AC 2011-1444: A RELEVANT, AUTOMOTIVE-THEMED EXPERIMENT THAT TEACHES FUNDAMENTAL FLOW RATE CONCEPTS AND EXPERIMENTAL UNCERTAINTY

Brian P Sangeorzan, Oakland University

Dr. Brian Sangeorzan, is registered Professional Engineer and an Associate Professor of Engineering at Oakland University in Rochester, Michigan, where he serves as the faculty advisor for the SAE student chapter and has taught graduate and undergraduate courses in thermodynamics, fluid mechanics, heat transfer and combustion for the past 27 years. His research interests generally include heat transfer and fluid mechanic phenomena in internal combustion engines, as well as the instrumentation and optical techniques for thermal/fluid measurements. Past research has included experimental methods for in-situ droplet and particle sizing in fuel sprays, high-speed flow visualization in I.C. engines, optical/thermal wave techniques for non-destructive measurements of thermal properties of thin films, and thermal system analysis of automotive paint curing ovens, the development of hydraulically-actuated turbines, and the study of exhaust gas heat transfer in engine exhaust manifolds. Current work includes thermal modeling of an engine lubrication system and engine piston heat transfer. He has served on a number of committees in the SAE International, and also serves on the SAE Detroit Section Governing Board. Dr. Sangeorzan has worked as a consultant to a number of companies and organizations, including Ford Motor Company, FEV Engine Technology, and General Motors Corporation.

Matthew Nathaniel Bruer, Oakland University

Matthew Bruer is the Assistant Lab Manager and the Electronics Technician, in the School of Engineering and Computer Science, at Oakland University. He acquired his Bachelors of Science in Engineering in June 21 2006 and is presently completing a Master's Degree in Electrical Engineering.

Laila Guessous, Oakland University

Laila Guessous, Ph.D. is an associate professor in the department of mechanical engineering at Oakland University (OU) in Rochester, MI. Her research and teaching interests lie in the areas of fluid mechanics and heat transfer, with an emphasis on computational methods. She is the program director for the NSF and DoD funded AERIM REU program at OU, as well as co-director of an NSF-funded International Research Experience for Students (IRES) collaborative research program with China on fluid and thermal transport in fuel cells.

Xia Wang, Oakland University

XIA WANG (wang@oakland.edu) is an assistant professor in the department of Mechanical Engineering at Oakland University. Her research and teaching interests lie in the areas of fluid mechanics and heat transfer, with an emphasis on fuel cell technology.

A Relevant, Automotive-Themed Experiment that Teaches Fundamental Flow Rate Concepts and Experimental Uncertainty

Abstract

It is a common experience, in undergraduate laboratories, that the students perceive the simple bench-top experiments to be boring or irrelevant to ***real*** engineering and societal problems. Without relevance, many students feel disconnected from the lab experience, lose interest in what they are doing and do not ***think*** while they are in the lab. If students do not think about the actual measurement, the measurement errors and how the measurements relate to an engineering model or to the information that they are trying to gain, then the lab experience has failed.

Described in this paper is an experiment designed for an introductory thermal science lab. The students are tasked with the job of calibrating one or more automotive fuel injectors, across a reasonable vehicle fuel demand schedule. In order to achieve an accurate injector calibration, the students must learn the fundamental concept of volumetric flow rate and the applied mathematical concepts associated with experimental uncertainty. They must apply what they have learned to design their test procedure and then conduct and validate the calibration. In addition, they will learn the purpose and function of an automotive fuel injector, and how the injector timing parameters can be used to match a typical engine fuel demand schedule – the relevance component.

This paper also describes the design and construction of a safe and inexpensive apparatus, which uses automotive fuel injectors and a simple microprocessor board for control and timing functions. A sample laboratory handout and some student results are given, along with some suggestions for semester-to-semester variety.

Introduction

Many of the experimental apparatus used in introductory, undergraduate laboratories are bench-top configurations that allow replicate stations, portability and efficient use of space. They are often commercial products that may interface with common data acquisition systems, power sources, or fluid supplies (in the case of many thermal science experiments). Often, they are focused on a particular device, such as a flow meter or pump or a heat exchanger, and not on how that device plays a role within a system. That purposeful focus is often good for practical and heuristic reasons, but may lack in relevance in the eyes of the students. Students perceive the experiment as "boring" and do not really focus on or think about what they are doing while in the laboratory. Additionally, even simple commercial lab apparatus can be quite expensive.

At Oakland University, we often begin our introductory thermal science laboratory with an experiment that uses flow meters to teach the concept of experimental uncertainty. In the past, we have used simple instrumentation, such as variable-area flow meters, mechanical timers and graduated cylinders, to allow the students to focus on the concepts rather than the instrumentation. In principle this is good, but the students' perceptions are that the experiment is "boring" and uses outdated equipment that they may never use in industry. It is common to witness students who are distracted with other things during the lab exercise, instead of thoughtfully focusing on the experiment. An interest in providing some relevance to the lab, while still teaching the same principles, prompted the fuel-injector-themed experiment described in this paper.

In this paper, an experiment is proposed to help teach the fundamental concepts of volumetric flow rate and experimental uncertainty analysis. This experiment requires a relatively simple laboratory apparatus, which is basically a fuel injector calibration bench, which can be constructed for a few hundred dollars (plus any machine shop charges). For the laboratory exercise, students are introduced to the concepts of volumetric flow rate, experimental uncertainty analysis, and the basic operation of an automotive fuel injector. They are asked to calibrate a fuel injector within a specified experimental uncertainty. There are a number of experimental variations, which keep the experiment 'fresh', from semester to semester.

This paper will provide a little background on the operation and flow requirements of automotive fuel (gasoline) injectors, discuss the design of the injector test bench, and show some injector calibration results and a sample student laboratory handout. The apparatus is a prototype, so suggestions for future improvements will be discussed.

Background

The course for which this experiment is designed is an introductory thermal engineering course. The course content includes concepts normally found in a first course in thermodynamics, plus an introduction to heat transfer. This is a required core course for all engineering majors and includes an integral laboratory component, which is typically synchronized with the lecture material. At the time they would be taking this course, our students would have completed a sophomore design experience and would have likely had an introductory circuits course and laboratory. The purpose of this introductory thermal science laboratory is to reinforce concepts in thermodynamics, as well as to teach experimental uncertainty analysis (and therefore thoughtful experiment design) and to develop some laboratory report writing skills.

We normally begin each laboratory exercise with an introductory lecture, and then students return in groups, at their convenience, to conduct the actual experiment. To provide some relevance during the laboratory introduction, students would be given some background information on the role of electronic fuel injectors in a modern gasoline engine, particularly in their role and function as precision fuel metering devices. A fuel injector must be able to precisely meter fuel over a wide range of engine operating conditions, from idle to rated power, and within the time constraints imposed, especially at higher engine speeds. In modern, electronic fuel-injected gasoline engines, the injectors are pulsed so as to control the timing and duration (amount) of fuel sprayed toward the intake valves. As a rough metric, the injector open-time varies from around 10 engine crank-angle degrees (CA°) at light load and low speed to about 300 CA° at rated power[1]. There is one combustion process and one injection event per cylinder, for every two engine revolutions (four-stroke engines). At rated power, a typical engine would convert about 28 percent of the fuel energy into shaft power[1]. Students would be given this information, along with the approximate energy in a gallon of gasoline (Table 1), and asked to estimate the maximum fuel flow rate required, and the period or frequency of injections at rated speed. These calculations would be done prior to conducting the lab experiments.

