AC 2008-329: A SIMPLE ANALYTICAL METHOD FOR FORCE ANALYSIS OF PLANAR FRICTIONAL TREE-LIKE MECHANISMS

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A Simple Analytical Method for Force Analysis of Planar Frictional Tree-Like Mechanisms

Abstract

A purely analytical method for force analysis of one degree-of-freedom planar frictional tree-like mechanisms (which constitute a large fraction of planar mechanisms) has been developed herein. The method includes linear friction (friction at sliders, not revolving joints) and uses the vectorial illustration of mechanisms, which is widely used for kinematic analysis of mechanisms too. In this method, a joint-force is determined either via its decomposition into the direction of its adjacent links or from the equilibrium equations of one of these links. Unlike the conventional analytical method which leads to a system of simultaneous equations, this method leads to only one linear algebraic-equation or one simple vectorial-equation at a time. Force analysis of planar frictionless mechanisms has always been tedious and time consuming, let alone frictional mechanisms, but this method has proved to be simple, straightforward and quick. It is therefore a most suitable tool not only for designers but for teaching force analysis of mechanisms too, as it downgrades the project-type problems to the level of classroom tutorials. The teaching significance of the method further surfaces when the reader would recall that textbooks have mainly focused on frictionless mechanisms due to the complexity of frictional mechanisms.

Keywords: Mechanisms, Planar mechanisms, Frictional mechanisms, Kinetic analysis, Force analysis, Kinetostatic analysis.

Nomenclature

α_i = angular acceleration of link i
ϕ_a = angular position of the velocity of joint, say, A
γ_i = angular position of the acceleration-vector of centroid of link i
γ_a = angular position of the acceleration of joint, say, A
λ_a = angular position of the force at joint, say, A
μ_i = coefficient of friction between link i and the foundation
μ_{ij} = coefficient of friction between links i and j
θ_i = angular-position of position-vector $\vec{R}_i$ depicting link i
ω_i = angular velocity of link i
$\vec{a}_a$ = acceleration vector of joint, say, A
$\vec{a}_i$ = acceleration vector of the centroid of link i
$\vec{A}$ = force at joint A (joint-force $\vec{A}$. Similarly joint-force $\vec{B}$ at joint B etc.)
$\vec{A}_i$ = component of joint-force $\vec{A}$ along link i
$\vec{f}_i$ = inertia-force vector of link i
f_i = inertia force of link i = $-m_i a_i$
$\vec{F}$ = input-/output- (external) force
$\vec{F}_i$ = normal constraint-force applied on link i by the foundation

$\vec{F}_{ij}$ = normal force applied on link j by link i

$\vec{F}_i'$ = frictional force applied on link (slider) i by the foundation (link 1)

$\vec{F}_{ij}'$ = frictional force applied on link j by link i

J_i = polar moment-of-inertia of link i about its centroid G_i

m_i = mass of link i

M = moment

n = total number of links including the foundation (link 1)

q_i = inertia torque of link i $= -J_i \alpha_i$

n_2 = rpm of link 2 (+ve if ccw, and –ve if cw)

$\vec{Q}_i$ = external force on link i, if any

R_i = length of link i

$\vec{R}_i$ = position vector of link i

T = input-/output- (external) torque

T_i = resultant external torque on link i, if any

$\vec{v}_a$ = velocity of joint, say, A

$\vec{v}_i$ = velocity of the centroid of link i

All angular positions are measured from x-axis.

Introduction

Analytical force (or kinetic or kinetostatic) analysis of mechanisms has always been one of the lengthy and time-consuming problems of mechanical engineering. In the past few decades, a number of methods have been developed for force analysis of frictionless mechanisms, but only a few researchers have embarked on frictional mechanisms due to its further complexity. The subject is still under research as no efficient and widely accepted method is yet available for this purpose, especially for teaching.

Lauw and Kinzel[1] developed an interactive computer-aided force analysis program (PORKIN), which includes Coulomb friction as well. Muir and Neuman[2] introduced a formulation for dynamic modeling of multibody robotic mechanisms incorporating friction (stiction, Coulomb, rolling and viscous friction), based on Newtonean dynamics, kinetics and the concept of force/torque propagation and frictional coupling at a joint, using extensive matrix-vector dynamics formulation to solve the systems of linear algebraic equation. Verriest[3] developed a method for kinematics and dynamics of a highly structured special-purpose robot, where direction-dependent friction allows such a structure to 'crawl' in various modes. Brost and Mason[4] described a graphical method for analyzing the motion of a rigid body subject to multiple frictional contacts in a plane. Kraus et al[5] simulated the dynamic systems using rigid body model with rolling and sliding unilateral contacts for planar systems. Song et al[6] employed a general model of contact compliance to derive stability criterion for planar mechanical systems with frictional contacts, introducing a smooth nonlinear friction law to approximate Coulomb's friction where the Coulomb's friction law is discontinuous. Stoenescu and Marghitu[7] investigated the effect of prismatic joint inertia on dynamics of planar kinematic chains with friction, using Lagrange's equations, exemplifying the effect of the prismatic joint inertia on the dynamic parameters of planar mechanisms.

