(x)$, it is a downwards oblique straight line.
- The bending moment diagram is a concave up parabolic curve for positive $\omega(x)$ and a concave down parabolic curve for negative $\omega(x)$

Equations (1) and (2) can be rewritten in an integral form, from the left end to the right end of the same segment as followings.

$$V_{\text{right end}} - V_{\text{left end}} = \int_{\text{left end}}^{\text{right end}} w(x) dx \quad (3)$$

$$M_{\text{right end}} - M_{\text{left end}} = \int_{\text{left end}}^{\text{right end}} V(x) dx \quad (4)$$

The physical meaning of Equation (3) is the shear force increment between the right end and the left end of the same segment is equal to the area under the distributed loading $\omega(x)$ over the segment. Similarly, the meaning of Equation (4) is that the bending moment increment between the right end and the left end of the same segment is equal to the area under the shear force diagram $V(x)$ over the segment.

The relation of the shear force and bending moment between two ends of the same segment

Rule #3: The increments of shear force and bending moment between two ends

- The change in shear force between two ends of the segment is equal to the area under the distributed loading.
- The change at the moment between two ends of the segment is equal to the area under the shear diagram.

The whole beam is divided into several segments. On the dividing point or the joint, we could have concentrated lateral forces and/or a concentrated couple moment. The shear forces and the bending moments of the left side of the joint can be different from the shear force and bending moment of the right side of the joint. Figure 2 is a differential block about the joint with a concentrated lateral force F_0 . Applying the equations of equilibrium, we have these two equations:

$$V_{\text{right side}} - V_{\text{left side}} = F_0 \quad (5)$$

$$M_{\text{right side}} = M_{\text{left side}} \quad (6)$$

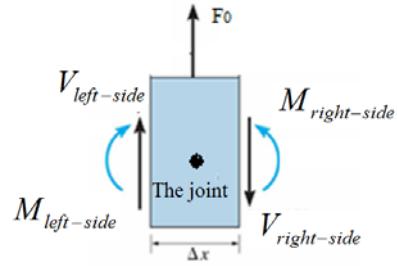


Figure 2 The Differential block around the joint with a concentrated lateral force

Figure 3 is the differential block around the joint with a concentrated couple moment M_0 .

Applying the equations of equilibrium again, we have another two equations:

$$V_{\text{right side}} = V_{\text{left side}} \quad (7)$$

$$M_{\text{right side}} - M_{\text{left side}} = M_0 \quad (8)$$

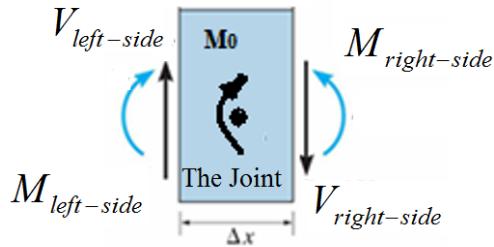


Figure 3 The differential block around the joint with a concentrated couple of moment

Equations (5) to (8) can be used to determine the values of shear force and bending moment on the right side of the joint.

The relation of the shear force and bending moment between two sides of the joint

Rule #4: The concentrated force at the joint

- The shear force change at the joint, where the concentrated force acts, is equal to the value of the concentrated force. When force is positive, the shear force jumps upwards. When force is negative, the shear force jumps downwards.
- There is no change in the value of the bending moment at the point, but the slope will be different.

Rule #5: The concentrated couple at the joint

- There is no shear force change at the point where a couple of moment acts.
- The bending moment change at the point where a couple of moment acts is equal to the value of a couple of moment. When couple moment is clockwise, the bending moment

jumps upwards. When couple moment is counter-clockwise, the bending moment jumps downwards.

The purpose of the S/B diagrams is to show the loading conditions of the beam, including the maximum or minimum loading conditions. When the bending moment is a parabolic curve, the maximum or minimum bending moment could happen between the two ends of the same segment. According to Equation (2), when the shear force is equal to zero, the bending moment must reach a maximum or minimum value. For example, Figure 4 shows the shear force and bending moment diagram of a segment with a downward uniform distributed loading. Based on the same concept of the Equation (4), the value of the maximum/minimum bending moment inside the segment will be equal to the value of the bending moment at the left end of the same segment plus the area under the shear force diagram between the left end and the shear-force zero-point.