Table 1. Approximate Properties of Gasoline[1]

Density	760 kg/m^3
Lower Heating Value (LHV)	44.0 MJ/kg

As an example of some fuel flow estimates and timing requirements, consider the following example of a 3.5 L, V6 engine found in a 2011 sport utility vehicle. This particular engine is rated at 290 hp, at 6500 rpm [2]. For the rated power and speed, the maximum fuel flow rate and the injector period are given in Table 2. The details of these calculations are given in Appendix A.

Table 2. Sample Laboratory Pre-Calculations

Injection Parameters for a 2011 Sport/Utility Vehicle	
Max. Fuel Flow Rate	23.1 cm^3/s
Flow Rate per Injector	3.85 cm^3/s
Injector Period	18,462 μs
Max. Injector Pulse Width	7692 μs

Following the discussion of fuel injectors, the students are then introduced to the concept of experimental uncertainty analysis. This is a brief introduction, usually with an example, which is based on the common method discussed in many textbooks and references [3, 4]. A sample laboratory handout, on *Uncertainty Analysis*, is given in Appendix D. Based on the principles explained in the handout, students are asked to derive equations for the relative and absolute uncertainties of the steady-flow volumetric flow rate, $\dot{V\!\!\!-}$, that could be indirectly measured using a volume-collection method,

$$\dot{V\!\!\!-} = \frac{V\!\!\!-}{t} \quad (1)$$

where $V\!\!\!-$ is the volume collected and t is the collection time. They are then asked to “flow” or ‘calibrate’ an injector, to a specified experimental uncertainty. The experimental details will be discussed in more detail following the section on Apparatus Design.

Apparatus Design

To conduct the calibration experiments, the students use a *Fuel Injector Test Bench*, shown schematically in Figure 1. The injectors are electronically-controlled and typical of what is used in modern gasoline “port injected” engines. They are mounted in a standard automotive fuel rail (obtained from a local car dealer). The microcontroller allows the students to select one of four different fuel injectors, and then set: the *Period* (time between start of injection pulses), the *Pulse Width* (injector open time), and the number of pulses.

The working fluid is stored in a plastic, marine-style fuel tank with a quick disconnect “fuel” line. The tank is red (meaning that it would normally be used with gasoline), but can be properly marked to indicate the actual fluid contents. This was a readily available and inexpensive solution. The fluid is pumped, with an aftermarket automotive fuel pump, to the fuel rail and through a regulator back to the “fuel” tank. The regulator allows adjustment of the fluid pressure to match that typically found in automotive systems (typically 45-60 psig). A damped pressure gage is mounted on the fuel rail to indicate the pressure.

Since this apparatus is intended to be used by students in a standard undergraduate laboratory, fluids safer than fuel must be used. Turpentine has a viscosity and density close to that of gasoline, but the vapors resulting from the liquid atomization are too powerful for a student lab. Water could be used, but the pump and fuel system would have to have the appropriate materials to avoid rust buildup in the system, and eventual pump failure. We have tried using water with a few different types of additives to prevent corrosion and offer some lubricity to the pumps and injectors. We are currently using an additive, called Grindzal[5], which is mixed with water at a ratio of 1 part Grindzal to 20 parts water, and produces minimal odor without foaming in the graduated cylinder and fluid lines.

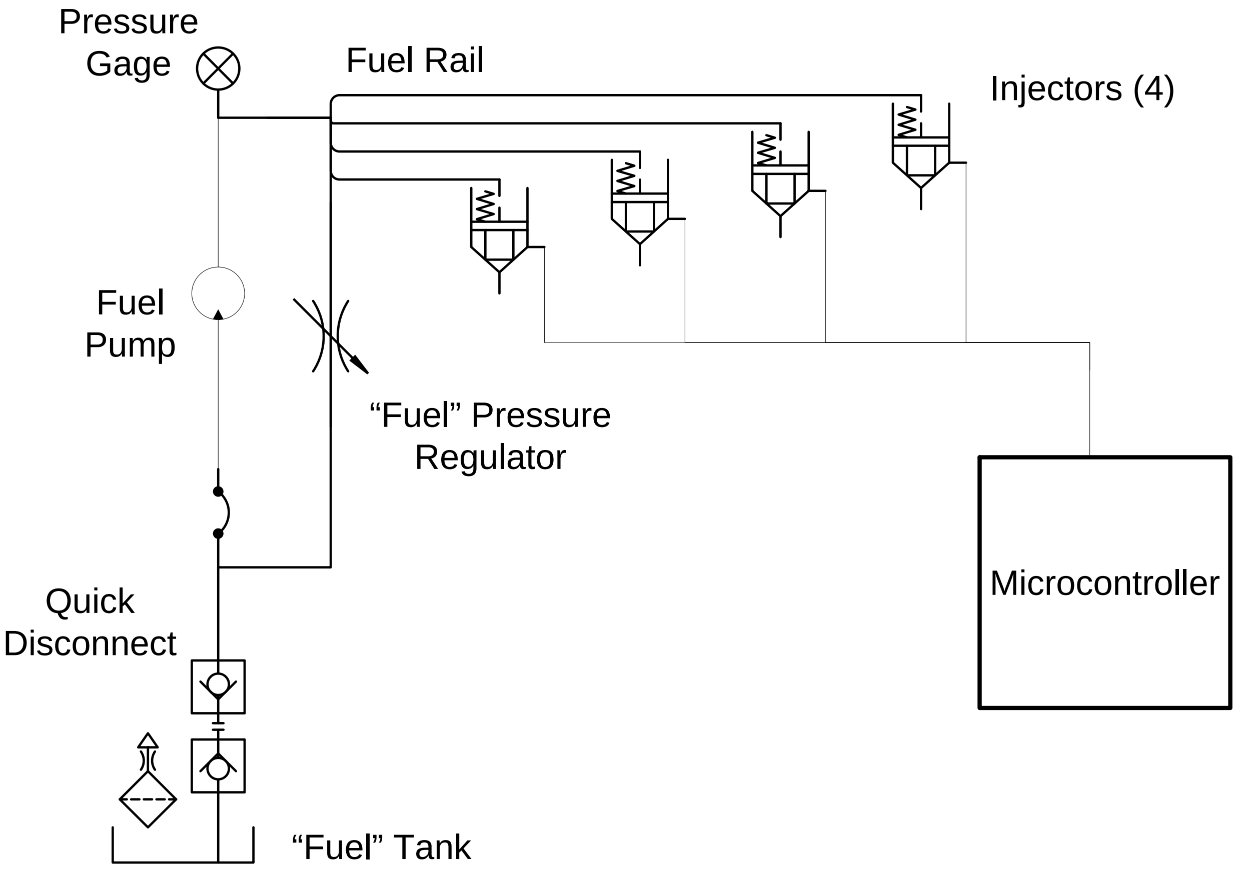

Figure 1. Schematic of Fuel Injector Test Bench

In the current prototype, the injectors are mounted behind a clear acrylic panel so that no spray splashes on any of the users. The height of the fuel rail is adjustable to accommodate various graduated cylinders. The injectors also have a small piece of clear tubing attached to the spray tip to agglomerate the atomized liquid and direct it down into the graduated cylinder. In our current prototype, the controller and key pad are mounted inside an enclosure, along with a power supply used for the pump, injectors and microcontroller. A photograph of the front or operator's view of the current prototype is shown in Figure 2, and a rear view, showing the adjustable fuel rail and graduated cylinder is shown in Figure 3. The clear tubes are removed from the injector tips for clarity.