The reviewed papers[1-7], though dealt with the investigation of friction on rigid body/multibody systems, illustrated no specific method of application to planar frictional mechanisms. Hence none of them were capable of being adopted as a foundation for this research.

In majority of textbooks on Theory of Machines and Mechanisms, graphical methods have been adopted as a major tool for force analysis of planar mechanisms, hardly touching on frictional mechanisms. This is due to the fact that analytical methods are lengthy and/or require computing, and no specific method has yet demonstrated a suitable and efficient capacity for class room applications.

With the advent of electronic computing devices some authors of textbooks were encouraged to describe analytical methods too. However, their approaches are mainly the analytical solution of the same equilibrium equations solved by graphical methods, applied to special cases of mechanisms, mostly four-bar mechanisms. For instance, Shigley and Uicker[8] demonstrated the force analysis of a four-bar linkage using a vectorial approach and Mabie and Reinholtz illustrated the application of a matrix method[9] and a complex-numbers method[10] to solve the equations of motion of some four-bar mechanisms. Norton[11] demonstrated the solution of matrix equation of motion to some slider-crank and four-bar mechanisms. Waldron and Kinzel[12] simply solved the system of simultaneous equations of motion for different links. In addition to graphical method, Erdman et al[13] also employed the solution of matrix equation of motion and demonstrated the method by applying to a four-bar mechanism; they approached the problem by superposition method as well, both graphically and analytically. Myszka[14], like Waldron and Kinzel[12], generated and solved equilibrium equations and demonstrated the method on an aircraft landing gear (again a four-bar mechanism).

It can be seen that these approaches generally suggest that for each individual mechanism the equations of motion have to be set up and organized such that they lend themselves to manual or computer solution. Needless to say that the manual solutions are lengthy and time-consuming while the necessity of computers in the classroom for computerized solutions discourages both teachers and students.

Among all teaching texts, the one by Hall, Jr.[15] more comprehensively covered the analytical approach to Kinematic and Force Analysis of Frictionless Mechanisms. On kinematic analysis, he extensively illustrated the application of the vector loop approach and on the force analysis he explained the method of equation of motion as well as the matrix arrangement of equations of motion. Unlike the authors of other books he extended the application of these methods beyond four bar mechanisms.

A purely analytical method developed by Abhary[16] is, by far, and according to the author's experience of teaching mechanisms for more than two decades, the most suitable method for teaching analytical approach to force analysis of planar frictionless mechanisms. The method for force analysis of planar frictional mechanisms developed herein is, in fact, the elaboration and refinement of this method for frictionless mechanisms, which proved to be quite capable of accommodating sliding frictions. Unlike the standard analytical method which leads to large systems of simultaneous equations, this method leads to only one linear algebraic/vectorial equation at a time. It is systematic and follows a general pattern. It is highly suitable as a standard technique for manual solution to the problem and could also be easily programmed as a computer oriented force analysis scheme for planar frictional mechanisms. To the best of the author's knowledge, this method is far easier to apply than any other existing method; hence it is not only a powerful design tool for analysis and design of planar frictional mechanisms but a most suitable method for teaching force analysis of frictional mechanisms.

Vectorial Illustration of Mechanisms

The method explained in this paper uses the well- known vectorial illustration of planar mechanisms, in which the geometry of a mechanism is defined by a number of vectors, whose unknown lengths and inclination angles are determined at the very first stage of analysis of planar mechanisms, i.e. position analysis.

