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A SIMPLE EXPERIMENT IN STRUCTURAL VIBRATIONS FOR CIVIL ENGINEERING STUDENTS

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Session

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Abstract

The study of and measurement of structural vibrations has important applications in various aspects of structural engineering for example in health monitoring of structures, establishing comfort levels for building occupants, minimizing floor vibrations in labs housing sensitive equipment, earthquake engineering, and ensuring that the natural frequency of flexible bridges does not match the resonant frequency from applied loads. In the United States, in the typical undergraduate civil engineering curriculum, students do not get courses or laboratory experiences in structural vibrations. The author has devised and implemented a simple experiment that can be understood and performed by senior civil engineering undergraduate students that will introduce them to this important topic. End-of-semester assessment found that this was a popular, stimulating, and beneficial experiment for the students.

The experiment is tied to one important application in the structural health monitoring of bridges. The theoretical natural frequency of a bridge is dependent on both the stiffness and the mass. As the structural condition of the bridge deteriorates with time, its stiffness decreases while mass remains constant. Even if mass were to change, for example due to an overlay, it can easily be quantified. Therefore, it follows that if the natural frequency is measured annually, the deterioration over time can be quantified.

Background knowledge that is expected prior to the lab includes Mechanics of Materials or Structural Analysis including how to calculate deflections on a cantilever beam, and a working knowledge of MS Excel. In the lab description, students are introduced to displacement, velocity, and acceleration relationships and the equation of motion is developed for a simple mass on a spring. The first fundamental frequency of a cantilever beam with a concentrated mass at the end is provided as a reference.

Three aluminum cross-sections having approximately equal masses are used as cantilever beams. Three-axis accelerometers are mounted at the free ends of the cantilever and connected to a data acquisition system. Students provide a simple excitation to the beams and data is collected. Students are given the raw acceleration vs time data. They must then perform fast Fourier

transform analysis to determine the fundamental frequency. This experiment provides a validation of how measuring natural frequencies can indicate stiffnesses and hence be used to track deterioration over time.

Introduction

Undergraduate engineering curricula are constantly constrained by the limit on credits for the degree (with a typical range of 124 to 130), and it is not possible to cover all the important topics. One such example is structural vibrations for civil engineers. While their counterparts in mechanical engineering get lab and classroom experiences in structural vibrations, the typical undergraduate civil engineering student does not. Any education for civil engineers on this topic seems to come at the graduate level only or via continuing education after graduation.

The study of and measurement of structural vibrations has important applications in various aspects of structural engineering for example in health monitoring of structures, establishing comfort levels for building occupants, minimizing floor vibrations in labs housing sensitive equipment, earthquake engineering, and ensuring that the natural frequency of flexible bridges does not match the resonant frequency from applied loads.

The author has devised and implemented a simple experiment that can be understood and performed by senior civil engineering undergraduate students that will introduce them to this important topic. The objectives of the experiment are to acquaint students with fundamental concepts in structural vibrations; to observe the variation of natural frequency with stiffness; and to familiarize students with advanced applications in structural health monitoring.

The experiment is tied to one important application in the structural health monitoring of bridges. The theoretical natural frequency of a bridge is of the form of equation 1

$$\omega = \sqrt{\frac{k}{m}} \tag{1}$$

where k is the stiffness and m is mass. As the structural condition of the bridge deteriorates with time, its stiffness decreases while mass remains constant. Even if mass were to change, for example due to an overlay, it can easily be quantified. Therefore, it follows that if the natural frequency is measured annually, the deterioration over time can be quantified.

Background Knowledge

Prior to the lab, students should review any method from Mechanics of Materials or Structural Analysis, for calculating beam deflections, particularly in cantilever beams. Examples of methods include: double integration, virtual work, conjugate beam, moment-area theorems etc. Since Structural Dynamics is usually not offered as a course in a typical undergraduate civil engineering curriculum, some basic background information must be presented and introduced to the students as follows:

Any object that is set into oscillatory motion by a short duration excitation force will vibrate at its own natural frequency. A good starting point for studying vibrations, would be a mass resting on a spring which is hit by a hammer. The displacement *Y(t)* would exhibit a sinusoidal variation. From basic physics, the velocity *V(t)*, and acceleration *A(t)* can also be obtained[1]. These are shown in Fig. 1.