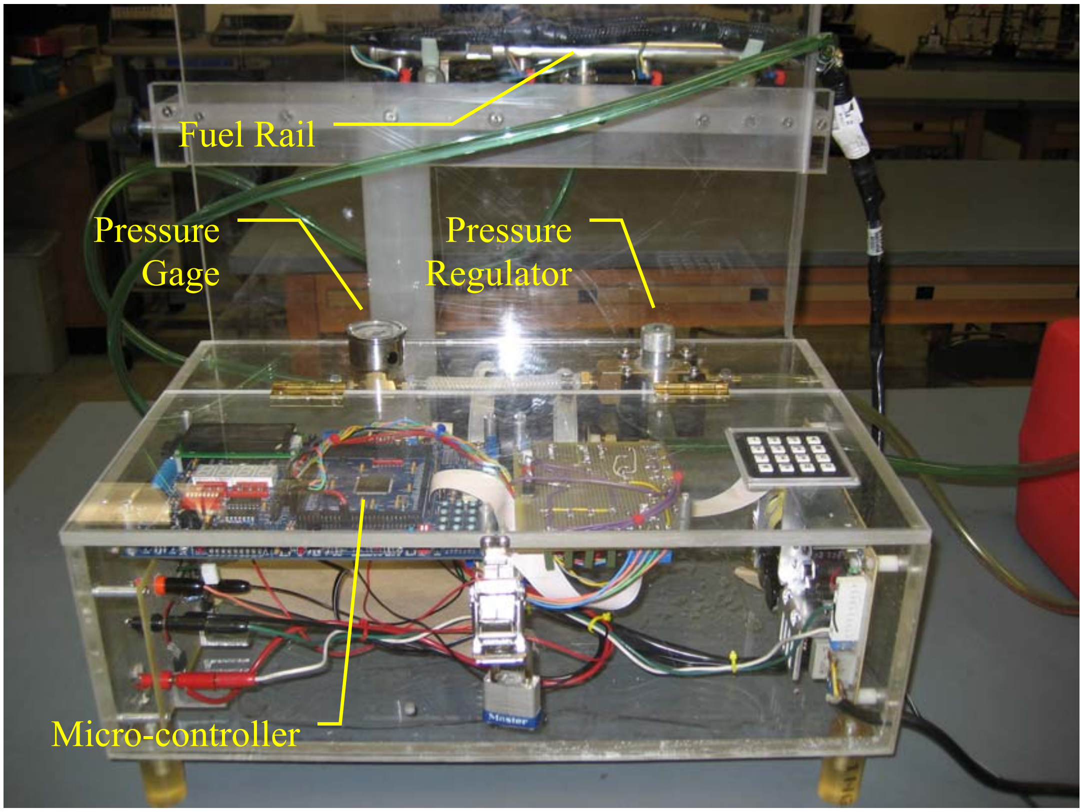

Figure 2. Prototype Fuel Injector Test Bench

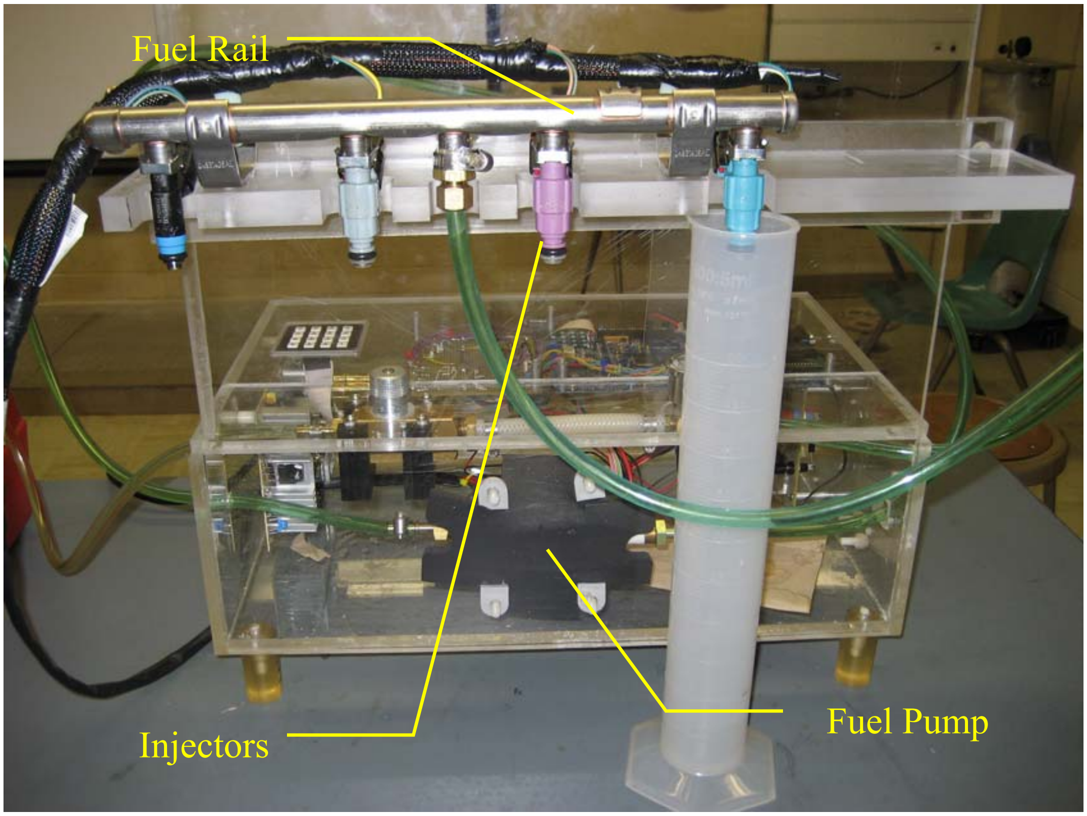

Figure 3. Rear View Showing Adjustable Fuel Rail

Baseline Injector Calibrations

Since we had no *a priori* knowledge about these injectors when we received them, some baseline testing was done before the apparatus was used by the students. Figure 4 illustrates the linearity in the fluid volume collected against the number of injection pulses. This data could also be presented as volumetric flow rate as a function of pulse width (injector open time), as shown in Figure 5. Fuel injector flow rates are very linear with pulse width, especially if the injector activation time (the time to physically move the solenoid) is accounted for. Figure 6 is a comparison of the four different injectors.

