A Recent Experience in Utilization of Online Resources in Teaching Undergraduate Dynamics

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Abstract

Undergraduate Engineering Dynamics (ENGR 2302) is one of the challenging courses in both Engineering and Engineering Technology curricula. Variety of topics related to Engineering Mechanics is covered in this course with varying degrees of difficulty students perceived to develop their understandings of the course topics. In particular, the relationship between force and potential Energy and physical meaning and significance of "Conservative forces" are among topics that typically undergraduate students struggle with in developing their understanding of mechanical systems dynamics. A recent experience gained by the author in teaching the course is discussed and examples are provided to help instructors who teach this course either online or in person.

Nomenclature:

m	*mass*	***U***	*Work done on an object by a force*
v	*velocity*	***F***	*Force*
V	*Potential Energy*	***r***	*displacement*
T	*Kinetic Energy*	***A***	*Area*

Introduction

Engineering Dynamics (ENGR 2302) course is a required course in both Mechanical Engineering Technology and Mechanical and Energy Engineering programs at University of North Texas. Author has taught the course for several semesters and would like to share with readers his experience gained in teaching the course. Among topics covered in the course, the one that appears to present a challenge to some students is the concept of "conservative forces" versus "non-conservative forces." The topic is covered in all respected and popular textbooks[1-4] in a nearly similar fashion. For example, Bedford and Fowler[1] states that if all forces that do work on an object are conservative the equation (1) can be applied:

$$\tfrac{1}{2}\, \boldsymbol{m v_1^2} + \boldsymbol{V_1} = \tfrac{1}{2}\, \boldsymbol{m v_2^2} + \boldsymbol{V_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (1)$$

They further indicated that frictional forces are not conservative.
Hibbeler[2] qualifies a conservative force by stating "If the work done by a force is independent of the path and depends only on the force's initial and final positions on the path, then we can classify this force as a conservative force." He suggests writing conservation of energy equation as:

$$\boldsymbol{T_1} + \boldsymbol{V_1} + (\textstyle\sum \boldsymbol{U_{1\text{-}2}})_{noncons.} = \boldsymbol{T_2} + \boldsymbol{V_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (2)$$

Where $(\sum \boldsymbol{U_{1\text{-}2}})_{noncons.}$ represent work done by all non-conservative forces such as frictional forces.

Gray et al.[3] offers a similar approach as Hibbeler did but Beer et al.[4] states that "for any conservative force we can write:"

$$\oint \mathbf{F} \cdot d\mathbf{r} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (3)$$

where the close circle indicates that the path is closed. Considering the mathematics proficiency of Mechanical engineering Technology students or even Mechanical Engineering ones, the concept of Green's Theorem where equation 3 originates from would be new to them and will have a challenge digesting it. So an easier understanding approach needs to be introduced to students in Mechanical discipline. Based on author's teaching experience, once the concept of conservative force is introduced along with the force–potential energy relationshiop given in equation 4 will ease understanding of the concept.

$$\boldsymbol{F} = -\, \partial \boldsymbol{V} / \partial \boldsymbol{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (4)$$

Equation (4) presents a definition of Force in terms of Potential Energy and can be used to effectively demonstrate the physical significant of conservative forces as shown below.
A hypothetical plot of a potential Energy- Distance relationship can be as shown in Figure 1.

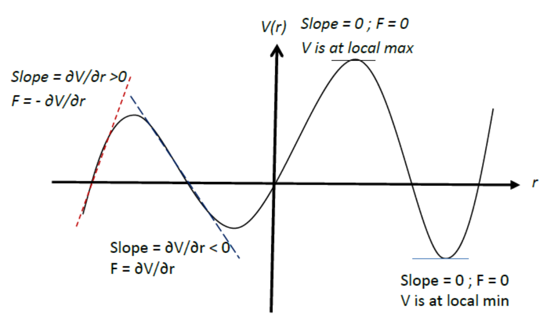

Figure 1: A hypothetical potential Energy (*V*) –Distance (*r*) relationship.

Assuming a ball is placed on the positive slope section of the plot (Figure 1) and since the force acting on the ball will be having negative value ($\partial V/\partial r > 0$) the ball will move to the left (left movement is given arbitrary sign of negative and ball movement to the right is considered positive). Alternatively, if a ball is placed on the negative slope of the plot (Figure 1), the ball will experience a positive force ($\partial V/\partial r < 0$) and will be moving to the right. At either local maximum or local minimum shown in Figure 1 the ball will not experience any force due to the fact that slope is zero in both cases ($\partial V/\partial r = 0$). The only difference between the two extreme equilibrium positions of maxima or minima will be that at the local maxima position, the ball will be in an "unstable" equilibrium position whereas at the local minima position the ball will be at the stable equilibrium.

