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A Simple Demonstration of the Power Factor

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Abstract

The power factor is a useful topic covered as part of the curriculum on alternating current circuits. The first exposure is typically in introductory courses. It is revisited in upper level network theory or circuit analysis courses in physics and engineering. It is standard practice to include the power factor in the curriculum of the circuits courses taught to all engineers.

While the power factor is simply the cosine of the phase angle between the voltage and current, it has practical application for circuits containing reactive loads. Working scientists and engineers are concerned about the power factor in a broad variety of contexts ranging from electromechanical systems, e.g. motors, to impedance matching networks in audio and broadcast systems. The power company cares about a user's power factor and sets rates accordingly.

We have developed a simple demonstration of an alternating current circuit using electric lamps as a proxy for resistive loads. The demonstration can be adapted to a laboratory experiment in either a second semester general physics course or a more advanced laboratory course on circuits. The pedagogical value of the activity is two-fold. First, it provides a clear visual representation of the power factor, and second it serves as a springboard for further discussion of the nature of the phase relationship of reactive circuit elements.

Introduction

A common introductory physics text book definition of the power factor is something along the lines of: "the voltage v has some phase angle ϕ with respect to the current i….The factor $\cos\phi$ is called the **power factor** of the circuit."[1] A more practical definition is adopted in advanced texts. There the power factor is defined as the cosine of the phase angle of the complex impedance. However it is defined, the power factor has practical application. It is difficult to supply real power to a resistive circuit element in a reactive load. If the current through and voltage across the load are nearly in phase then the average power dissipated in the load is maximized. This has broad applications in power distribution systems.

We believe the following demonstration of the power factor and power factor correction to be easy to perform and valuable from a pedagogical viewpoint. The utility of the demonstration depends somewhat on prior student knowledge. We assume students have developed a model of electric current flow that exhibits the following, directly observable, behavior; the brightness of an incandescent lamp is an indication of the amount of current flowing through it. That is, the brighter the lamp-the greater the current flow.[2] It is reasonable to assume such prior knowledge because direct current circuits always precede alternating current (ac) circuits in the physics curriculum. Furthermore, we assume that the demonstration is performed in the context of the discussion of ac circuit topics such as root-mean-square values (rms), impedance, reactance, phase angle, and average power. It is not necessary at the introductory level to introduce either the phasor notation or the concept of complex power.

Demonstration

We have developed a simple circuit where the effects of power factor correction can readily be observed. A schematic of the circuit is shown below in Figure 1. The circuit consists of a 120V power source at 60 Hz, two ordinary, unfrosted incandescent lamps rated at 40W, a variable capacitor 1-20µF, and an inductor with a nominal value of 1 H. This circuit represents a common situation since the reactive loads in commercial, industrial, and residential applications are typically inductive due to the presence of motors and other coils.

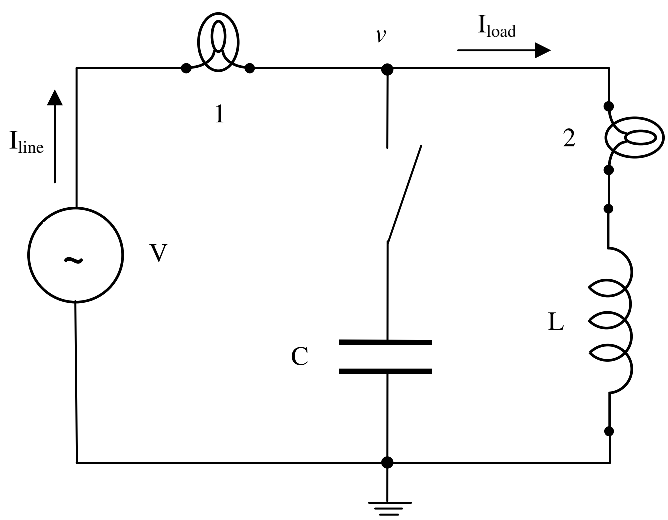

Figure 1. Schematic diagram of the circuit used to demonstrate the power factor.

Consider the operation of the circuit shown in Fig. 1. We identify two currents in the circuit, I_{line} is the current flowing from the source V through lamp 1, and I_{load} is the current flowing through lamp 2 and the inductor L. When the switch is open the capacitor is not part of the circuit. All of the current from the source flows through lamps 1 and 2 thus $I_{load} = I_{line}$. The lamps have the same brightness providing visual reinforcement of the concept of current flow in a series circuit.[3] Lamp 1 is in series with the source so that its brightness represents the average power loss in the supply lines. Lamp 2 is in series with the inductor and its brightness represents the average power dissipated in the load. The key observation is that the lamps are equally bright. The inductor can be cautiously shorted to see that both lamps brighten; the inductor *impedes* the flow of current in the ac circuit.