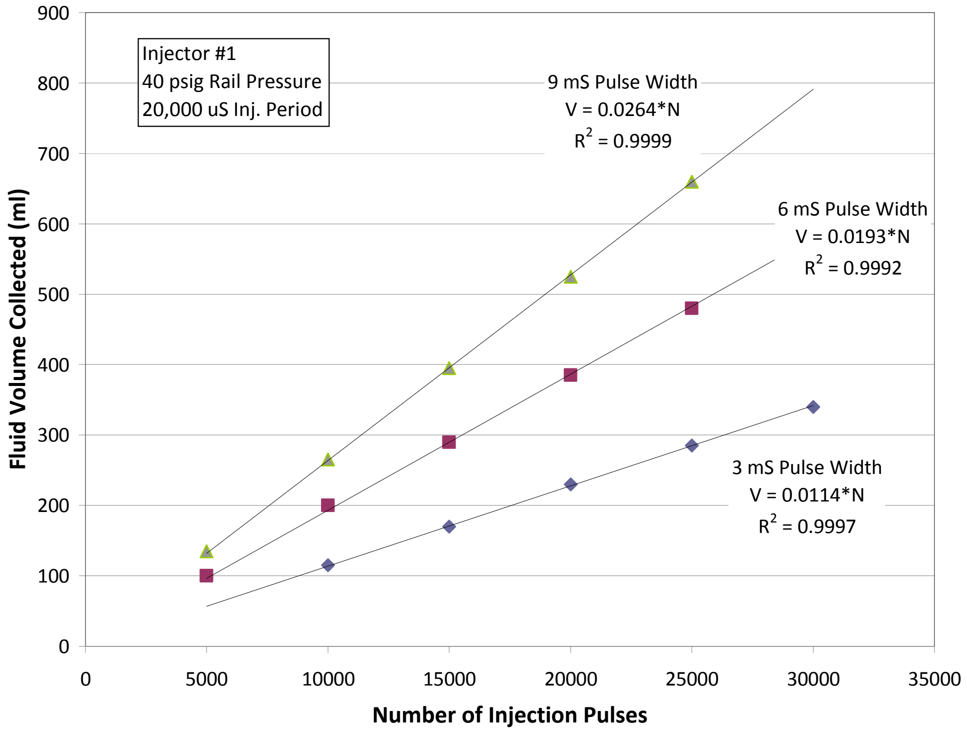

Figure 4. Sample Fuel Injector Baseline Test

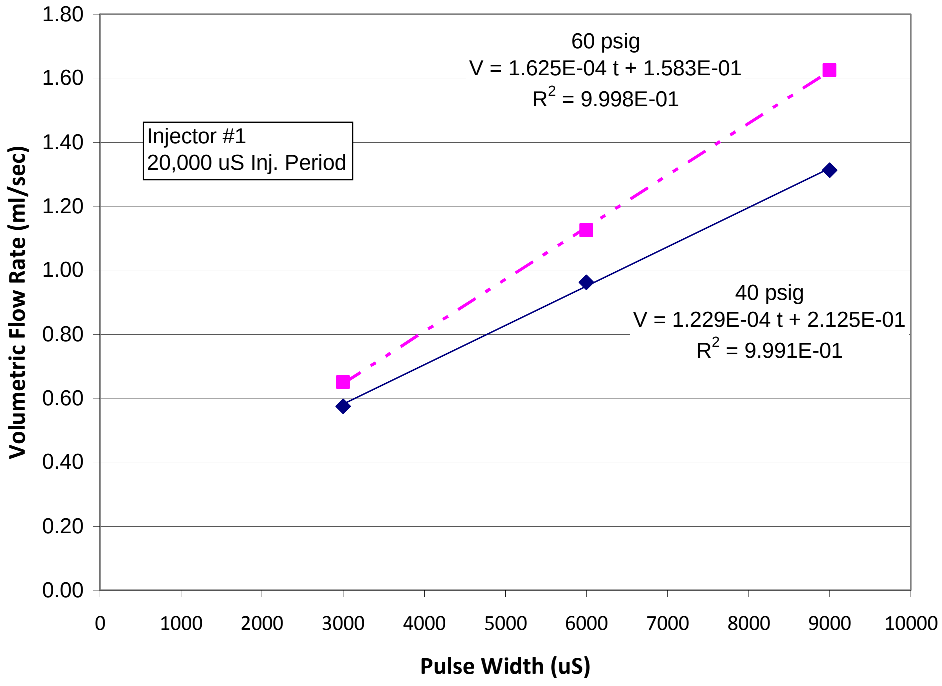

Figure 5. Injector Linearity with Pulse Width

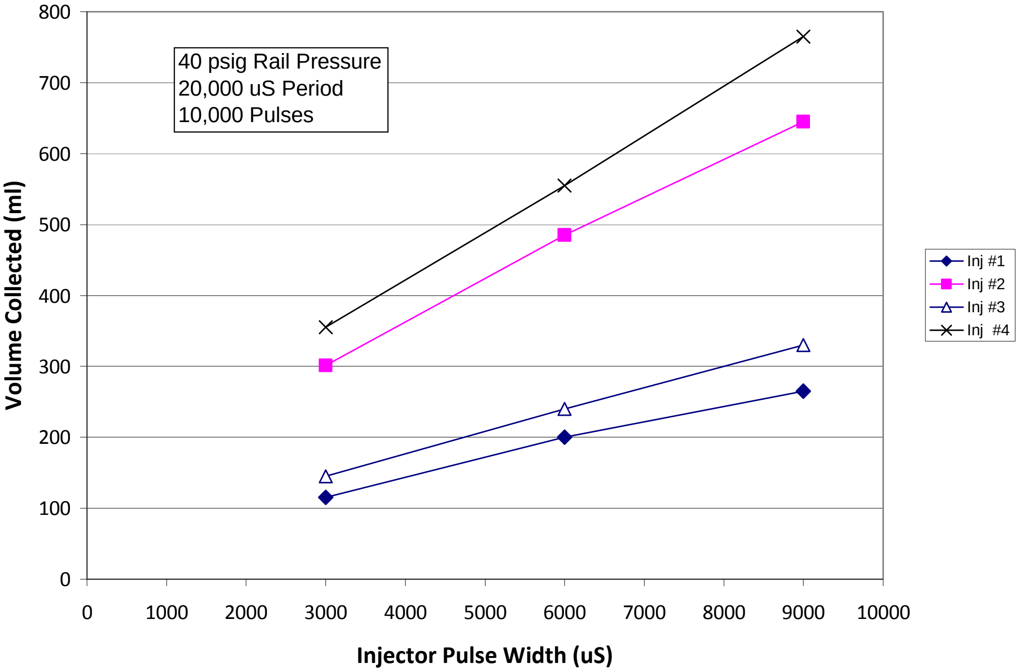

Figure 6. Comparison of the Four Injectors

Sample Laboratory Results

The goal of the student assignment would be to 'calibrate' or measure the fuel flow rate of a specified injector, as a function of pulse width, and to estimate the experimental uncertainty of their measurements. Alternatively, students may be asked to calibrate an injector at a specified flow rate, to a specified experimental uncertainty. Students would pre-calculate the injector period to match a specific engine speed; for example, 6500 rpm corresponds to an 18,462 μS period (20,000 μS used in the sample below). There are two options for the time measurements. Students can use a lab timer, with about ±0.15 second uncertainty (including human error), or they can set a specified number of pulses and calculate the time very accurately using the period and number of pulses. Students would have to select from several graduated cylinders, ranging in size from 100 ml to 1000 ml, knowing that the larger graduated cylinders have larger uncertainties (larger smallest divisions). Hence, they are required to make decisions on their own as they go about deciding what volume to collect, etc.