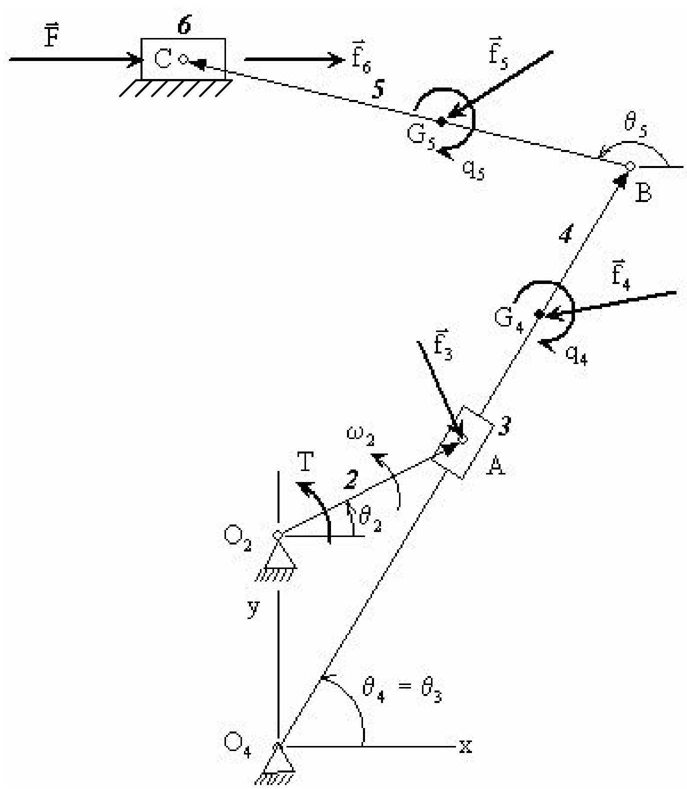

Figure 1 A Quick-Return Mechanism

Figure 1 is the vectorial illustration of a quick-return mechanism where G_i is the centroid of link i and the position vectors are

$$\vec{R}_2 = \overrightarrow{O_2 A} = \overline{O_2 A} \cdot e^{j\theta_2} \quad (1)$$

$$\vec{R}_3 = \overrightarrow{O_4 A} = \overline{O_4 A} \cdot e^{j\theta_4} \quad (2)$$

$$\vec{R}_4 = \overrightarrow{O_4 B} = \overline{O_4 B} \cdot e^{j\theta_4} \quad (3)$$

$$\vec{R}_{g_4} = \overrightarrow{O_4G_4} = \overline{O_4G_4} \cdot e^{j\theta_4} \tag{4}$$

$$\vec{R}_5 = \overrightarrow{BC} = \overline{BC} \cdot e^{j\theta_5} \tag{5}$$

$$\vec{R}_{g_5} = \overrightarrow{BG_5} = \overline{BG_5} \cdot e^{j\theta_5} \tag{6}$$

In this paper the velocity of a joint, say B, is denoted by $\vec{v}_b = v_b e^{j\phi_b}$ and its acceleration by $\vec{a}_b = a_b e^{j\gamma_b}$, and the inertia force of link i

$$\vec{f}_i = -m_i \vec{a}_i = -m_i a_i e^{j\gamma_i} = f_i e^{j\gamma_i} \tag{7}$$

It must be mentioned that in a mechanism, usually either the input- or the output-load is unknown. For example, in Figure 1 the input torque T on link 2 is the unknown.

Convention to Define a Joint-Force

To facilitate and systematize the procedure, the following convention is observed herein to identify and denote joint-forces of a planar mechanism. This convention, with the aid of a portion of a hypothetical mechanism illustrated in Figure 2a, is stated as follows:

> If links i and j (i < j) of the mechanism are pivoted together at joint, say, B as illustrated in Figure 2a, the force at joint B on the free body diagram of link i, is denoted by $\vec{B}$; hence, by $-\vec{B}$ on link j, Figure 2b.
>
> In Figure 2

$$\vec{R}_{g_i} = \overrightarrow{HG_i} = \overline{HG_i} \cdot e^{j\theta_i} \tag{8}$$

$$\vec{R}_{g_j} = \overrightarrow{G_jD} = \overline{G_jD} \cdot e^{j\theta_j} \tag{9}$$

Denotion of Frictional Forces

The frictional force on a slider always opposes the relative velocity of the slider with respect to its mating link, and its magnitude is the product of the coefficient of friction and the normal reaction between the slider and its mating link.

Therefore, the frictional force on link 6, Figure 1, as depicted in its free-body-diagram, Figure 4a, is

$$\vec{F}_6' = -\mu_6 F_6 e^{j\phi_c} \tag{10}$$

and the frictional force on link (slider) 3 is opposite to the velocity of the slider with respect to its carrier, link 4 , i.e. opposite to

$$\overrightarrow{d(O_4A)/dt} = \dot{R}_3 e^{\theta_4} \tag{11}$$

Therefore, defining

$$\begin{cases} \delta_{34} = -1 & \text{if} \quad \dot{R}_3 > 0 \\ \delta_{34} = +1 & \text{if} \quad \dot{R}_3 < 0 \end{cases} \tag{12}$$