In Fig. 2 below we show an unretouched photograph taken of the circuit. The variable capacitor has a bank of switches to allow 1,2,4,5, or 8 µF to be added in parallel. In the picture all switches are in the off position and the capacitor is not affecting the circuit. Note that the two lamps exhibit equal brightness; the current through the supply lines equals the current in the reactive load. From this observation no conclusions can be drawn regarding the phase relationship between the voltage across any element and the current through it.

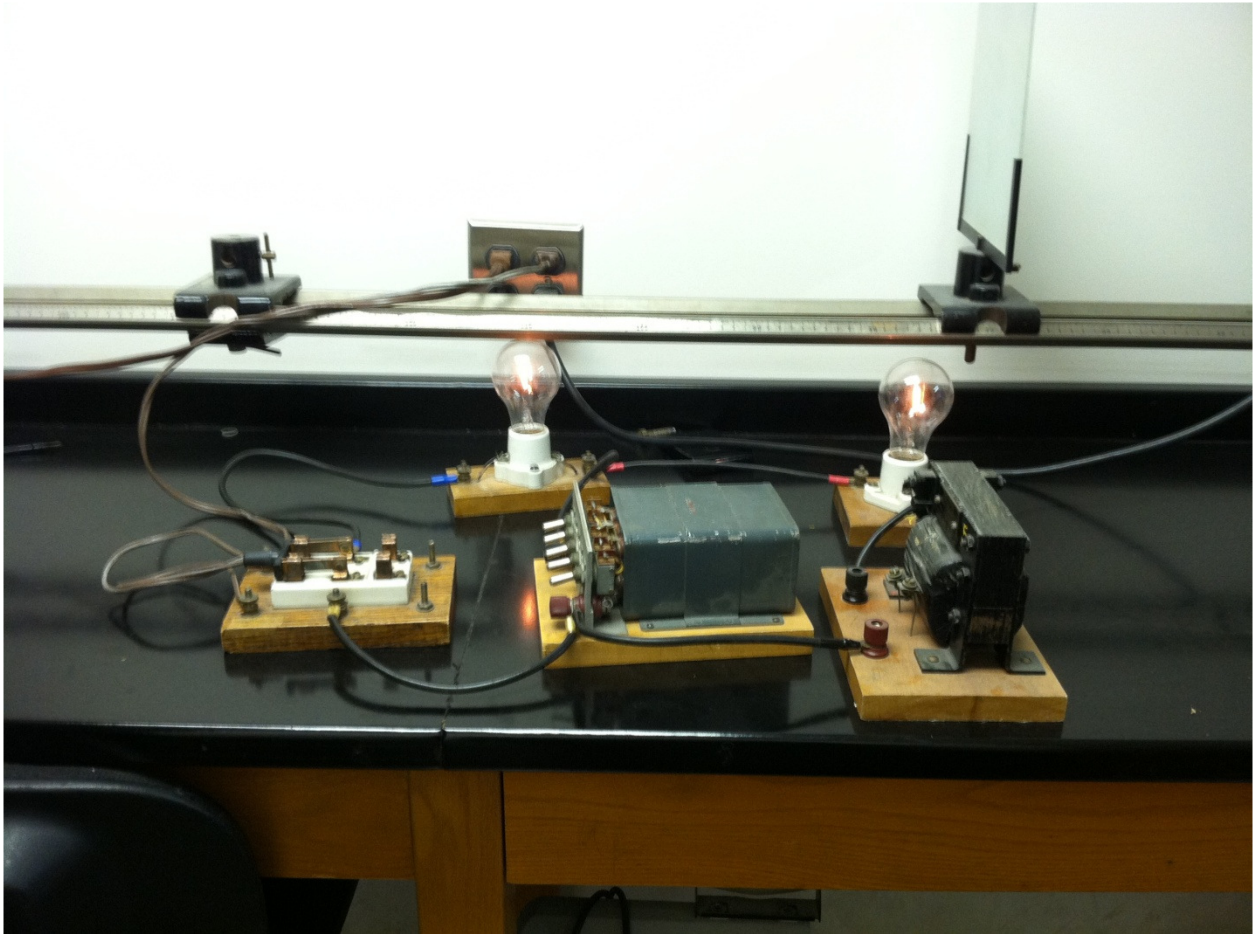

Figure 2. An unretouched photograph of the circuit used to demonstrate the power factor. The circuit consists of two 40W unfrosted incandescent lamps, a variable capacitor (not part of the circuit) and an inductor.

