

A shaft design aid for integrating basic elements of introductory machine design.

Ernest W. Tollner and H.Y. Shadow Huang  
Biological and Agricultural Engineering Department  
Driftmier Engineering Center  
University of Georgia  
Athens, GA 30602

### Abstract

Many of the concepts one learns in a two course sequence in machinery design culminates in the design of power transmission shafts and supporting elements ranging from various drives, supporting bearings and other supports. By the time one reaches the end of the sequence, there is little time to develop a program that can enable the average student to achieve the benefit of the integration advantage. A Microsoft Excel-based spreadsheet platform has been developed to this end. Using flexure, shear and superposition, the model can accommodate design of a simply supported circular solid or hollow shaft with simultaneous overhung normal and lateral thrust loads on each end and two loads interior to the supporting bearings. Loads may originate from the x-y and x-z planes, with a provision for shaft tension-compression forces being included. Dynamic loads are not specifically addressed. The model is organized into respective pages in which x-y, y-z and resultant forces are computed. The model also has a page devoted to showing various shear and bending moment diagrams representing the various loading planes and resultants. Stress concentration factors and associated calculation aids are included on a page. Development assumptions and comments are organized into a page. Finally, there is an input-output page walking students through the process of developing inputs and organizing key outputs for use. During the period of instruction on shaft design, students are provided with the model and do some hand calculations enabling checking the model performance as particular topics are introduced. As a project or exam, students can then use the tool to do realistic power transmission problems where they design a variety of drives, the bearings and the shaft.

## Introduction

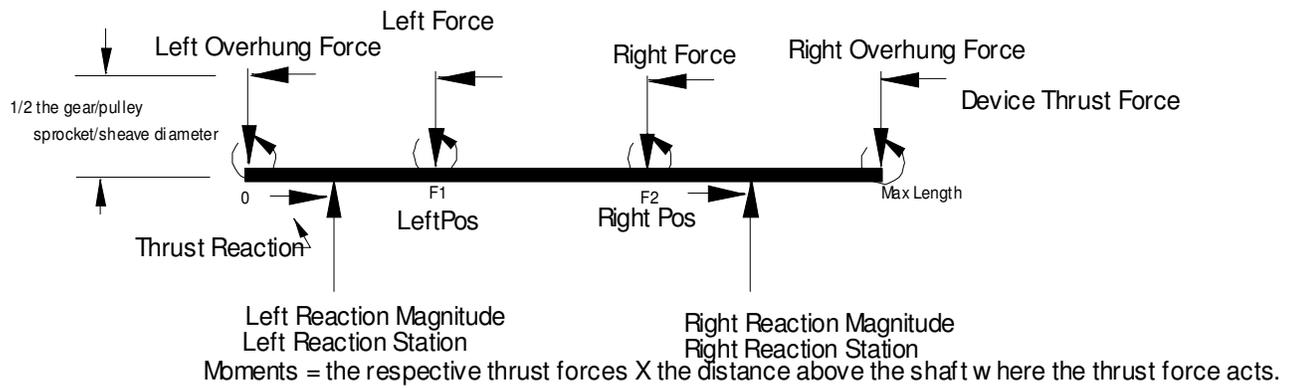
Power transmission shaft design is a recognized culmination in introductory machine design courses. Shigley and Mische (2001) places this topic at the end of their text. Norton (2002) places the topic in the middle of their text (beginning of the second course in a two-course sequence) where topics can then be added as they are introduced. Spotts (2004) provides a solid treatment of shaft design fairly early in his text. He also has an Excel Spreadsheet model that is available for student use. Many of the concepts one learns in a two course sequence in machinery design culminates in the design of power transmission shafts and supporting elements ranging from various drives, supporting bearings and other supports. The shaft as an integrator of many different aspects of machine design from bearings, various drives and supports can then be advantageously used.

Developments in shaft design continue to appear in the literature. A search of recent data bases brought forth several contemporary developments. For example, high speed dynamic loading is addressed by Jorgensen and Shin (1995); specialty shafts for specific applications are discussed by Jeftenic et al. (2002) and Figueiredo et al. (1999). Non circular shaft design with finite elements using the Timoshenko element is being explored (Karch, 1993; Smaili and Tylicki, 1993). That shaft design has proceeded from the introductory level is well established. The focus of this tool is not so much production design but as a pedagogical tool for introductory purposes.