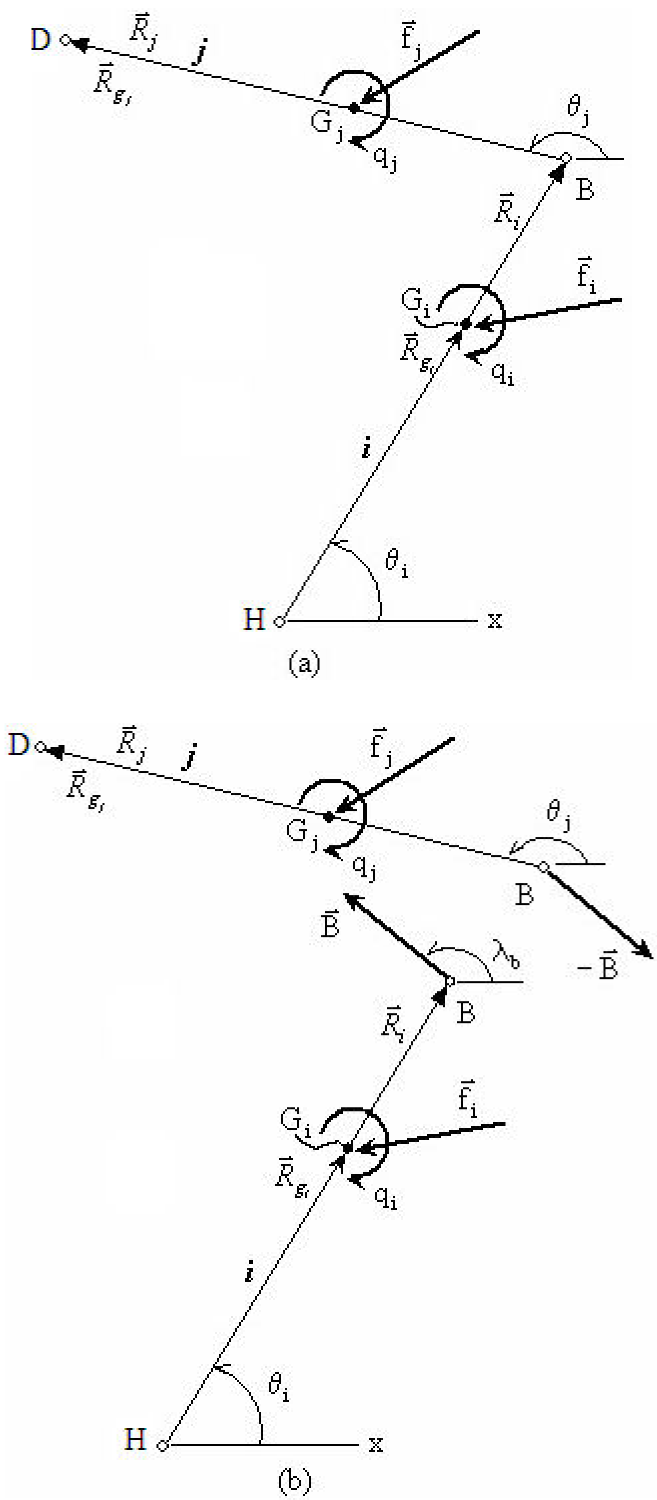

Figure 2 Illustration of joint-forces

then the frictional force on link 3, i.e. $-\vec{F}'_{34}$ in Figure 6, is

$$-\vec{F}'_{34} = \delta_{34}\mu_{34}F_{34}\angle\theta_4 \tag{13}$$

so

$$\vec{F}'_{34} = -\delta_{34}\mu_{34}F_{34}\angle\theta_4 \tag{14}$$

Analytical Force Analysis of Frictional Planar Tree-Like Mechanisms

Force analysis of a mechanism uses the results of the kinematic analysis, i.e. positions, velocities and accelerations, as the data.

The method developed herein, for determining the joint-forces, consists of an algorithm as follows:

i. In a mechanism either the input load or the output load is known. In the former case start the analysis from the input link, otherwise from the output link.

ii. To determine a joint-force, construct the free body diagram of the joint's adjacent links using the convention previously stated in this paper, starting with the joint on the input or output link as described in Step i.

   As a guide, consider the sub-mechanism in Figure 2a of a hypothetical mechanism where the general picture of the two adjacent links of a joint, B, is depicted along with their free body diagrams, Figure 2b.

iii. Determine the current joint-force from either the force- or a moment-equilibrium equation of an adjacent link, if possible, then go back to step ii to determine the next joint-force. Otherwise reconstruct the free body diagram of the joint's adjacent links by decomposing the joint-force into the direction of these links.