In Fig. 3 we show another unretouched photograph taken of the circuit when 5 μF are added in parallel to the combination of lamp 2 and the inductor. It is readily apparent from the photograph that lamp 1 dimmed significantly while lamp 2 remains bright. While it is difficult to see from the picture, lamp 2 increases in brightness when the power factor is corrected. These results indicate that the amount of current flowing from the source through the lines decreases while the current flowing through the load increases. When the current through the lines decreases, the resistive loss also decreases. This decrease is readily observed via the brightness of the lamp. This is a manifestation of power factor correction. The circuit in Fig. 2 has a low power factor while the circuit shown in Fig. 3 has a power factor near unity.

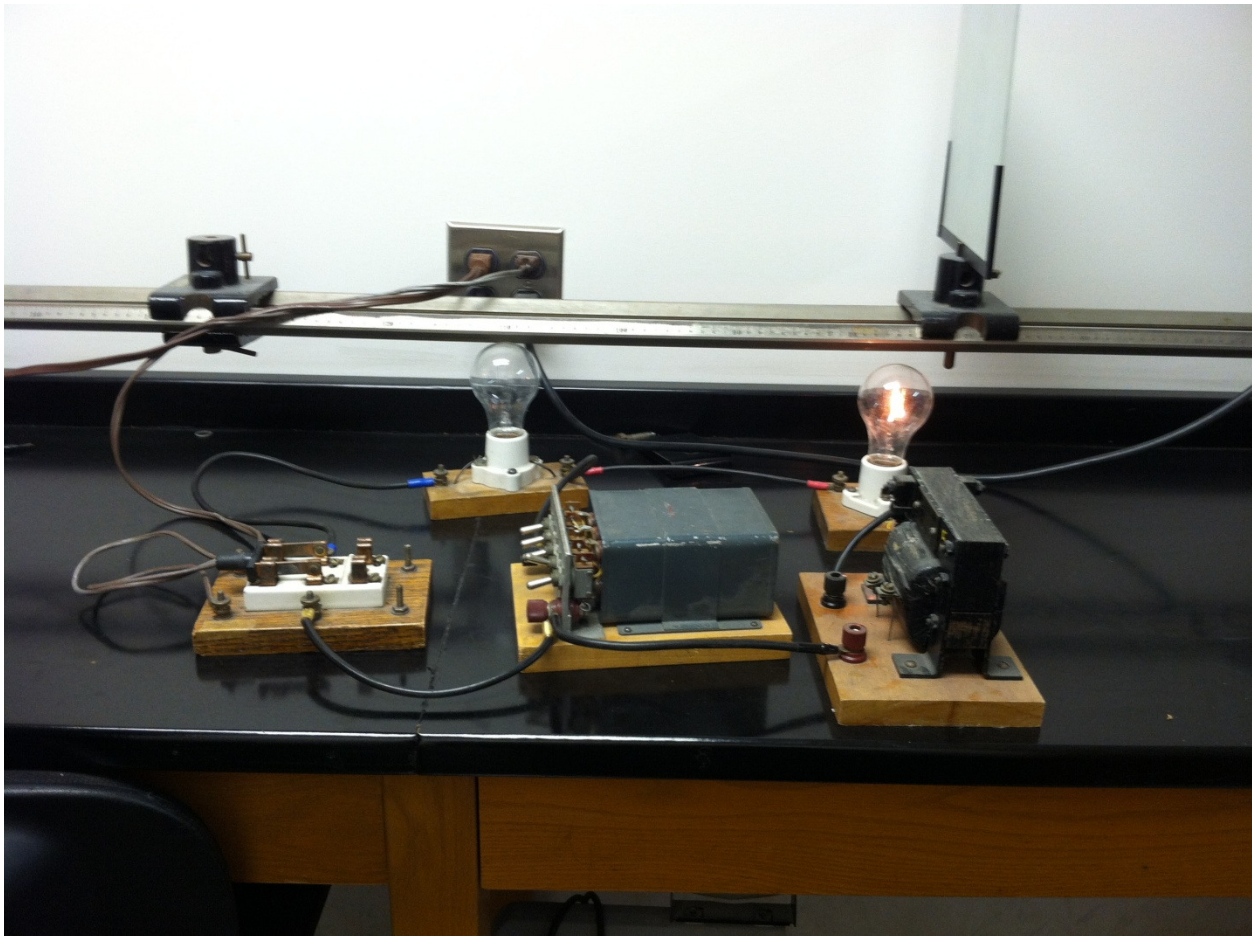

Figure 3. An unretouched photograph of the circuit used to demonstrate the power factor. The circuit consists of two 40W unfrosted incandescent lamps, a 5 μF capacitor and an inductor.