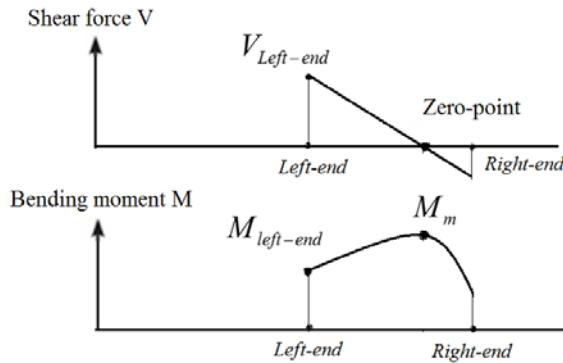


Figure 4 An example of bending moment with a maximum value

The maximum/minimum bending moment

Rule #6: For a segment with a constant distributed loading $\omega(x)$, if there is a shear-force zero-point, the bending moment at this point will be maximum/minimum bending moment, the value of which will be equal to the value of the bending moment at the left end of the same segment plus the area under the shear force diagram between the left end and the shear-force zero-point.

2.3 The practical graphical procedure for drawing the S/B diagrams

After the above six rules for the graphical procedure for drawing the S/B diagrams are presented, the practical executable graphical procedure is as follows:

- (1) Determine reaction forces of the beam at supports and then use the reaction forces to replace the supports. The reaction forces will be treated as external forces now.
- (2) Divide the beam into several segments at the points (joints) with a concentrated load, concentrated moment, start and end points of the constant distributed loads.

- (3) Create the S/B diagrams coordinates which align with the beam
- (4) Draw the first point of the shear force and the first point of the bending moment at the left end of the beam according to the corresponding values at the left end of the beam.
- (5) Draw the shear force diagram in the most left segment and then continuously move to the next adjacent segment. In each segment, do the following:

- (a) Use Rule #1 and Rule #2 to determine the shapes of the shear force diagram,

According to Rule #1, for no load in the segment, the shear force diagram is a horizontal line.

According to Rule #2: for a constant (uniform) distributed the load $\omega(x)$, the shear force diagram is an oblique straight line. For positive $\omega(x)$, it is an upwards oblique straight line, for negative $\omega(x)$, it is a downwards oblique straight line.

- (b) Use Rule #3 to determine the shear force at the right end of the same segment

According to Rule #3, the change in shear forces between two ends of the segment is equal to the area under the distributed loading of the same segment.

- (c) Use Rules #4 and #5 to determine the shear force at the right side of the joint, which will become the value of the shear force of the left end of the next adjacent segment.

According to Rule #4, for the concentrated force at the joint, the shear force change at the joint is equal to the value of the concentrated force. When force is positive, the shear force jumps upwards. When force is negative, the shear force jumps downwards.

According to Rule #5, for the concentrated couple at the joint, there is no shear force change across the joint.

- (d) Repeat (a) to (d) for the rest of the segments.

- (6) Draw the bending moment diagram at the most left segment and then continuously move to the next adjacent segment. In each segment, do the following:

- (a) Use Rules #1 and #2 to determine the shapes of the bending moment diagram

According to Rule #1, for no load in the segment, the bending moment is oblique straight line/a constant line. For positive V, it is an upwards oblique straight line; for negative V, it is a downwards oblique straight line; and for a zero-shear force line, it is a horizontal line.

According to Rule #2, for a constantly distributed load, the bending moment diagram is a concave parabolic curve for positive $\omega(x)$ and a convex parabolic curve for negative $\omega(x)$.

- (b) Use Rule #3 to determine the bending moment at the right end of the segment

According to Rule #3, the change in the bending moment between two ends of the segment is equal to the area under the shear diagram between the two ends of the same segment.

- (c) For the segment with a constant distributed loading, use Rule #6 to determine the maximum/minimum bending moment.

According to Rule #6, for a segment with a constant distributed loading $\omega(x)$, if there is a shear-force zero-point in the segment, the bending moment at the point will be maximum/minimum bending moment. This value will be equal to the value of the bending moment at the left end of the same segment plus the area under the shear diagram between the left end and the shear-force zero-point.

- (d) Use Rules #4 and #5 to determine the bending moment at the right side of the joint.

According to Rule #4, for the concentrated force at the joint, there is no change in the value of the bending moment at the point, but the slope will be different.

According to Rule #5, for the concentrated couple at the joint, the bending moment change at the point where the couple moment acts is equal to the value of the couple moment. When couple moment is clockwise, the bending moment jumps upward. When couple moment is counter-clockwise, the bending moment jumps downward.

- (e) Repeat (a) to (e) for the rest of segments

3. Implementation, assessment, survey and discussions

3.1 Implementation

In the spring and fall semesters 2017, we used around 2-hours lecturing to explain and implement the practical graphic procedure for drawing the S/B diagrams, which was the same amount of time delivered in the traditional procedure provided in the textbook [2]. At the beginning of the lecture, we gave the handout of the procedure to students, the copy of which is in section 2 of this paper. One-hour lecturing was used to explain the six rules which mainly determined the type of curves, the values on the right end of the same segment, and the values of the right side of the joint in the next adjacent segment. Because of the visual Figures 1 to 4, these six rules were easily accepted and understood by students. We then used another one hour of lecture to go over four examples. The first three simple examples as shown in Figure 5 were used to explain and demonstrate the six rules and the proposed procedure. The fourth example was a general example including at least two types of loadings and was completed by students. The proposed practical graphical procedure for drawing the S/B diagrams described in section 2.3 was straightforward and stand-alone steps, and were easy to follow, after a few examples.