## Model description

The shaft design considers the shaft with loadings of the type shown in Figure 1. In this sketch,  $x$  is collinear with the shaft,  $y$  is vertical and  $z$  is into the plane of the sketch. Loadings in the  $x$ - $y$  and  $x$ - $z$  planes are possible, along with thrust in the  $x$ -direction (all the shaft axis). Normal loadings in both planes arise from the various gear, belt or chain drives. Thrust forces may arise from misaligned belts or helical gear drives, or other shaft-powered applications.

The Microsoft excel spreadsheet is arranged by functional pages. Each page will be described beginning with the calculation pages. The fundamental input-output page will be described in the summary.



Sign Conventions:

Forces and reactions positive as shown

Moments Positive counter clockwise as shown

Deflections are positive upward and negative downward

Programmable Thrust (not shown) is positive in Tension

### General Forces Case -- Two Reactions

Figure 1. Schematic sketch of the modeled shaft and loadings in the x-y plane. Similar loadings are allowed in the y-z plane.

#### Torque and Force calculator page

Most shaft design problems do not start with known normal and shear forces. They must be computed from various power requirements. With a known power, specified angular velocity and pitch diameter, the model contains modules for computing forces and torques associated with spur, helical, bevel and worm gears. The model can also calculate forces and torques with standard chain, v-belt and flat belt drives. Students can easily build this page as the material is covered. These parameters may be manually ported to the main input page.

#### X-Y, X-Z and resultant forces computations pages

These pages contain the heart of the model, organized along the lines of columns described in Table 1. The shaft is modeled in units of 10 stations. These serve as reference indicators for the supports and the forces and are used in logic guiding the integration of the deflection equations. One should locate a station where there is a support, force or stress concentration. Distances between stations is given in column 2. The interval is variable. The capability of the spreadsheet to link to various pages is exploited wherever possible.

Table 1. Layout of columns on the x-y, x-z and resultant forces pages.

1	Station
2	Distance along the beam. Distance interval between stations may be variable. One should use small intervals at reactions and forces, with larger intervals between spaces. The beam may be any length not compromising linearity, superposition and location in the elastic region.
3	Bending moment -- calculated
4	Outside shaft diameter -- needed
5	Moment of Inertia -- calculated
6	M/EI - calculated
7	First integral
8	Second integral
9	Deflection -- calculated
10	Slope
11	Notch factor - use calculator above; use charts for von Mises stresses.
12	Same as above, for shear
13	Alternating torque, usually very small or zero. Input in column F on the input page. Numbers here are from corrected alternating loads due to shear. Equivalent alt. Torque due to shear forces is calculated for each plane and added to the torsion. The maximum is used in the resultant sheet.
14	Mean torque, user input; occurs between the input and output points, zero otherwise.
15	Alternating moment, computed from bending moments
16	Mean moment - computed to the right of workspace based on converting thrust forces into an equivalent moment based on $F/A = Mc/I$ . A moment curve from a centrifugal force can also be computed and placed in the moment column. For most shaft designs, this can be ignored and thus set to zero.
17	A factor in the DE-Gerber equation
18	B factor in the DE-Gerber equation
19	r: if r greater than rcrit above, then the shaft may fail by fatigue; if r is less than rcrit the shaft fails in yield.
20	Computed diameter to avoid failure
21	Computed diameter to avoid failure with a hollow shaft. The hollowness factor k must be compatible with the chosen outside diameters and inside diameter. $K=0$ and $D_{inside}=0$ for most designs. Note: Column 22 is to the right beyond the work space around column AT
22	Thrust force is the side thrust force due usually to a helical gear. This force must be resisted by an opposing gear and/or a thrust bearing at one of the reactions.
23	Moment arising from centrifugal force. Do a separate moment calculation with the force located on the beam on the x-y plane to compute the moment. Copy this moment to column 23. Then go ahead and represent the other forces and moments. One can generally ignore this by setting to zero.
24	Equivalent moment arising from thrust due to devices (conservatively distributed along the entire shaft. Thrust in Tension is treated as positive. Extra thrust may be programmed. Distribute along the shaft depending on which reaction resists the thrust.
25	Equivalent total moment, bringing in the potential imported moments in #35. If imported moments are used, all forces must be zero.
26	XZ bending shear force
27	Input alternating uncorrected shear stress computed from input torque
28	XZ equivalent shear stress based on XZ shear loading
29	XY equivalent shear force based on XY loading
30	Equivalent shear stress based on XZ loading
31	Sum of actual and equivalent shear stresses
32	Conversion of equivalent shear stress sum to an equivalent alternating torque
33	The original input alternating torque shown for comparison purposes
34	Percent difference to show the impact of shear compared to the mean torque. Shown on the mean torque since alternating torque may be very small in many applications.
35	Imported Moments if using a non simple reaction. Zero if simple reaction; All forces on input page should be zero if the moments are imported.
36	Imported Deflections
37	Imported Slopes.