For this sample experiment, students were asked to measure the flow rate of a specified injector, over a range of injector pulse widths, and for a specified 'equivalent' engine speed of 6499 rpm (~ 20 mS injection period). In a commercial laboratory, this is often called 'flowing' or 'calibrating' the injector. This would be the simplest of the experiments mentioned, and would be very similar to the procedure in a commercial calibration laboratory. They would set the injection period, the pulse width, and a number of injections, and measure the fluid collected and the total collection time. From the collection volume and time, they would calculate the flow rate and the uncertainty in the flow rate. Note that time could be measured with a stopwatch or by multiplying the total number of pulses by the period. The stop watch is a more crude measure, but provides an illustration of relationship between the measurement uncertainty and the accuracy of the flow rate measurement.

The uncertainty analysis (Appendix D) suggests that the uncertainty in the volumetric flow rate can be determined by:

$$\Delta \dot{\mathrlap{\,-}V} \cong \frac{\Delta \mathrlap{\,-}V}{t} + \frac{\mathrlap{\,-}V\,\Delta t}{t^2} \tag{2}$$

Also, the relative uncertainty can be determined by:

$$\frac{\Delta \dot{\mathrlap{\,-}V}}{\dot{\mathrlap{\,-}V}} \cong \frac{\Delta \mathrlap{\,-}V}{\mathrlap{\,-}V} + \frac{\Delta t}{t} \tag{3}$$

where:

$\dot{\mathrlap{\,-}V}$ = the volumetric flow rate
$\Delta \mathrlap{\,-}V$ = the uncertainty in the volume measurement
Δt = the uncertainty in the time measurement
$\mathrlap{\,-}V$ = the actual collection volume
t = the actual collection time

A sample student data set is shown in Table 3. For this data set, the students simply calibrated one injector, at three pulse widths and two rail pressures. The data is also plotted in Figure 7. Notice the relatively small uncertainty bars in the figure. The small uncertainty is due to the relatively large collected volume and the relatively long collection times. Students would not have a sense of the required collection volumes or times without having conducted the uncertainty analysis before conducting the experiments. In fact, it is very easy to have relative uncertainties of less than one percent with a little planning.

Table 3. Sample Injector Calibration Data

Injector #1						
20,000 μS Period						
Volume Uncertainty = +/- 5 ml; Time Uncertainty = +/- 0.3 sec						
Pulse Width (μS)	**Rail Pressure (psig)**	**Collection Volume (ml)**	**Collection Time (sec)**	**Vol. Flow Rate (ml/sec)**	**Uncertainty (ml/sec)**	**Uncertainty (%)**
3000	40	115	200.3	0.57	0.0258	4.50
6000	40	200	199.7	1.00	0.0265	2.65
9000	40	265	200.3	1.32	0.0269	2.04
3000	60	130	200.3	0.65	0.0259	4.00
6000	60	230	199.7	1.15	0.0268	2.32
9000	60	330	200.0	1.65	0.0275	1.67

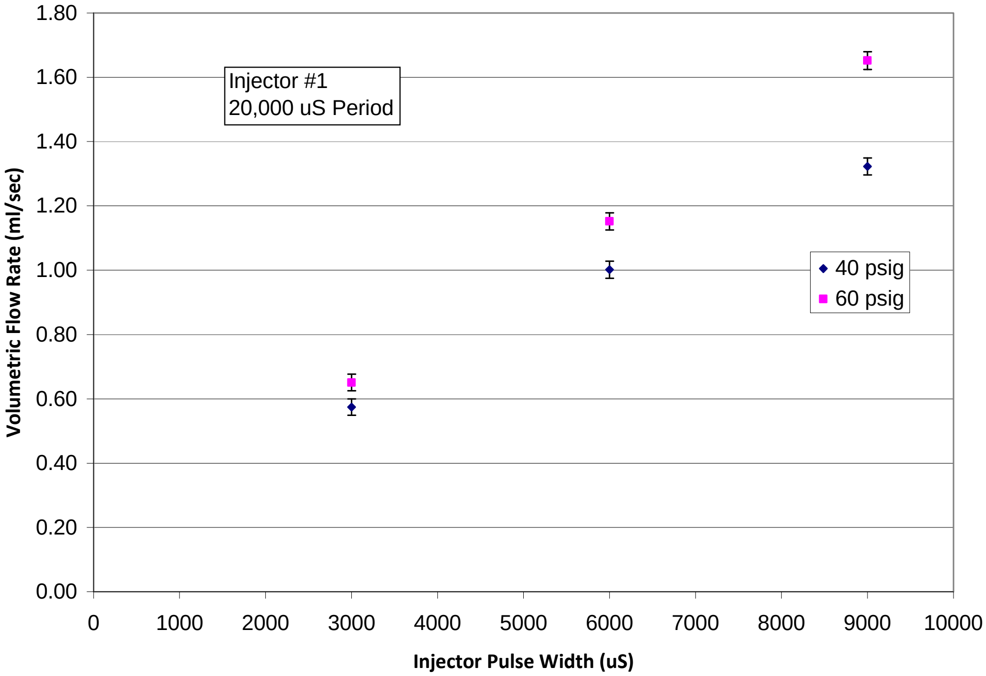

Figure 7. Sample Injector Calibration with Uncertainty Bars

Conclusions and Recommendations

At this point in time, the prototype Fuel Injector Test Bench has been built and tested in the student lab. Students find the concept of calibrating a fuel injector much more interesting than simply making volumetric flow rate measurements with a flow meter. The injector calibration also provides some relevance for the concept of experimental uncertainty. There are no formal student assessments done for this experiment, or for this course laboratory; we have been using our second thermal science laboratory for assessment purposes. Anecdotally however, the students are much more interested in this exercise than the simple flow meter experiment which it replaces. They are interested to learn about automotive fuel injectors, and to learn about industrial practice for calibration. In fact, several students, who are not in this course, have asked about the 'new' experimental apparatus, having seen it in the lab. The students on our Formula™ SAE team were wondering whether they could use the apparatus to calibrate the fuel injectors for their competition vehicle! We do plan to include a student assessment of the lab experience in this course in some future semesters.

Important for a required student laboratory is the ability to conduct a variety of experiments or at least vary the experimental parameters. This apparatus has four different injectors which can be calibrated under a variety of equivalent engine speed conditions and necessary fuel flow rates. In the example above, flow rate in ml/sec was determined, however, there are several other variations. For example, using the assumptions about engine power stated in the Background section (see Appendix A), students could be asked to determine the injector pulse width required to provide the proper amount of fuel for peak engine power. As another alternative, the injectors could also be calibrated in terms of fuel delivery per injection event (ml/injection), and the uncertainty analysis can be done for that quantity as well.

The apparatus is simple and safe when properly used, however some additional safety considerations will be adopted in the next version of the apparatus. Currently the operator is on the opposite side of the apparatus, separated from the injectors by and acrylic panel, but we will add additional protection. We plan to include a guard with a micro-switch, over the injectors and graduated cylinder, so that no hands may be below the injectors when the injectors are operating. Secondly, some of the soft fuel lines will be replaced with hard lines to better package the apparatus and minimize the chance to snag or pull on a soft line. Some transparent lines are required to inspect for air in the system. Also, the electronic components will be better sealed against any liquid intrusion in the next generation. The current injection fluid additive is relatively safe, odor-less and non-irritating, but we continue to look for possibly better alternatives.