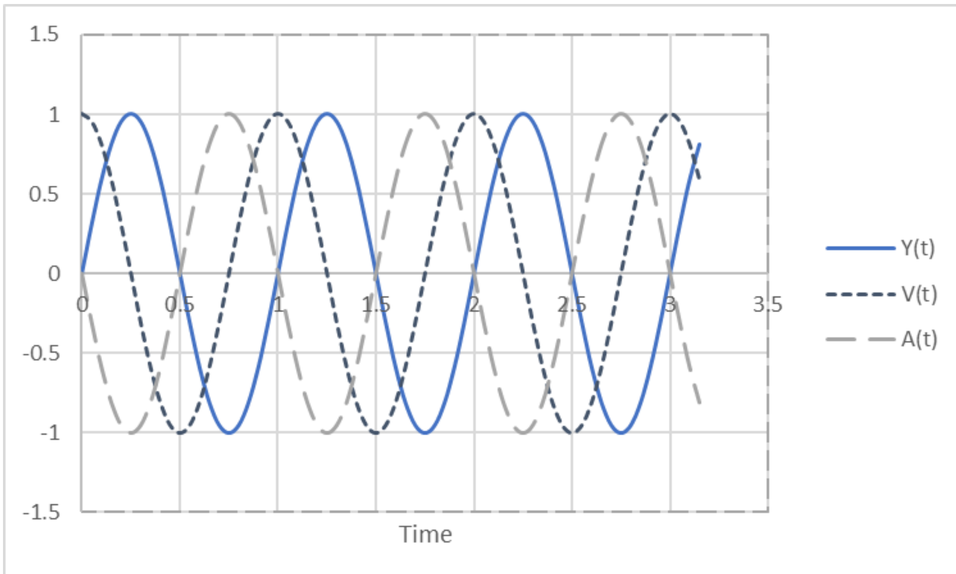

Fig. 1. Displacement, Velocity, and Acceleration graphs versus Time for a simple mass on a spring which is struck by a hammer.

Notice that the velocity is 90° out of phase with the displacement, and the acceleration is 180° out of phase with the displacement. For experimental purposes, acceleration is often the easiest of the three parameters to measure using an *accelerometer*.

The displacement can be written as

$$Y(t) = Y_{\max} \sin(\omega t) \tag{2}$$

The acceleration is the second derivative of displacement with respect to time and is given by

$$A(t) = \ddot{Y} = -\omega^2 Y_{\max} \sin(\omega t) \tag{3}$$

Consider the idealized model diagram of a mass resting on a spring (Fig. 2). Assuming that the mass is traveling in the upward direction at some point in time, the free body diagram shows the forces acting on the mass. It will be subjected to an inertial force, F_i (opposite to the direction of motion) which is equal to the mass times acceleration, and a reaction force from the spring, F_s which is equal to the spring stiffness times the displacement. Thus, if the mass is traveling upwards, both the inertial force and spring reaction will be downward.