   For example, in the hypothetical sub-mechanism, Figure 2a, the joint force $\vec{B}$ in Figure 2b is decomposed into $\vec{B}_i \parallel \vec{R}_i$ and $\vec{B}_j \parallel \vec{R}_j$, Figure 3.

iv. Determine the non-parallel component of the joint-force on any adjacent link from the moment-equilibrium equation of the link about its other end.

   In the hypothetical sub-mechanism, Figure 3, this means that $\vec{B}_j$ is determined from the moment-equilibrium equation of link i about H

$$\begin{aligned}\sum M_h &= \left|\vec{R}_i \times \vec{B}_j + \vec{R}_{g_i} \times \vec{f}_i\right| + q_i = \left|R_i e^{j\theta_i} \times B_j e^{j\theta_j} + R_{g_i} e^{j\theta_i} \times f_i e^{j\gamma_i}\right| + q_i \\ &= R_i B_j \sin(\theta_j - \theta_i) + R_{gi} f_i \sin(\gamma_i - \theta_i) + q_i = 0\end{aligned} \tag{15}$$

Therefore

$$B_j = -\left\lfloor R_{gi} f_i \sin(\gamma_i - \theta_i) + q_i \right\rfloor / R_i \sin(\theta_j - \theta_i) \tag{16}$$

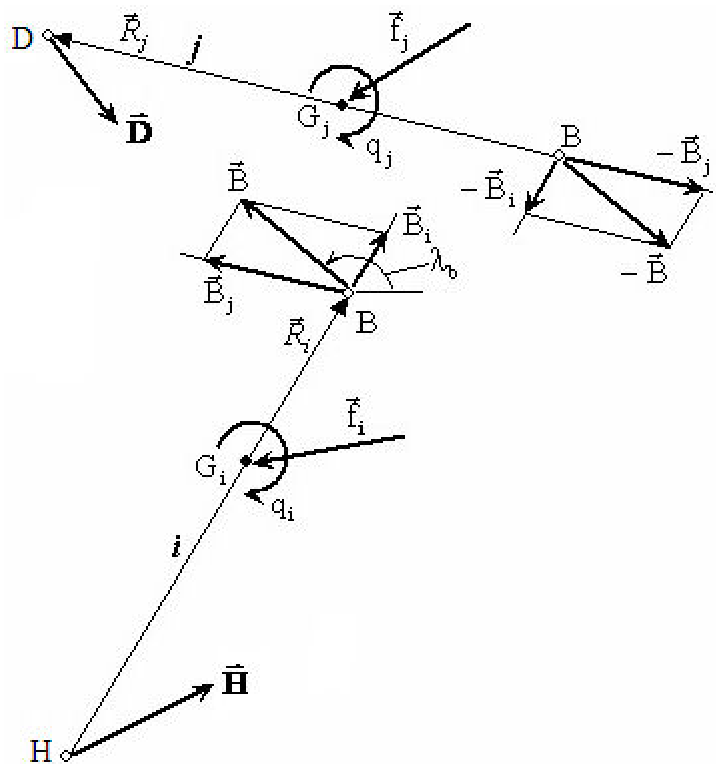

Figure 3 Decomposition of a joint-force

Similarly, $\vec{B}_i$ is determined from the moment-equilibrium equation of link j about D, Figure 3

$$\sum M_d = \left|-\vec{R}_j \times \left(-\vec{B}_i\right) - \vec{R}_{g_i} \times \vec{f}_j\right| + q_j = \left|R_j e^{j\theta_j} \times B_i e^{j\theta_i} - R_{g_j} e^{j\theta_j} \times f_j e^{j\gamma_j}\right| + q_i$$
$$= R_j B_i \sin(\theta_i - \theta_j) - R_{g_j} f_j \sin(\gamma_j - \theta_j) + q_j = 0 \quad (17)$$

i.e.

$$B_i = \left\lfloor R_{g_j} f_j \sin(\gamma_j - \theta_j) - q_j \right\rfloor / R_j \sin(\theta_i - \theta_j) \quad (18)$$

Therefore the joint-force $\vec{B}$ is

$$\vec{B} = Be^{j\lambda_b} = \vec{B}_i + \vec{B}_j = B_i e^{j\theta_i} + B_j e^{j\theta_j} \quad (19)$$

v. Once a joint-force is thus determined, the other joint-forces on the adjacent links can be easily determined from their force-equilibrium, i.e. $\sum \vec{F} = 0$.