References

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2. Ford Motor Company Website: http://www.ford.com/suvs/explorer/specifications/, accessed on 1/18/11.
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4. Fox, R.F., Pritchard, P.J., and McDonald, A.T., “Introduction to Fluid Mechanics”, 7th ed., (see Appendix F), Wiley, 2009
5. Grindzal™ is available from Synthetic Lubricants, Inc., Greenville, MI, http://www.synlube-mi.com/
6. National Instruments LM 1949, Injector Drive Controller, http://www.national.com/mpf/LM/LM1949.html#Overview
7. Motorola 68HCS12 Microcontroller, http://www.alldatasheet.com/datasheet-pdf/pdf/118982/MOTOROLA/68HC12.html
8. National Instruments MM74C922, http://www.datasheetcatalog.com/datasheets_pdf/M/M/7/4/MM74C922.shtml

Appendix A. Sample Fuel Injector Calibrations

Following a brief introduction to engines and fuel injectors, students would be given the basic engine parameters, and would then be asked to calculate the expected maximum injector flow rates and time between the start of injector pulses (period). Approximating the injector open-time at about 300 CA° at rated power, and one injection for every two engine revolutions (four-stroke engines), and about 28 percent of the fuel energy into shaft power, the following table summarizes the injector parameter estimates. Sample calculations are given below. Also note the gasoline properties were given previously in Table 1.

Injection Parameters for a 2011 Ford Explorer™	
Displacement, Engine Type	3.5 L, V6
Number of Injectors	6
Rated Power	290 hp
Rated Speed	6500 rpm
Max. Fuel Flow Rate	23.1 cm^3/s
Flow Rate per Injector	3.85 cm^3/s
Injector Period	18,462 μs
Max. Injector Pulse Width	7692 μs
Min. Injector Pulse Width	2083 μs

Maximum Fuel Flow Rate =

$$\dot{V} = \frac{290hp}{0.28\dfrac{hp}{Fuel\,hp\,in}} \times \frac{0.7457\,kW}{hp} \times \frac{1kg}{44\,MJ} \times \frac{1MJ}{10^3\,kW-s} \times \frac{1m^3}{760\,kg} = 23.1\times10^{-6}\,\frac{m^3}{s} = 23.1\frac{cm^3}{s}$$

Period for One Injection Cycle = $\dfrac{2\,{}^{rev}\!/_{cycle}}{6500\,{}^{rev}\!/_{\min}} \times \dfrac{60\sec}{\min} = 18{,}462\,{}^{\mu s}\!/_{cycle}$

Injector Pulse Width for 300 CA° = $300\dfrac{CA}{pulse} \times \dfrac{1rev}{360\,CA} \times \dfrac{1\min}{6500\,rev} \times \dfrac{60\sec}{\min} = 7692\,\mu\,s$

Injector Pulse Width for 10 CA° (at 800 rpm idle condition) =

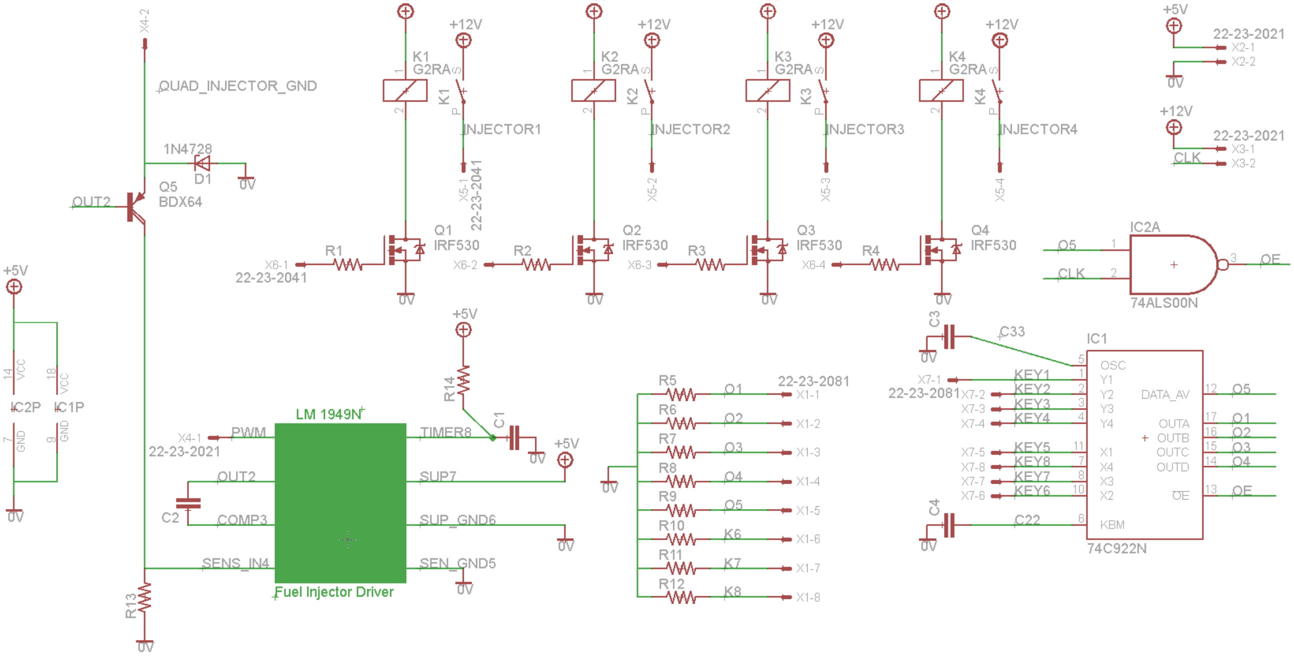
$$= 10\frac{CA}{pulse} \times \frac{1rev}{360\,CA} \times \frac{1\min}{800\,rev} \times \frac{60\sec}{\min} = 2083\,\mu\,s$$