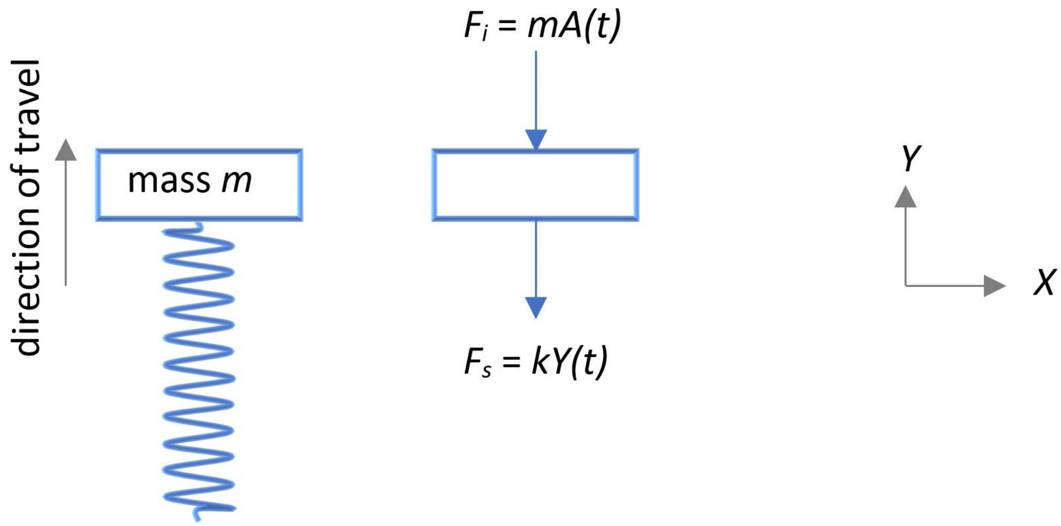

Fig. 2. Idealized model and free body diagram of the mass at some point in time.

From force equilibrium in the *Y* direction, considering the free body diagram above

$$kY(t) + mA(t) = 0 \tag{4}$$

Substituting equations (2) and (3) into (4)

$$kY_{\max} \sin(\omega t) - \omega^2 m Y_{\max} \sin(\omega t) = 0 \tag{5}$$

The nontrivial solution of equation (5) requires that

$$\left(k - \omega^2 m\right) = 0 \text{ and hence } \omega = \sqrt{\frac{k}{m}}$$

It should be noted that even when the motion of a vibrating structure is complex, characterized by irregular movements, that motion can still be understood as a superposition of many different sine waves, each having different amplitudes and frequencies. Also, in any real-world system, there is an energy-loss mechanism (e.g. friction, drag, sound etc.) referred to as *damping* which will cause the amplitude to decay and the vibrations to eventually die out.

Now consider the case of a cantilever beam. The mass is continuously distributed across the length of the beam and not lumped at a point. This increases the complexity because the problem must now be formulated in terms of partial differential equations (since the displacement is a function of both time and position) rather than ordinary differential equations as discussed previously. This requires more advanced mathematical techniques. Formulas for the natural frequencies of beams can be obtained from references[2,3]. The first natural frequency (in Hertz) of a cantilever beam with an additional concentrated mass at the free end can be taken as

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{3EI}{\left(0.2235\rho L + m\right)L^3}} \quad (6)$$

where ρ is the mass per unit length of the beam in kg/m, m is the concentrated mass at the end of the cantilever, E is the modulus of elasticity, I is the moment of inertia, and L is the length of the cantilever.

Most single-valued functions may be written as the summation of a series of simple sine and cosine waves within a desired range. This series is called the Fourier Series. The Fourier Transform is an extension of the Fourier Series where the period of the function may extend to infinity. The Discrete Fourier Transform (DFT) helps us analyze data (say consisting of N data points with a sampling time of Δt) in the frequency domain.

$$X\left(f_n\right) = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{-i2\pi n\Delta t} \quad (7)$$

The DFT may be evaluated at discrete frequencies $f_n = \frac{n}{N\Delta t}$

This allows us to evaluate the dominant frequencies in a waveform. The Fast Fourier Transform (FFT) is a more computationally efficient way of evaluating the DFT and there are several different algorithms for FFT that are in use. Most of the FFT algorithms require the number of data points to be a power of 2. FFT analysis can easily be performed with the aid of Microsoft Excel for example. Instructions on performing FFT in Excel can be found in various references for example Klingenberg[4]. The maximum limit of data points for FFT in Excel is 4096.

Procedures

Obtain three cantilever beams of approximately the same length. The cantilever beams should be made of the same material. The author chose three cross-sections that have approximately the same area, but different moments of inertia. For example, three cantilever beams made of Aluminum 6061 were chosen with cross-sections and orientations as shown in Fig. 3.