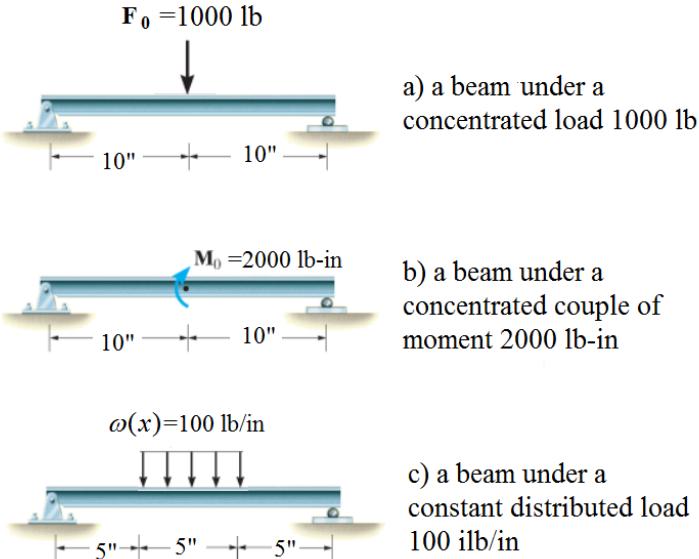


Figure 5 three demonstration examples

3.2 The assessment results of the quiz

We used a 20- minutes-quiz with one question as shown in Figure 6 to assess the effectiveness of the proposed procedure on two group of students. One group of students were taught with the proposed procedure. Another group of students was taught using the traditional procedure [2].

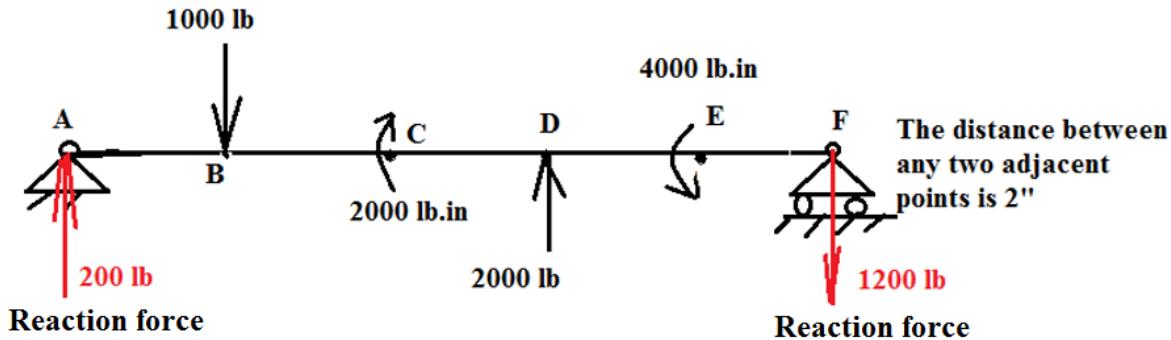


Figure 6 the quiz question

The means and standard deviations of the grades of the quiz are listed in Table 1. Students of group A and B in Table 1 were taught with the proposed graphical procedure. The students of Group C in Table 1 were taught with the traditional graphical procedure provided in the textbook [2].

The $t - test$ method is used to check whether there is a significant difference between average scores of Group A in spring 2017 and Group B in fall 2017. For the confidence level 95%, the $t_{critical}$ is 1.688 for the degree of freedom 36. The $t_{experiment}$ based on the collected data listed in Table is 1.079. Since $t_{experiment} = 1.079$ is less than $t_{critical} = 1.688$, the student scores between Group A in spring 2017 and Group B in fall 2017 are statistically almost the same.

Now, the scores from the Group A and Group B are combined, and its average score and standard deviation are also listed in the column Group A+B of Table 1. The $t - test$ method is again used to check whether there is a significant difference between average scores of Group A+B and Group C in fall 2017. For the confidence level 95%, the $t_{critical}$ is 1.676 for the degree of freedom 50. The $t_{experiment}$ based on the collected data listed in Table 1 is 2.600. Since $t_{experiment} = 2.600$ is larger than $t_{critical} = 1.676$, the student scores between Group A+B and Group C in fall 2017 are statistically different. Therefore, for the same quiz question as shown in Figure 6, the quiz grades for the students taught with the proposed procedure were significantly higher than the quiz grade for the students taught with the traditional graphical procedure.