The information on this page is referenced from the input page. Cells on this page is locked to keep from inadvertently changing the contents. The password freely available to the students. The remainder of Table 1 steps through the computations to do the double integration of the deflection equation. The approach suggested by Shigley and Mische (2002) was implemented for deflection magnitude and slope calculation.

An attempt was made to accommodate effects of thrust by computing an equivalent bending moment to achieve the desired level of thrust stress. Equivalent shear forces from loads in the other plane were considered. There is space to input shear and bending moment data for shafts that are supported by more than two supports. These values must be computed externally. The D-E Gerber equation (Shigley and Mische, 2002) was used to compute the diameter required at each station. The cylindrical shaft was simulated by adjusting the D-E Gerber equation parameters as shown by Shigley and Mische (2002).

The x-z page is identical to the x-y page in terms of computational details. The resultant page looks similar to the other pages but does some appropriate Pythagorean sums to get at the resultant forces and moments. Students can easily implement strength and deflection calculations for at least one plane as the course progresses.

#### Notch factors

A summary of notch factor charts are included on a separate page. This data, from Norton (2001) covers all the standard keyway and drilled sources of stress concentration. A calculation aid for computing the appropriate endurance stress, stress concentration and notch fatigue factors is included based on Shigley and Mische (2002). The notch factors must be manually entered at the appropriate station. This may be quickly implemented by students as it is covered.

#### Charts

A series of bending moment and shear force relations are included on a separate page for each plane and resultant forces. Shear and bending forces were calculated for each type of loading and superimposed. Basic relations in Shigley and Mische (2002) were used to compute these forces as functions of positions along the shaft. This can be computed by students and provides a tool that can be of interest to students in other courses in solid mechanics as well.

#### Development notes

A page summarizing key assumptions, information in Table 1 and other usage notes is provided. Students are encouraged to further customize this page for their use.

## Model summary

The input-output page of the model provides a convenient framework for presenting a summary of the shaft design model. Figure 1 is reproduced at the top of the page. The numbered items are shown on this page, with space provided for the input and outputs. A color coding scheme was developed to enable easy identification of input fields and output fields. The items are summarized in Table 2, which also contains comments that expand on each item. Example outputs are given for a 10 inch shaft loaded on both overhangs (6 inch pulleys) with loading of 1200 lbs (x-y) and 100 lbs(x-z) normal on each shaft end. Thrust loadings of 50 and -50 (x-y) and 80 and -80 (x-z) were included on the respective pulleys.

Some outputs are shown in Figures 2, 3 and 4. Deflection is zero at the supports and maximum in the middle as one would expect. The resultant calculation effectively removes the negative deflection from the support to the overhang. The beam slope is zero in the middle as one would expect for a two overhang shaft. The slopes are of key interest at the support points (bearing design considerations) and at the overhangs (gear, belt alignment, chain alignment). The diameter plot shows the manually input shaft diameter used for deflection and slope considerations. One should adjust the programmed shaft diameter to satisfy these criteria at the outset. One must then ensure that the shaft diameter computed from loadings does not exceed the programmed diameter.

Particularly if weight is a problem, one may hollow the shaft. The hollow cylindrical shaft may be compared with the solid cylindrical shaft. The model shows that one may eliminate a large fraction of the interior cross section with little effect on shaft strength. If the shaft is not hollow, this curve will be superimposed on the solid cylinder shaft. One should not overlook adjusting support locations in order to make an economical design.

Table 2. Primary input-output items.