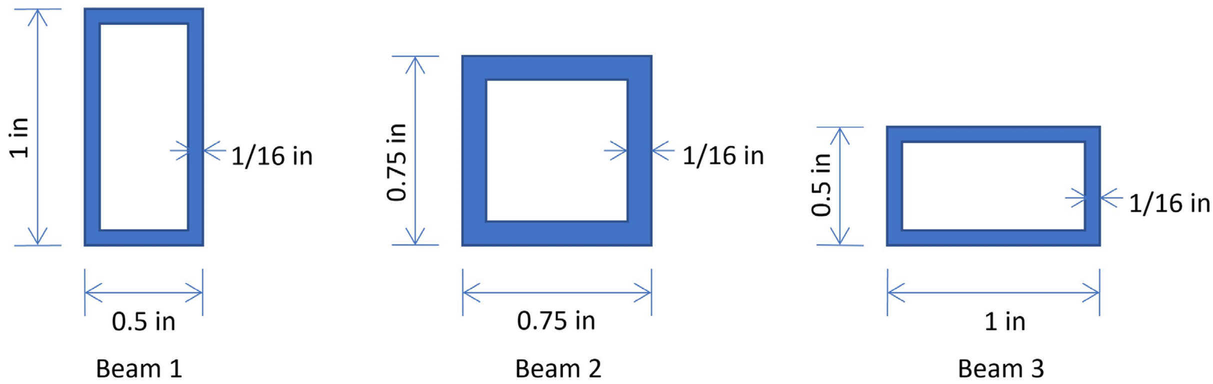

Fig. 3. Cross-sections of cantilever beams

A hole should be drilled at the free end of the cantilever. Place a bolt through the hole, so that the accelerometer can be mounted on the beam. A three-axis accelerometer was used. Note that a research or high quality accelerometer is not necessary. An educational grade accelerometer should be sufficient, for example the sensors produced by PASCO. It is highly recommended to collect and graph the data in real time for troubleshooting and also as a visual reference for students when performing the experiment. For example, the PASCO Capstone software could be used to collect and plot the data. In order to minimize the setup time and reduce errors, it is recommended that the cantilever beams should not be moved once they are set up. Three accelerometers should be used (one for each beam). The length of the cantilever, and the mass of the cantilever beam, accelerometer and mounting bolt should be provided to the students.

Three cantilever beams will be set up in the lab. The lengths of the cantilevers are all approximately the same. They are all made of the same material (Aluminum 6061), and their cross-sectional areas are approximately the same. Therefore, the overall mass of the beam is approximately the same for all the beams.

The stiffness k of the beam is given by

$$k = \frac{3EI}{L^3} \tag{8}$$

Where E is the modulus of elasticity, I is the moment of inertia, and L is the length of the cantilever. The modulus of elasticity and the length will be approximately the same for each beam. Only the moment of inertia will be different. An accelerometer will be attached to the free end of the beam and monitored via the PASCO Capstone Software. Only the vertical acceleration (along the x axis of the accelerometer) will be evaluated in this lab.

1. Begin recording data in Capstone, and strike the end of the cantilever beam. A finger tap should be sufficient. Then stop recording the data. Note that we will be collecting data at 500 samples per second.

2. Write down the approximate first natural frequency for each beam read from the Capstone FFT chart.
3. Perform FFT analysis with the raw *x*-axis acceleration data in Excel. Note that the number of data points must be a power of two and the limit is 4096, so you may have to truncate the data set.
4. Compare the measured natural frequency with the theoretical one calculated using equation (6).
5. Make a plot of natural frequency versus stiffness.
6. Comment on the use of this technique in the real world to monitor deterioration over time.

Data that you will need for the lab: (please convert lengths to *m* where applicable)

Table 1. Property Data Provided to the Students

Parameter	Beam 1	Beam 2	Beam 3
Width (in)	0.5	0.75	1
Depth (in)	1	0.75	0.5
Thickness (in)	1/16	1/16	1/16
ρ (kg/m)	0.29837	0.28364	0.29765
End mass *m* (accelerometer plus pin) (kg)	0.06539	0.05921	0.05818
Length of cantilever (m)	0.8763	0.8763	0.8763

Results

Although students observe the FFT graph in real time, they are given the raw acceleration data and are still asked to perform the FFT analysis on their own and obtain the first natural frequency. An example of the results from the FFT analysis from one of the beams is shown in Fig. 4. The first natural frequency clearly stands out.