Appendix B. Microcontroller System Description and Schematics

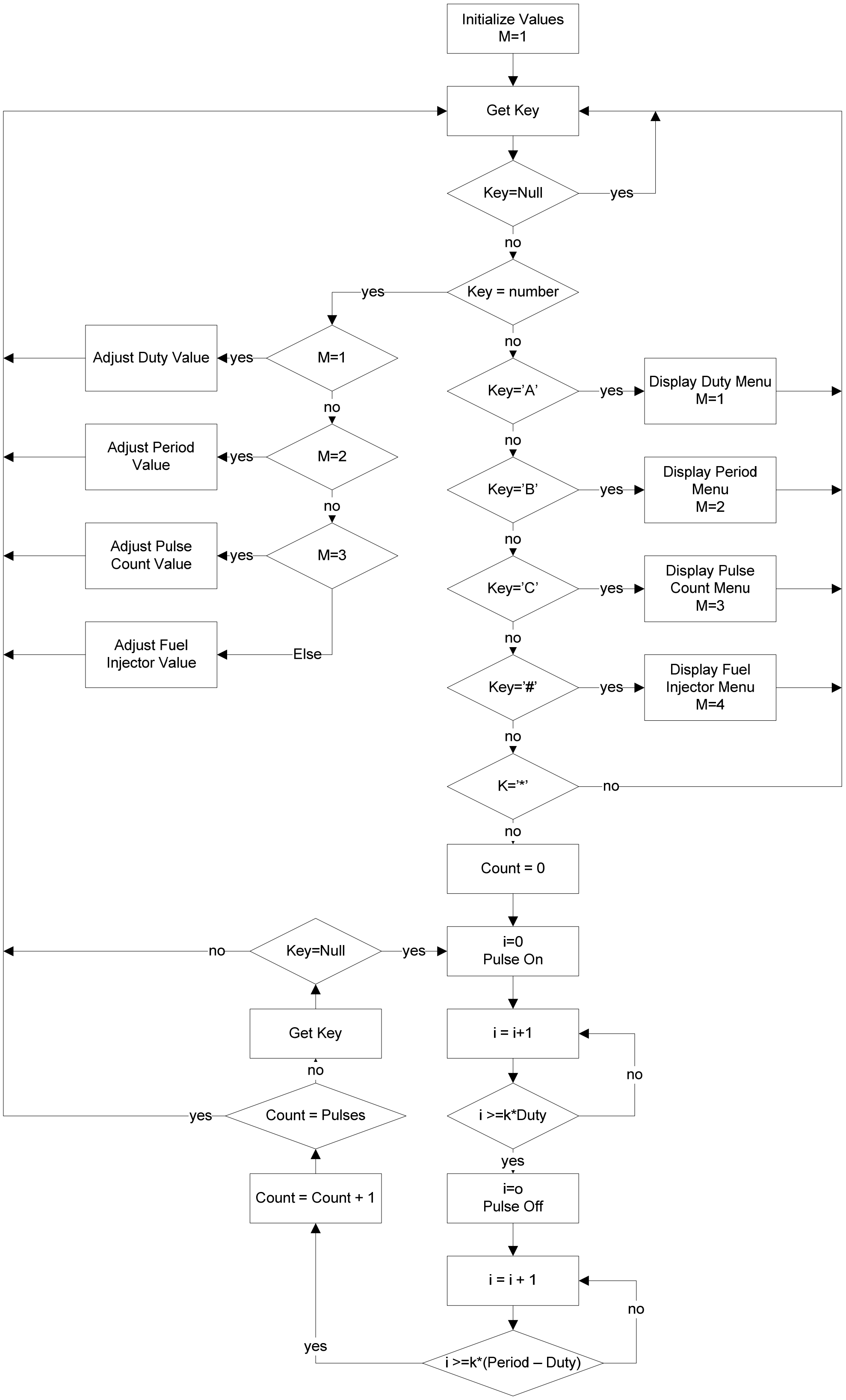
The following circuit is designed as a fuel injector driver. There are four relays that supply power to one of four individual fuel injectors at a time. The NPN Darlington transistor serves as the sink for all of the fuel injectors. The National Instruments LM 1949 chip[6] serves as the injector driver/controller. By directly sensing the initial current through the injector solenoid via R13, the LM 1949 initially saturates the NPN Darlington base to supply all of the current required to initially open the injector solenoid. After the solenoid opens, the LM 1949 tapers off the current to the minimum needed to hold the solenoid open. By conserving power and responding fast to changes in current demand, the LM 1949 provides a strong correlation with the average voltage or the duty of the input signal.

The relay and LM 1949N control signals were interfaced with a HCS12[7] micro-controller. The circuit includes an encoder, a National Instruments MM74C922[8], to interface a hex keypad with the HCS12 microcontroller. The accompanying components were those suggested in the application note for this device. This input to the microcontroller uses pull-down resistors on all of the eight bits. The microcontroller provides the necessary clock for the encoder.

Apparatus Electrical Schematic



Appendix C. Software Flow Chart



Appendix D. Uncertainty Analysis Handout

UNCERTAINTY ANALYSIS

Uncertainty analysis is concerned with the evaluation of uncertainties (inaccuracies, errors) in experimental results and determining how they propagate through the calculation of other results. The uncertainty of a measurement, x, is designated as Δx. It refers to the possible value of the error and indicates the degree of accuracy with which the measurement is believed to have been made[1]. For example, if a volume measurement is expressed as
$\forall$ = 500 ml ± 10 ml, the experimenter believes that the measurement could be off by as much as $\Delta\forall$ = 10 ml (over or under). The value of the measurement as indicated by the instrument is called the nominal value. Thus, for this example, the nominal value is 500 ml.

It is often convenient to represent uncertainty values in terms of percentages. We thus define relative uncertainty as $\Delta x/x$, and percent relative uncertainty as $\Delta x/x \times 100\%$. For the volume listed above, the percent relative uncertainty would simply be $10/500 \times 100\% = 2\%$, and the result could equivalently be reported as $\forall$ = 500 ml ± 2%.

Unless otherwise noted, the uncertainty in a value recorded by an instrument is typically estimated to be half of the smallest scale marking, or half of the last digit that can be read on a digital readout. Thus, if the scale on a pressure gage has its smallest increments of 10 psi, then the uncertainty is estimated to be ± 5 psi. In the case of mechanical timers that are activated or deactivated manually by the experimenter, an additional uncertainty accounting for reaction time might need to be considered.

Suppose you measure *n* independent variables, x_1, x_2, …, x_n, and use them to calculate some other quantity, F, which is a function of these *n* variables, i.e.,
$F = F(x_1, x_2, \ldots, x_n)$
The x_i quantities are measured and thus have associated uncertainties Δx_1, Δx_2, … , Δx_n. These errors propagate into the calculation of F, causing an error ΔF in the computed quantity F. This error must be estimated.

If we consider the Δx_i's as absolute limits on the individual errors in the x_i measurements, the absolute uncertainty in F may be approximated using the concept of an exact differential and neglecting higher order terms in a Taylor series expansion[2] can be expressed as:

$$\Delta F \approx \frac{\partial F}{\partial x_1}\Delta x_1 + \frac{\partial F}{\partial x_2}\Delta x_2 + \ldots + \frac{\partial F}{\partial x_n}\Delta x_n$$

The magnitude of the partial derivative relative to x_i can be interpreted as a measure of the influence of x_i; the larger its value, the larger the influence of x_i on quantity F.

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[1] If we knew the "exact" errors in a measurement, we wouldn't need to talk about "errors." Uncertainties are *estimates* of error.

[2] For more details on the derivation, refer to pages 58-60 in "Measurement Systems: Applications and Design," 4th ed., by Ernest O. Doebelin, McGraw-Hill, 1990.

Some of the partial derivatives might be negative, which might at first glance appear to *reduce* the error. However, since the x_i measurements are independent, and the errors are random (uncorrelated and both positive and negative), there are several ways of handling the signs. The simplest way is to take the absolute value of each term so as to "stack" the errors. Thus, the maximum value of the uncertainty in F can be computed as:

$$\boxed{\Delta F \cong \left|\frac{\partial F}{\partial x_1}\Delta x_1\right| + \left|\frac{\partial F}{\partial x_2}\Delta x_2\right| + \ldots + \left|\frac{\partial F}{\partial x_n}\Delta x_n\right|}$$

Note: When taking the partial derivative of F relative to a variable x_i, all of the other variables are treated as "constants." For example, if

$$F(x, y) = x^2 y + x + 3$$

then,

$$\frac{\partial F}{\partial x} = 2xy + 1 \qquad \frac{\partial F}{\partial y} = x^2$$

The uncertainty in F would then be approximated as

$$\Delta F \cong \frac{\partial F}{\partial x}\Delta x + \frac{\partial F}{\partial y}\Delta y = (2xy + 1)\Delta x + x^2 \Delta y$$

Example: We wish to calculate the power input, P, to an electrical circuit by measuring the voltage, E, and resistance, R, through the circuit. The voltmeter indicates a reading of 10 V and the ohmmeter indicates a value of 100 Ω. The voltmeter and ohmmeter have accuracies of ± 0.1 V and ± 1 Ω, respectively. Determine the uncertainty in the power "indirect" measurement.