Equation (3) is actually the heart of Green's Theorem that is given as follows:
Let C be a peicewise-smooth, simple closed curve that bounds a region R in the plane[5]. If P and Q have continuous partial derivatives on an open set that contains R, then

$$\int_C P\,dx + Q\,dy = \int_R\int \{(\partial Q/\partial x) - (\partial P/\partial y)\}\,dA \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(5)}$$

Equation (5) represents a closed curve region that for a conservative vector field represent a circle if the integral equals zero as equation (3) indicated but if it is a non-zero for a vector field such as (*F*=*yi* –*xj*) it can represent a non closed circle such as a spiral shown bellow (Figure 2)

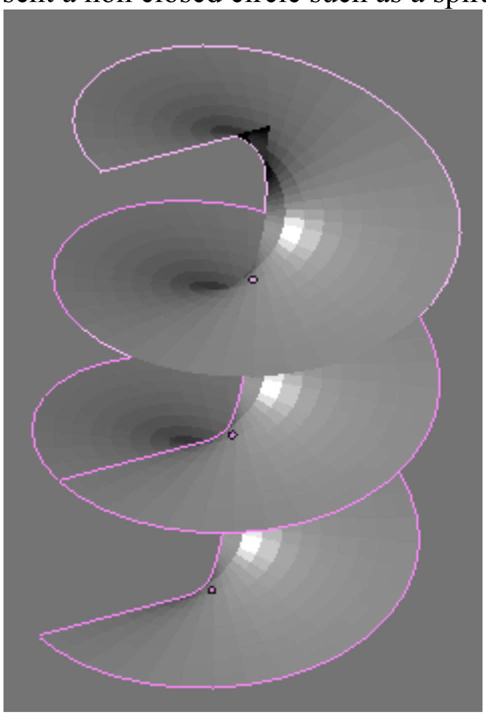

Figure 2: Spiral force function represented by (*F*=*yi* –*xj*)[6].

The best test to determine whether a vector field is conservative is to determine whether the curl of the function is equal to zero or not. All conservative forces meet the following condition:

$$\nabla x F = 0 \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where $\nabla = (\partial/\partial x + \partial/\partial y + \partial/\partial z)$ is just an operator and $F = F_x\, i + F_y\, j + F_z\, k$ is the force vector. For example, while $F = (3x^2 - 2xy)i - x^2\, j$ is conservative, $F = (x - xy^2)i + x^2yj$ is nonconservative because the curl of the former is equal to zero where as that of the later is not.

$$\nabla x\, F = \text{Curl of F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2 - 2xy & -x^2 & 0 \end{vmatrix} = (0)i + (0)j + (-2x+2x)k = 0$$

($\therefore$ conservative)

$$\nabla x\, F = \text{Curl of F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x - xy^2 & x^2y & 0 \end{vmatrix} = (0)i + (0)j + (2xy + 2xy)\, k = 4xy\text{k} \neq 0$$

($\therefore$ nonconservative)

Summary and Conclusions

In summary, we have described the conservative and nonconservative vector fields in relation to dynamic systems to be included in Engineering Mechanics Dynamics (ENGR 2302) course. Green Theorem is utilized to best describe the mathematics behind the concept and a relatively easy test based on the curl of a given vector field is highlighted to be used in determining whether a vector field is continuous or noncontinuous. In addition, a relationship between force vector field and its potential energy was described for a ball that is physically located in an uphill or downhill slope as shown in Figure 1 so that students can intuitively understand the main concept by following the direction of a force the ball feels. The force exerted on the ball located at a slope indicated the potential energy associated with the ball causing to move the ball to the right or left depending on whether the ball is located on the uphill or downhill slope type. Such a simple explanation along with mathematical backing of the topic should fully educate Engineering and Engineering Technology students on this important topic that may have been chosen to either not fully or lightly explain it in a given ENGR 2302 course.

References

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Dr. Nasrazadani currently serves as Professor of Mechanical Engineering Technology at the University of North Texas. He has taught Mechanics course (Statics, Dynamics, Mechanics of Materials, etc.) for more than 30 years at various institutions. He teaches Engineering and Engineering Technology students with an intuitive approach that he found very helpful in educating undergraduate students. His research interests include Engineering Materials, Corrosion and Degradation of structural Materials.