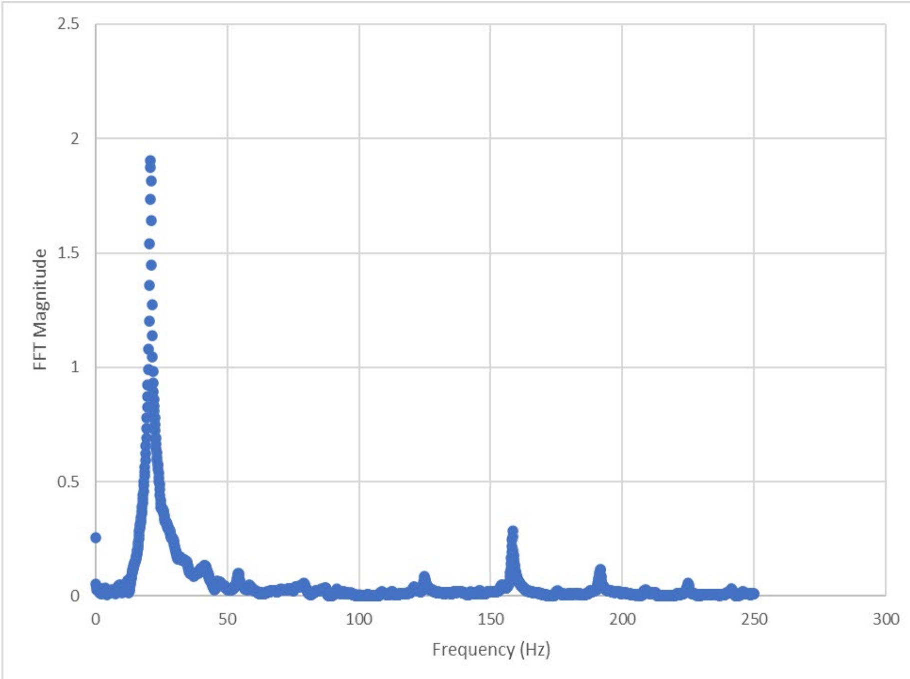

Fig. 4. Example of results from FFT analysis

Students are asked to make a plot of first natural frequency versus the beam stiffness. An example of such a plot is shown in Fig. 5. It can be clearly seen that as stiffness decreases the natural frequency also decreases. Similarly in a bridge situation, as a bridge's structural condition deteriorates over time, it follows that if natural frequency measurements are made on a regular periodic basis, it may be possible to detect damage and assess the condition of the bridge.