Applying this rule to link i, Figure 3

$$\sum \vec{F} = \vec{H} + \vec{f}_i + \vec{B} = 0 \quad (20)$$

so

$$\vec{H} = -\vec{f}_i - \vec{B} \quad (21)$$

and to link j

$$\sum \vec{F} = -\vec{B} + \vec{D} + \vec{f}_j = 0 \tag{22}$$

so

$$\vec{D} = \vec{B} - \vec{f}_j \tag{23}$$

vi. Repeat step ii to v for all moving- as well as fixed-joints.

It can be seen that a joint-force is determined either by its components (along its adjacent links, such as joint-force $\vec{B}$ determined out of its components $\vec{B}_i$ and $\vec{B}_j$, Figure 3) via two simple algebraic linear equations (such as Equation 15 & 17), or from the vectorial force-equilibrium equation of the links (such as joint-force $\vec{H}$ via Equation 20 and $\vec{D}$ via Equation 22).

The application of the method is fully demonstrated through an example in the next section.

Example

Consider the quick-return mechanism illustrated in Figure 1, where the input link 2 rotates at n_2 rpm and the output link 6 delivers force $\vec{F}$ opposite to the velocity of point C during the forward stroke, while input-torque T is to be determined. The assumption is that the length, mass and centroid of all links are known, and kinematic analysis of the mechanism (which is a prerequisite for the force analysis) is already fully performed; that is to say positions, velocities, accelerations, and consequently inertia forces and inertia torques of all links are already determined.

The specifications of the mechanism are as follows:
$O_2A = 200$, $O_4B = 700$, $BC = 600$, $O_4O_2 = 300$, $O_4G_4 = 400$, y-coordinate of joint C = $y_C = 900$, $BG_5 = 300$ mm; $\theta_2 = 0$ deg, $m_3 = 0.5$, $m_4 = 6$, $m_5 = 4$, $m_6 = 1$ kg; $J_4 = 10$, $J_5 = 6$ kgm^2; F = 1kN; $\mu_6 = \mu_{34} = 0.5$; $n_2 = 150$ rpm ccw and link 2 is dynamically balanced about O_2. The kinematic analysis of the mechanism at $\theta_2 = 0^\circ$ produced the corresponding data as per Table 1, also $R_3 = 361$ mm and $\dot{R}_3 > 0$.

Table 1 Kinematic data for the mechanism in Figure 1

Link	θ deg	ω rad/s	α rad/s2	Point	v m/s	ϕ deg	a m/s^2	γ deg
2	0	15.708	0.000	A	3.142	90	49.348	180
3		4.833	43.800	B	3.383	146	34.748	174
4	56	4.833	43.800	C	3.986	180	27.092	180
5	148	3.687	-1.796	G_4	1.933	146	19.856	174
6		0.000	0.000	G_5	3.528	164	30.884	177

The force analysis of the mechanism is performed as follows:

a. Apply step i to determine where to start the analysis
   Since the output load $\vec{F}$ is known, the analysis must begin with the output link, i.e. link 6.

b. Apply step ii to illustrate the components of joint-force $\vec{C}$.
Construct the free body diagram of the adjacent links of joint C, link 5 and 6, Figure 4.

c. Apply step iii
Joint-force $\vec{C}$ cannot be determined from only one equilibrium equation of its adjacent links, hence it is decomposed into $\vec{C}_5$ and $\vec{C}_6$ parallel to these adjacent links, respectively.
(Note: according to the convention proposed herein, the joint-force $\vec{C}$ is applied to link 5 and its reaction to link 6, because 5<6).

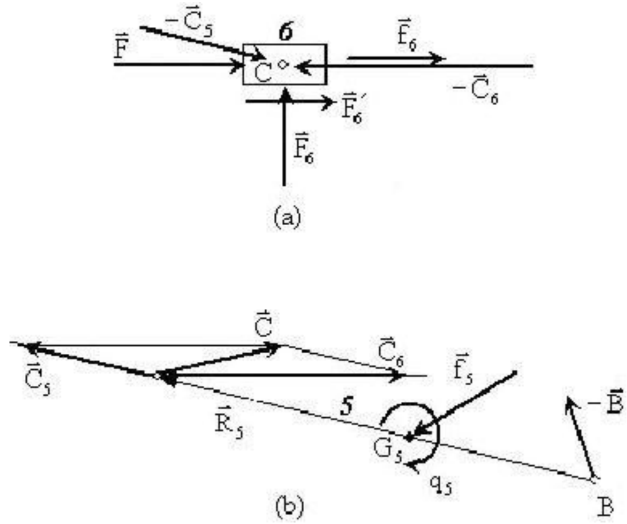

Figure 4 Free body diagram of link 5 and 6

d. Apply step iv to determine joint-force $\vec{C}$ via its components
$\vec{C}_6$ is obtained from the moment-equilibrium equation of link 5, Figure 4b, about joint B