Circuit Analysis

The node-voltage circuit analysis method was used to calculate the voltage at the node labeled *v* in Fig. 1. From this value, it is straightforward to calculate all of the voltages, currents and average powers in the various circuit elements. The power factor can also be easily determined. The load impedance is the complex number Z_L. The phase angle of the impedance, ϕ, is given by,

$$\tan \emptyset = \frac{Im(Z_L)}{Re(Z_L)},$$

where *Im* is the imaginary part and *Re* is the real part of the load impedance. The power factor is the cosine of the phase angle.

Voltages and currents were measured using a true-rms reading multi-meter. The results of the current and voltage measurements are shown below in Table 1. From the measured values of voltage and current, capacitive and inductive reactances, X_C and X_L respectively, were found using $X = {}^{V}/_{I}$. These measured values were used to calculate the impedance and corresponding power factor.

Without power factor correction the lamps had resistances of 200Ω and the inductive reactance was about 500Ω. This reactance corresponds to a 1.3H inductor. With power factor correction the currents through the respective lamps change, and a concomitant change occurs in their resistance. As the lamp current goes up, so does the resistance. The resistance of lamp 1 was measured to be 100Ω while lamp 2 measured 230Ω. For reference, the capacitive reactance was about 500Ω, corresponding to a 5.4 μF capacitor, within the 10% accuracy range. The circuit analysis showed the uncorrected power factor to be 38% while the corrected power factor is greater than 99%.

	38% power factor		99% power factor	
	V (volts)	I (mA)	V (volts)	I (mA)
Vsource	122	184	122	115
Lamp 1	37	184	12	115
Lamp 2	38	184	46	200
C	0	184	112	230
L	91	184	99	200

Table 1. Measured values of the current through and voltage across the various elements in the circuit. All values are rms.

Phase angle

It is certainly difficult for students to understand what the phase angle means. Experiments to measure it usually involve complicated circuits utilizing oscilloscopes or other "black-box" equipment. It is possible to envision an experiment with two variable-phase, constant-frequency, constant-amplitude power sources, V_1 and V_2. Connect them to an incandescent lamp and vary the phase of one of the supplies. When the relative phase is zero, no current flows through the lamp, while it would glow brightly if the phase difference were 180°. This circuit for is shown below in Fig. 4.

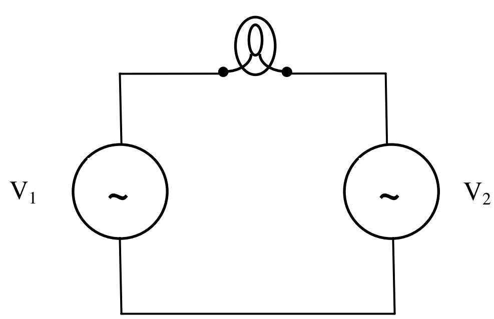

Figure 4. An incandescent lamp connected to two variable-phase, constant-frequency , constant-amplitude voltage sources.

An analog to this experiment is occurring here. If students think of the capacitor and the inductive load to be in parallel; then they can visualize that the currents in these two lines are practically anti-parallel – flowing in opposite directions. When one goes "up", the other goes "down", and vice versa. Evidence for this can be had by reasoning along the following lines. Consider the currents in the circuit of Fig. 1. Suppose at some time, the current is flowing from the source right-ward toward the top node and also through the inductive load, clockwise around the loop. The current through the capacitor must flow upwards toward the top node to increase the current through the inductive load while decreasing the current from the source. Everything inverts during the next half-cycle. This reinforces that the currents through parallel inductors and capacitors are 180° out of phase with each other, and bolsters the old adage of "ELI the ICE man". In this mnemonic, E represents the voltage, I the current, L the inductance, and C the capacitance. "ELI" refers to the voltage leading the current in an inductor and "ICE" refers to the current leading the voltage in a capacitor.

Conclusion

A simple circuit that demonstrates the power factor has been developed. By using incandescent lamps students can visualize the effect of the amount of current flowing and deduce that by inserting a parallel capacitance into a circuit with an inductive load the source current decreases while the load current increases. Concomitant changes to the delivered power are easily inferred. By using a true-rms reading multi-meter the activity can easily be incorporated into a laboratory activity.

Acknowledgements

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