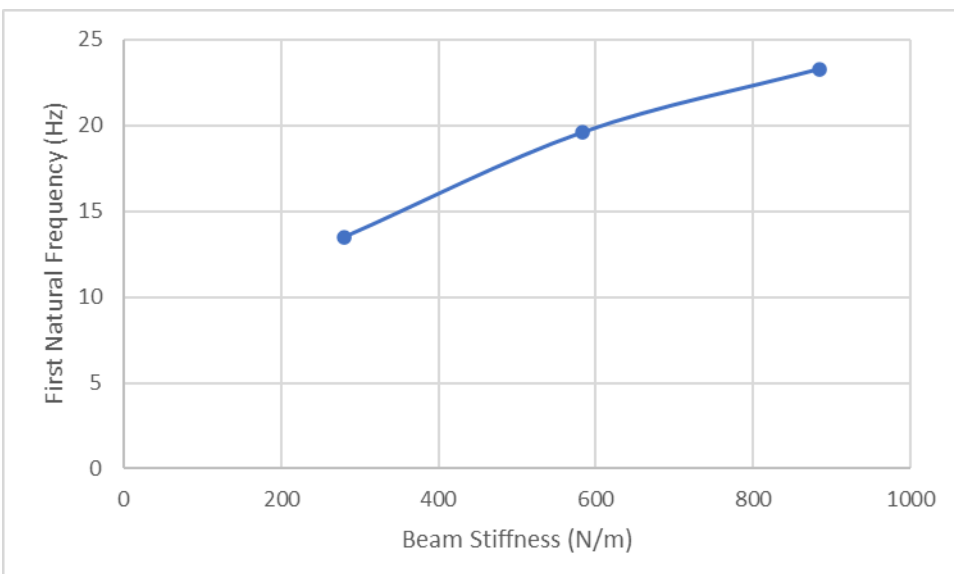

Fig. 5. First natural frequency versus beam stiffness

Assessments

There are multiple levels of assessment that were done to assess student learning. The author performed a pre-lab assessment prior to conducting the lab. The pre-lab is a simple question to test the fundamental knowledge about periodic variations, fundamental period, and frequency that a student may possibly have picked up e.g. in a Physics course, Differential Equations course, or Dynamics course for example.

Pre-Assessment

Question: An acceleration vs time graph was recorded and showed a simple sinusoidal variation as shown below. Estimate the frequency in Hz of this signal.

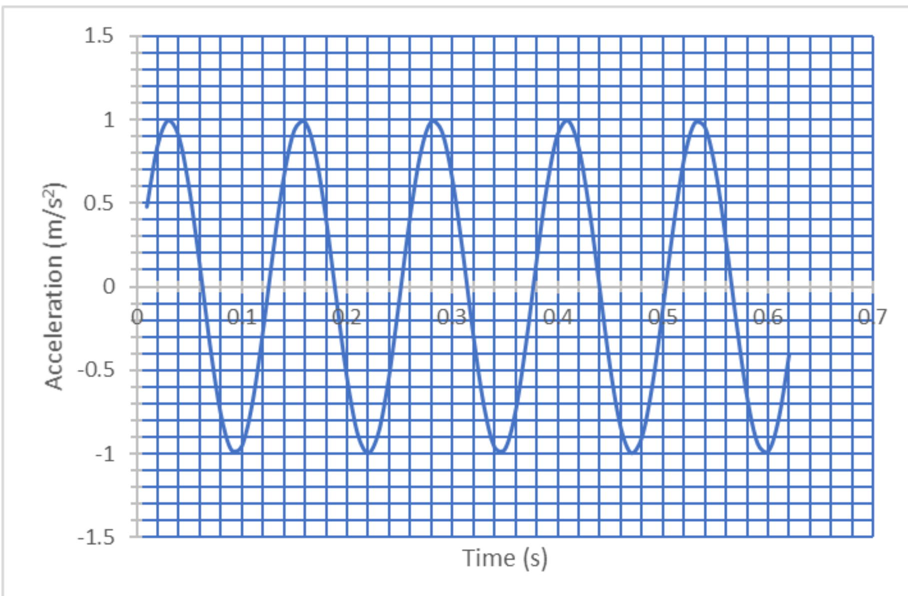

Fig. 6. Accompanying graph for Pre-Lab Assessment

Post Assessment

There is an exam given at the end of the Civil Engineering Experimentation I course. The author has used one exam question to assess the outcome that by the conclusion of the lab students will become familiar with concepts of period and frequency; can recognize that complex waveforms can be represented by a series of sine waves; and can use FFT to determine natural frequencies.

Question: A complex waveform is shown below. Estimate the lowest fundamental frequency. The raw data are also available in an Excel file on D2L (a Learning Management System)