Item number	Major categories of the input-output page	Comments
1	Compute loads for the x-y and x-z planes.	This is on the torque-force calculator page.
2	Input the Material properties and Shaft inside cylinder diameter, hollowness Kcyl factor, design factor.	This requires geometric knowledge and materials properties.
3	Input Forces and Force Positions -- XY and XZ Planes; Read net thrust.	Manually bring forces from the forces-torque calculator page or other source
4	Input reaction positions (stations) and read reactions -- Put stations at the reaction points.	Define the stations where shaft is supported. Stations should also correspond where diameter changes and notch features occur.
5	Program the shaft outside diameter, notch factors, torques and thrust/centrifugal forces as a function of length (Left end is zero as shown above)	The key features of the shaft are defined along the length here.
6	Check Beam Length and Computed DE-Gerber Strength Properties	This enables a check of some key input parameters
7	View the charts to ensure 1) the programmed shaft diameter above satisfies deflection slope criteria; and 2) strength criteria.	Some example charts are shown in Figures 2 through 4. The particular examples shown are for loading at the overhang points defined on the shaft endpoints.
8	When shaft diameter is adequate, read reactions above and read slope at reactions to proceed with bearing selection.	One inputs a diameter for the deflection calculation in step 5. This input diameter must result in a slope suitable for the application (guide lines are given on the page at this point which come from Shigley and Mische (2001). One then compares the computed diameter based on moments and forces loads with the input diameter, which may vary with position along the shaft in step 5. The shaft must be as large as the largest diameter to satisfy both deflection and stress requirements.

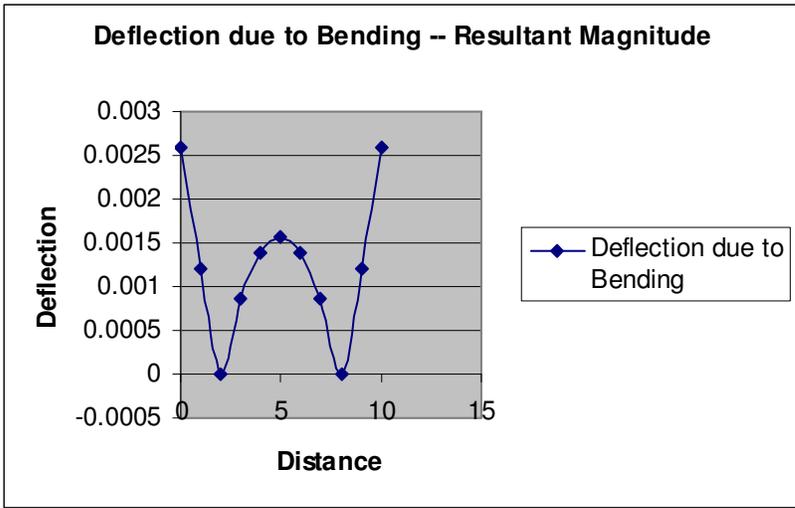


Figure 2. Typical resultant deflection vs. bending as a function of distance along a 10 inch shaft supported at the 2 inch and 8 inch points.

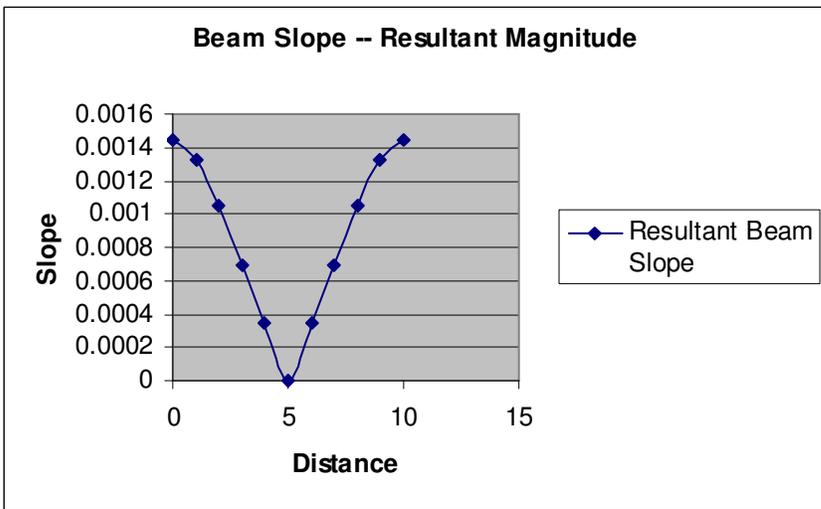


Figure 3. Typical resultant beam slope vs. distance along a 10 inch shaft supported at the 2 inch and 8 inch points.