Table 1 The means and standard deviations of the quiz grades

	Group A Spring 2017	Group B Fall 2017	Group A+B	Group C Fall 2017
Sample size	21	17	38	14
Mean	94.4	97.1	95.6	86.1
Standard deviation	9.20	5.16	7.77	18.8

3.3 Survey results

We conducted a student survey in the classes where students were taught with the proposed graphical procedure. The survey question was “The graphical procedure for drawing the shear force and bending moment diagrams facilitated me in drawing the shear force and bending moment diagrams of a beam” with five choices (a) Strongly agree; (b) Agree; (c) No opinion; (d) Disagree; and (e) Strongly disagree.

The survey results about the question are shown in Table 2. 85.7% of the students strongly agreed or agreed that the provided graphical procedure facilitated them to draw the shear force and bending moment diagrams.

Table 2 the survey results of the survey question

	Spring 2017	Fall 2017	Total	Percentage
Strongly agree	7	5	12	85.7%
Agree	9	9	18	
No opinion	2	0	2	5.7%
Disagree	2	1	3	8.6%
Strongly disagree	0	0	0	0%
Sample size	20	15	35	

Here are some comments student made on the graphical procedure: “Rules and procedure are clean and concise. No problem occurs when following the procedure.”; “The graphical approach makes it visually very easy to determine and solve for values related to the shear force /bending moments.”; “The procedure is straightforward and easy to follow which makes the process simple.”. But some students also said, “The rules for the graphical procedure are not necessary

because the relationship equations between the distributed loading, shear force and bending moment already tell everything.”

4 Discussions and Conclusions

The ability to draw shear force and bending moment diagrams on beam-like components is an important skill for mechanical engineering students. There are no technical difficulties in drawing the S/B diagrams on beams because the shear force and bending moment equations for the S/B diagrams can be fully obtained by using the method of sections. When several external typical loading conditions on beam-like components are applied, the graphical approach for drawing the S/B diagrams by a free-hand sketch is a good alternative.

It has been observed that some students had difficulty to effectively draw the S/B diagrams of beams. They had indicated that they did not know how to start a problem and could not follow the graphical procedure. To help these students, we scrutinized every necessary step of drawing the S/B diagrams and proposed a practical graphical procedure. This has been successfully implemented in the spring and fall 2017 in the Mechanics of Materials course. The assessment data based on a given quiz showed that the average grade of the students taught with the proposed procedure was significantly higher than the average grade of students taught with the traditional procedure. So, students' score had been significantly improved by the proposed procedure. The student survey indicated that 86 percent of students agreed that the proposed graphical procedure helped them to effectively draw the S/B diagrams. Students indicated that the proposed procedure was straightforward and easy to follow.

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6 Appendix A

Procedure for analysis (Page 273 of the reference 2)

The following procedure provides a method for constructing the shear and moment diagrams for a beam based on the relations among distributed load, shear and moment.

Support Reactions

- Determine the support reactions and resolve the forces acting on the beam into components that are perpendicular and parallel to the beam's axis

Shear Diagram

- Establish the V and x-axis and plot the known values of the shear at the two ends of the beam
- Notice how the values of the distributed load vary along the beam, such as positive increasing, negative increasing, etc., and realize that each of these successive values indicates the way the shear diagram will slope ($dV/dx=W$). Here W is positive when it acts upward. Begin by sketching the slope at the end points.
- If a numerical value of the shear is to be determined at a point, one can find this value either by using the method of sections and the equation of force equilibrium or by using $\Delta V = \int wdx$, which states that the change in the shear between any two points is equal to the area under the load diagram between the two points

Moment Diagram

- Establish the M and x-axes and plot the known values of the moment at the ends of the beam
- Notice how the values of the shear diagram vary along the beam, such as positive increasing, negative increasing, etc., and realize that each of these successive values indicates the way the moment diagram will slope ($dM/dx=V$). Begin by sketching the slope at the end points
- At the point where the shear is zero, $dM/dx=0$, and therefore this will be a point of the maximum or minimum moment.
- If a numerical value of the moment is to be determined at the point, one can find this value either by using the moment of sections and the equation of moment equilibrium

or by using $\Delta M = \int V dx$, which states that the change in moment between any two points is equal to the area under the shear diagram between the two points.

- Since ω must be integrated to obtain ΔV , and V is integrated to obtain M , then if ω is a curve of the degree n , V will be a curve of degree $n+1$ and M will be the curve of degree $n+2$. For example, if ω is uniform, V will be linear, and M will be parabolic.