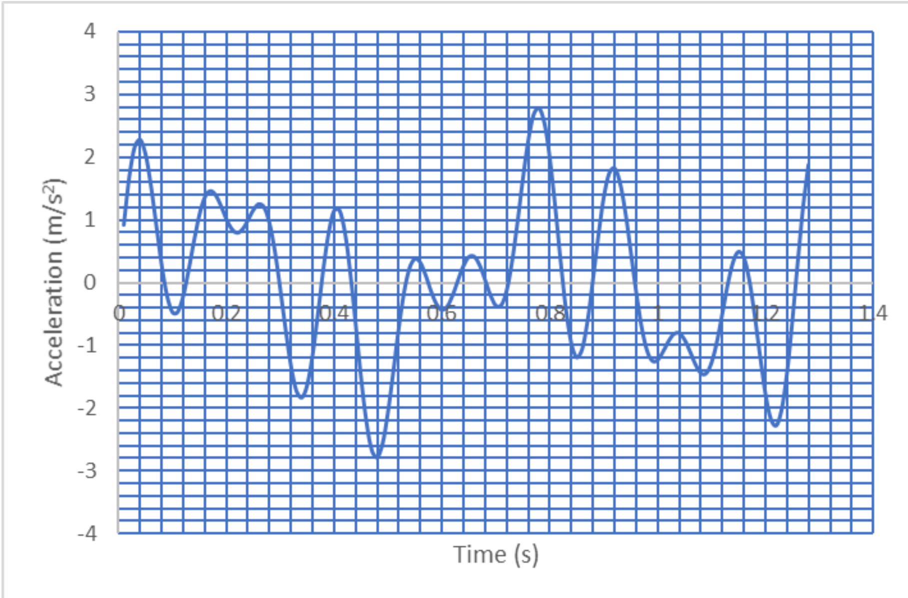

Fig. 7. Accompanying graph for post assessment exam question

A rubric was used to help grade the students' performance on the assessment questions as shown in Table 2.

Table 2. Rubric for Grading Pre and Post Assessment Questions

Assessment	4=Good	3=Marginal	2=Adequate	1=Unacceptable
Pre-Assessment	Shows good knowledge of the topic. Has detailed calculations shown. Units correct	Shows adequate knowledge of the topic. Recognizes f=1/T but may have error in reading off the graph or in calculations	Shows little knowledge of the topic. Tries to estimate peak-to-peak period	Shows no knowledge of the topic Leaves question blank
Post-Assessment	Shows good knowledge of the topic. Correctly performs the FFT. Is able to read off the lowest fundamental frequency.	Shows adequate knowledge of the topic. Recognizes that complex waveforms can be decomposed into sine waves. Attempts to run FFT	Shows little knowledge of the topic. Tries to estimate periods off the original graph.	Shows no knowledge of the topic Leaves question blank

The results from three years of data are shown in Table 3. As can be seen, the mean assessment scores improved from pre to post assessment proving the success of the learning outcomes. A statistical analysis of hypothesis testing was performed. The null hypothesis $H_o : \mu_1 - \mu_2 = D_o$ was set up as $H_o : \mu_{post} - \mu_{pre} = 0$

That is the null hypothesis was that there is no difference between the population means. The *z*-test statistic is

$$\frac{\mu_1 - \mu_2 - D_o}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{14}$$

This results in test statistics of 1.17, 2.10, and 3.63. Since this exceeds $z_{0.025}$ which is 1.96, for both 2015-16 and 2016-17, we can reject the null hypothesis at the 5% significance level. That is, the evidence suggests that the difference between the means is real, and the students' performance is better in post assessment than pre assessment. For the 2017-18 year, the test statistic is below 1.96 and therefore the null hypothesis cannot be rejected.

Table 3. Results from Pre and Post Assessments

	Academic Year:	2017-18	2016-17	2015-16
	Total No. of Students	17	17	36
Pre-Assessment	Good	2	0	0
	Marginal	11	9	12
	Adequate	2	3	18
	Unacceptable	2	5	6
	Variance	11.1	13.1	17.0
	Standard Deviation	0.81	0.88	0.69
	Average Score	**2.8**	**2.2**	**2.2**
Post Assessment	Good	5	4	2
	Marginal	9	6	19
	Adequate	3	7	13
	Unacceptable	0	0	2
	Variance	7.8	10.5	18.4
	Standard Deviation	0.68	0.79	0.72
	Average Score	**3.1**	**2.8**	**2.8**
Ho: μ_{post}-μ_{pre}=0 test statistic		1.17	2.10	3.63
$Z_{0.025}$ (5% SL)		1.96	1.96	1.96
Result		Do not Reject Null Hypothesis	Reject Null Hypothesis	Reject Null Hypothesis