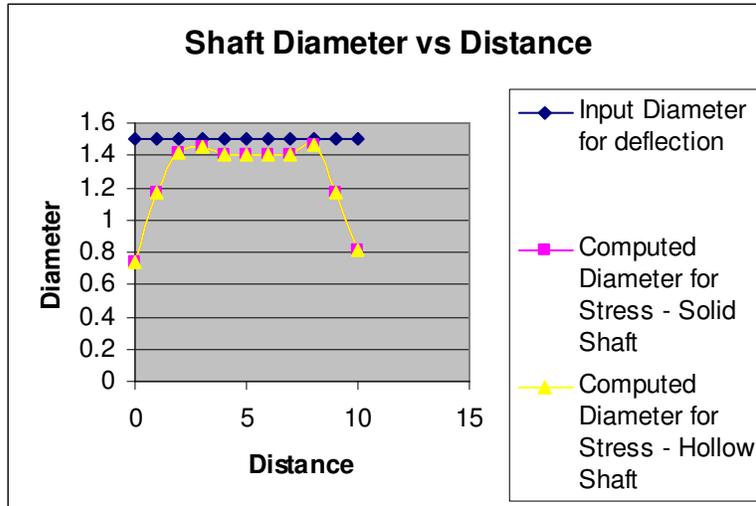


Figure 4. Computed diameter vs. length compared to the input diameter used in the deflection calculations for the 10 inch shaft supported at the 2 inch and 8 inch points. Solid shafts and cylindrical shafts overlap because the input specification was for solid shaft in this problem.

#### Spreadsheet validation

Parametric studies were conducted by using finite element simulations in order to verify the equations from the spreadsheet. An Euler-Bernoulli beam model was generated using a commercial finite element program ABAQUS CAE (ABAQUS, Inc., Pawtucket, RI), with length of 10 in and unit cross-section area. The boundary conditions of this model were varied according to the functional-input page, i.e., by changing the station locations, forces and moments on the shaft and overhung in the x-y and x-z planes. Results of parametric studies were generated in terms of deflection along the model axis, and thus, equations were verified by comparing the deflections from the simulations to the functional-output page. Agreements of deflections were observed in all the studies, indicating using flexure, shear and superposition in the spreadsheet, the simply supported shaft model performed appropriately.

#### Spreadsheet application

The following student problem was given as a take home project,  
 Design the belt drive using agricultural belts. Specify the minimum initial belt tension and the tight and loose side tensions when operating. Increase the initial tension if needed.

Design the gear drive using products from Browning

Specify gear pitch, pitch diameters, part numbers face widths and materials for both gears.

Design the shaft with a design factor of 2.5

Select material for solid cylindrical shaft (ignore the fact that some adjustment may be required to accommodate gears, bearings and sheave).

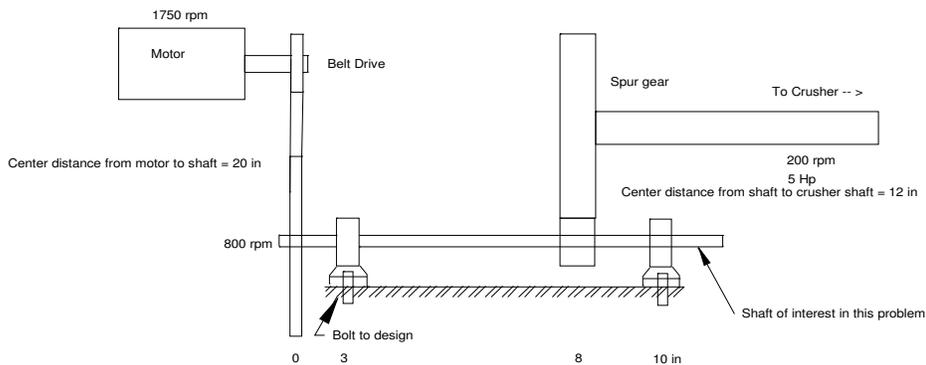
Use keyways at each Sheave/gear

Specify the diameter of the shaft as a solid diameter

Specify forces and reaction magnitudes

Design the bearings, using basic pillow blocks. Specify Browning parts.

Specify the minimum grade 5 bolt diameter to hold the indicated shaft bearing in place.



Final Project Schematic

Figure 5. Take-home design project. The class used the shaft design tool for this purpose.

## Conclusion

Students helped build and check the spreadsheet over a month's time at the end of a two-course sequence in machine design. They enjoyed the experience of seeing many components of machine design mesh together with this project.

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