$$\sum M_b = \left|\vec{R}_5 \times \vec{C}_6 + \vec{R}_{g_5} \times \vec{f}_5\right| + q_5 = \left|R_5 e^{j\theta_5} \times C_6 e^{j0} + R_{g_5} e^{j\theta_5} \times f_5 e^{j\gamma_5}\right| + q_5$$
$$= -R_5 C_6 \sin\theta_5 + R_{g_5} f_5 \sin(\gamma_5 - \theta_5) + q_5 = 0 \tag{24}$$

from which

$$C_6 = \left[R_{g_5} f_5 \sin(\gamma_5 - \theta_5) + q_5\right] / R_5 \sin\theta_5 = -23\text{N} \tag{25}$$
$$\vec{C}_6 = C_6 e^{j0} = -23\angle 0^\circ \text{ N} \tag{26}$$

And the force-equilibrium equation of link 6, Figure 4a, is

$$\sum \vec{F} = -\vec{C}_5 + \vec{F} + \vec{f}_6 + \vec{F}_6 + \vec{F}_6' - \vec{C}_6 = 0 \tag{27}$$

or

$$\vec{C}_5 - \vec{F}_6 - \vec{F}_6' = \vec{F} + \vec{f}_6 - \vec{C}_6 \tag{28}$$

or

$$C_5 e^{j\theta_5} - F_6 e^{j90} + \mu_6 F_6 e^{j\phi_c} = -F e^{j\phi_c} + f_6 e^{j\gamma_c} - C_6 e^{j0} \tag{29}$$

The expansion of Equation 29 into a system of two linear equations

$$C_5 \cos\theta_5 + \mu_6 F_6 \cos\phi_c = -F \cos\phi_c + f_6 \cos\gamma_c - C_6 \tag{30}$$

$$C_5 \sin\theta_5 - F_6 = 0 \tag{31}$$

facilitates the determination of $\vec{C}$ and $\vec{F}_6$ as follows

$$C_5 = \frac{-F\cos\phi_c + f_6 \cos\gamma_c - C_6}{\cos\theta_5 + \mu_6 \sin\theta_5 \cos\phi_c} = -943 \text{ N} \tag{32}$$

$$F_6 = C_5 \sin\theta_5 = -500 \text{ N} \tag{33}$$

hence

$$\vec{C} = \vec{C}_5 + \vec{C}_6 = C_5 e^{j\theta_5} + C_6 e^{j0} = -943\angle 148^\circ - 23\angle 0^\circ = 924\angle -33^\circ \text{ N} \tag{34}$$

$$\vec{F}_6 = 500\angle -90^\circ \tag{35}$$

e. Apply Step v to determine joint-force $\vec{B}$
From the force-equilibrium equation of link 5, Figure 4b

$$\sum \vec{F} = \vec{C} + \vec{f}_5 - \vec{B} = 0 \tag{36}$$

$\vec{B}$ is determined

$$\vec{B} = \vec{C} + \vec{f}_5 = B e^{j\lambda_b} = 1{,}033\angle -30^\circ \text{ N} \tag{37}$$

where λ_b is the inclination or angular position of $\vec{B}$.

Now apply step vi, i.e. repeat step ii to v, to other joints as follows:

f. Determine joint-force $\vec{F}_{34}$

$\vec{F}_{34}$ is the normal force applied by link 3 on link 4. Apply step ii, i.e. construct the free body diagram of links 4 and 3, Figure 5 and Figure 6 respectively. From Figure 5

$$F_{34} = -\frac{R_{g4} f_4 \sin(\gamma_4 - \theta_4) + R_4 B \sin(\lambda_b - \theta_4) + q_4}{R_3} = -1{,}499 \text{ N} \tag{38}$$

$$\vec{F}_{34} = F_{34} e^{j(\theta_4 + 90)} = 1{,}499\angle -34^\circ \text{ N} \tag{39}$$