Survey:

An additional form of assessment was done in a survey to the students. This new lab was replacing a previous lab on metal strength testing which was moving to the Civil Engineering Materials course. For this reason, the author wished to gain some feedback from the students. The survey questions were:

On a scale of 1 to 4 with 4 being the highest, please rate

1. The level of challenge presented by the lab
   4 Very Challenging 3 Moderately Challenging 2 Mildly Challenging 1 Too easy
2. How valuable do you feel the lab is to prepare you for real-world applications in civil engineering
   4 Very valuable 3 Moderately valuable 2 Slightly valuable 1 Unrelated to civil engineering
3. How interesting the lab felt to you
   4 Very interesting 3 Moderately interesting 2 Slightly interesting 1 Boring
4. Please provide any comments

The average results from three years' worth of data were Level of Challenge 3.2; Valuable 3.5; and Interesting 3.4. This seems to indicate a good level of learning challenge and perceived benefit of the lab. A few examples of the comments were:

"This lab was sort of techie compared to the others. I enjoyed the lab and learned a lot from it."

"I felt that this lab was useful to prepare us for the real-world practice."

"I liked being able to see and feel the vibrations. I could relate to walking across a rope bridge."

The lab report associated with this lab was also assessed as part of ABET Outcome 6: an ability to develop and conduct appropriate experimentation, analyze and interpret data, and use engineering judgment to draw conclusions. The rubric used to assess the lab report is shown in Table 4.

Table 4. Rubric to assess lab report against ABET Outcome 6

Performance Indicator	5=Exemplary	4=Good	3=Adequate	2=Marginal	1=Unacceptable
Conduct experiment	Highly efficient use of tool. Able to produce accurate results.	Efficient use of tool. Able to produce accurate results.	Able to use tool to produce appropriate results.	Minimal knowledge of how to use tool and/or inaccurate results.	Unable to use tool; inaccurate results.
Analyze results	Experimental results are explained in	Experimental results are explained in	Experimental results are explained in	Experimental results are explained in	Experimental results are not explained in

	terms of appropriate models. Analysis detailed and correct.	terms of appropriate models. Analysis is correct.	terms of appropriate models with only minor errors.	terms of appropriate models but with major errors.	terms of appropriate models and major errors exist.
Evaluate significance	There is complete and detailed treatment of variability, error, significance, and agreement with hypothesis.	There is substantial treatment of variability, error, significance, and agreement with hypothesis.	There is treatment of variability, error, significance, and agreement with hypothesis.	There is partial treatment of variability, error, significance, and agreement with hypothesis.	There is no treatment of variability, error, significance, and agreement with hypothesis.
Present results	Presentation of results is detailed, well organized and clear, and reflects significant thought and/or interpretation of results.	Presentation of results is detailed, and clear, and reflects appropriate thought and/or interpretation of results.	Presentation is clear and reflects some thought and/or interpretation of results.	Presentation has errors, is not detailed, well organized, or clear, and has little reflection or interpretation of results.	Presentation has significant errors, is not detailed, well organized, or clear, and has no reflection or interpretation of results.

The expected level of outcome is 3 which is at the level of application. The average results from three years of data were 2.7, 2.6, 2.7. The lab report is written by a group of three to four students typically. While most groups are at the 3 level, a few are at the 2 level, which drives the average score down a bit. A presentation is given at the beginning of the semester on the expectations of a good lab report as well as examples of past exemplary work. Often this comes down to group dynamics and not everyone on the group performing to their peak level. This is often manifested in a rushed submission to meet the deadline.

Conclusions

There is a need for undergraduate civil engineering students to have some exposure to the field of structural vibrations since it has many important applications in civil engineering. In this paper, the author has presented a simple experiment that could be performed by senior civil engineering students. During the experiment, they learn important concepts and appreciate the role that could be played by structural vibrations in one application, that is monitoring the deterioration of structural condition over time. The lab is also relatively inexpensive with current one-time investment costs estimated to be around $350. End-of-semester assessment found that this was a popular, stimulating, and beneficial experiment for the students.

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Biographical Information

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