The power is given by: $$P = \frac{E^2}{R} = P(E, R)$$

The uncertainty in P can be estimated as: $$\Delta P \approx \frac{\partial P}{\partial E}\Delta E + \frac{\partial P}{\partial R}\Delta R$$

Where $$\frac{\partial P}{\partial E} = \frac{2E}{R} \qquad \frac{\partial P}{\partial R} = -\frac{E^2}{R^2}$$

Thus, $$\Delta P \cong \frac{2E}{R}\Delta E - \frac{E^2}{R^2}\Delta R \text{ or } \qquad |\Delta P| \cong \left|\frac{2E}{R}\Delta E\right| + \left|\frac{E^2}{R^2}\Delta R\right|$$

i.e., $$|\Delta P| \cong \frac{2 \times 10V}{100\Omega} \times 0.1V + \frac{(10V)^2}{(100\Omega)^2} \times 1\Omega = 0.03W$$

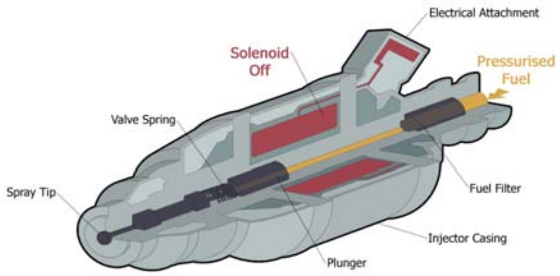
The nominal value of the power is: $$P = \frac{(10V)^2}{100\Omega} = 1W$$

Thus, $P = 1 \pm 0.03$ W

Appendix E. Sample Student Lab Assignment

Introduction to Thermal Engineering
Fall 2010 **EGR 250**
Laboratory Assignment #1

Fuel Injector Calibration: Experimental Uncertainties



Section through an Automotive Fuel Injector
(Taken from Wikipedia: http://en.wikipedia.org/wiki/Fuel_injection)

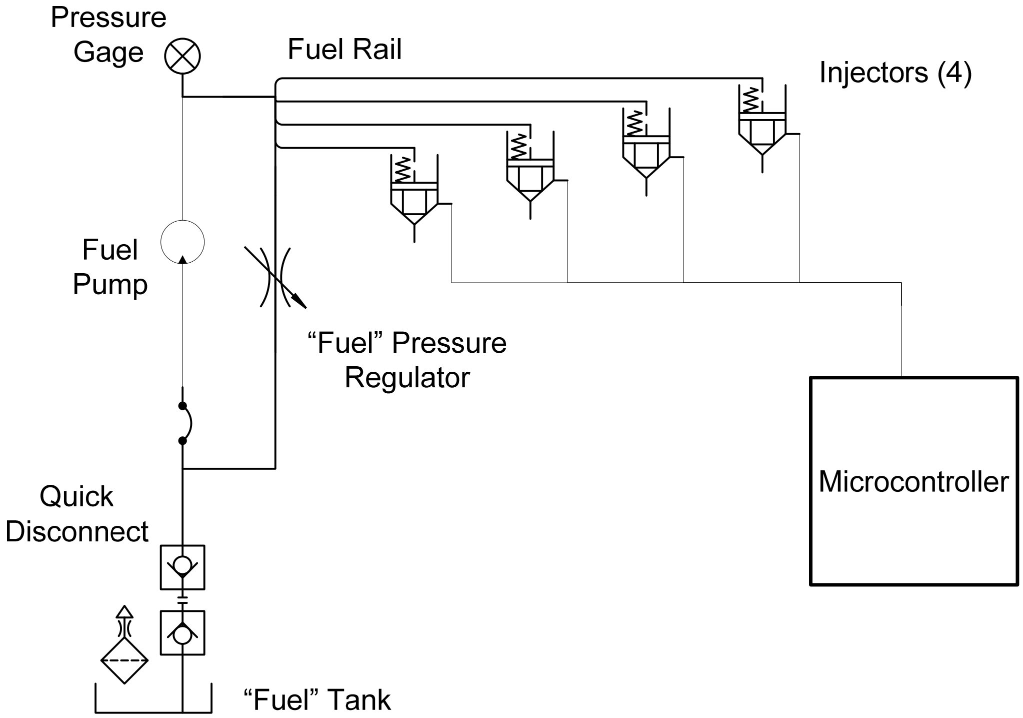
The purposes of this laboratory exercise are to introduce you to fluid flowrate measurements, calibration, and associated experimental uncertainties. During the laboratory introduction, your instructor will be discussing the application of fuel injectors in a modern gasoline engine, including some important timing and flow rate parameters. You will also be introduced to the operation of the *Fuel Injector Test Bench* and any other required equipment, and you will design and conduct experiments to calibrate an automotive fuel injector. A schematic of the test bench is shown on page 2 of this handout.

The target engine conditions for this lab are: V-6 Engine, <290> hp at <6500> rpm.

Experiment:

1. Before coming into the lab, pre-calculate the injection period necessary to model the target engine speed and the required fuel flow rate for the engine specified above. Also, conduct an uncertainty analysis so that you have some reasonable collection volumes and times in mind.
2. Use the volume collection method, along with the Fuel Injector Test Bench, to calibrate your fuel injector over a reasonable range of pulse widths. To insure repeatability, repeat each test two times (for a total of three trials each). What injector pulse width is required to provide the fuel flow rate for the target engine power?
3. Plot volumetric flowrate, $\dot{\forall}$, ml/sec, as a function of injector pulse width. Also tabulate your results
4. Using your equations for the uncertainty <u>and</u> relative uncertainty of the measured volume flow rates, tabulate the uncertainty and relative uncertainty values and show the uncertainty for each data point on the above graph (using error bars or error bands). Comment on the results and discuss how the experimental uncertainties associated with the method can be improved.

5. Suppose now that you wish to calibrate the injector to within 1% of its target flow rate. Design an experiment to do so. Make sure that you can **justify** the experimental conditions (i.e., collection time or collection volume selected) based on your uncertainty calculations.



Schematic of Fuel Injector Test Bench

Test Bench Operation: **SAFETY GLASSES REQUIRED**

1. Place a graduated cylinder under the fuel injector to be tested. Lower the fuel rail so that the injector is as deep into the graduated cylinder as the apparatus allows. The injector rail should be away from you behind the acrylic panel as you face the micro-controller panel.
2. Plug the power cords for the micro-controller and the fuel pump into the wall. You should hear the pump motor and see the display light up.
3. Press key "A" to display and set the *Pulse Width*. Then press key "B" to display and set the *Period*. Then press key "C" to display and set the number of pulses. Press key "#" to select the fuel injector.
4. Press the "*" key to start the injection process. Before you collect data, run the fuel injector to purge any air from the system. Once no bubbles are visible in the fuel line, you can drain the graduated cylinder and begin your experiment.
5. Always unplug the apparatus when you are finished. You may pour the collected fluid back into the fuel tank when you are done.