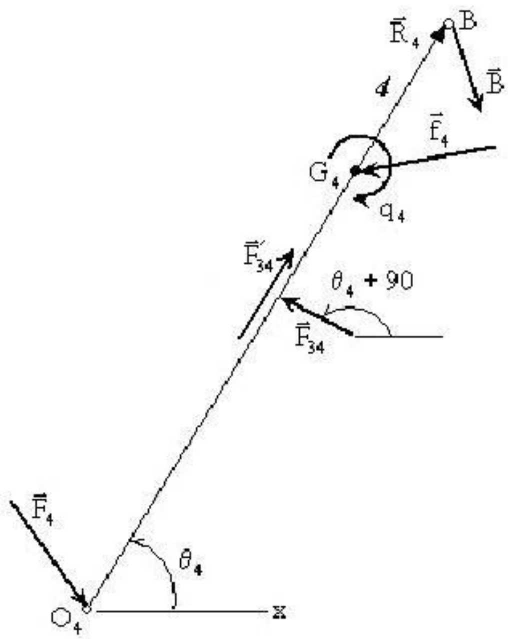

Figure 5 Free body diagram of link 4

The constraint force $\vec{F}_4$ is then determined from the force-equilibrium equation of links 4 (step v), Figure 5

$$\sum \vec{F} = \vec{F}_4 + \vec{F}_{34} + \vec{F}'_{34} + \vec{f}_4 + \vec{B} = 0 \tag{40}$$

where $\vec{F}'_{34}$ is as per Equation 14; so

$$\vec{F}_4 = -\vec{F}_{34} - \vec{F}'_{34} - \vec{f}_4 - \vec{B} = 1{,}809\angle 125^\circ \text{ N} \tag{41}$$

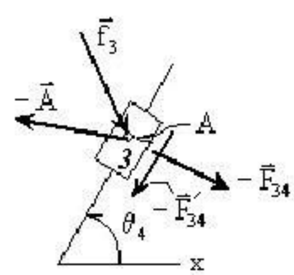

Figure 6 Free body diagram of link 3

g. Determine joint-force $\vec{A}$
Apply step iii to the free body diagram of link 3, Figure 6

$$\sum \vec{F} = -\vec{A} + \vec{f}_3 - \vec{F}_{34} - \vec{F}'_{34} = 0 \tag{42}$$

hence

$$\vec{A} = \vec{f}_3 - \vec{F}_{34} - \vec{F}'_{34} = 1{,}608\angle 119^\circ \text{ N} \tag{43}$$

h. Determine constraint-force $\vec{F}_2$

Apply step iii to the free body diagram of link 2, Figure 7

$$\sum \vec{F} = \vec{F}_2 + \vec{A} = 0 \tag{44}$$

hence

$$\vec{F}_2 = -\vec{A} = 1{,}608\angle -61^\circ \text{ N} \tag{45}$$

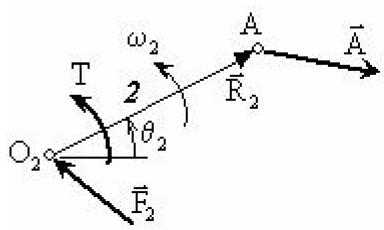

Figure 7 Free body diagram of link 2

Comparison with the Conventional Analytical Method

The quick-return mechanism analyzed in the previous section contains thirteen unknowns $\vec{F}_2$, $\vec{F}_4$, $\vec{F}_6$, $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{F}_{34}$ and T whose determination via the conventional analytical method would lead to a system of thirteen simultaneous equilibrium-equations: three equations for each of links 2, 4 and 5, and two equations for each of links 3 and 6. Needless to say that, on the one hand the parametric (general) solution of thirteen simultaneous equations can be achieved only manually, hence it is very tedious and time consuming, and on the other hand its numerical solution implies computation, therefore makes it unsuitable specially for teaching and class room tutorial purposes. In contrast, the method developed herein generates the general (parametric) solution for each load via only one linear algebraic or one vectorial equation at a time as demonstrated in the previous section; and pocket calculators suffice the manual solution.

Conclusion

The purely analytical method developed herein for the force analysis of one degree-of-freedom frictional planar tree-like mechanisms is general and, as demonstrated, can be systematically applied to generate the parametric solution. Hence it can be efficiently employed as a standard technique for manual or automatic solution to the problem.

After a little acquaintance with the method, the user would realize that the moment- and force-equilibrium equation (such as, say Equation 24 & 36 for link 5) could easily be set up without even resorting to the free body diagram of links. In other words, the convention proposed for depiction of joint-forces helps the user easily generate the conceptual image of the free body diagrams in the mind. In addition to that, each link is solved independently for which, only one simple linear algebraic or vectorial equation is to be solved at a time.

The superiority of the method over the conventional analytical one was demonstrated in the previous section. Compared with graphical methods, not only is the method much more precise (due to its analytical nature) but it is certainly less time consuming too. This is due to the fact that, setting up and solving the equilibrium equations (as functions of kinematic parameters already determined in the kinematic analysis of the mechanism) is much easier to manage than drawing numerous precise scaled figures.

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