# AC 2007-102: A STATISTICAL METHOD, USING LABVIEW SOFTWARE, TO DETERMINE MISSILE DEFENSE SYSTEM LOCATIONS

**Charles Bittle, University of North Texas**

CHARLES C. BITTLE has been a Lecturer at the University of North Texas since 1997. He earned his B.S.E.E. at Lamar State School of Technology in 1960 and his M.S.E.T. at the University of North Texas in 2000. Mr. Bittle served in the U.S. Federal Service for 32 years as System Engineer, Program Manager and General Manager. He is a registered Professional Engineer in Texas.

**Mitty Plummer, University of North Texas**

MITTY C. PLUMMER is an associate professor at the University of North Texas since 1992. He earned his BSEE, MENE, and PhD from Texas A&M. He worked in a variety of industrial positions for 22 years before joining UNT.

# A Statistical Method, Using LabVIEW® Software, To Determine Missile Defense System Locations

Introduction

Universities should offer an elective course covering missile defense technology. This course should cover subsystems needed for a ballistic missile defense engagement during powered, ballistic, and re-entry flights. A text book for the course should be written to include all the subsystems needed for these engagements. These subsystems are search, acquisition, track and target subsystems. In the early 1970's, the first author was evolved with designing, building and installing successful ground based missile locating and tracking systems for the Department of Defense. Funds for additional ground based missile locating and tracking systems were not allocated because a decision was made to deploy satellite missile defense systems. The 1972 Antiballistic Missile (ABM) Treaty with the Soviet Union delayed development of missile defense systems by the United States (U.S.). Now, the U.S. has a National Missile Defense (NMD) program. The most pressing concern today is the feasibility of an attack by North Korean ballistic missiles bearing nuclear or biological weapons. Hypothesizing that a North Korean missile destroys a city like San Francisco or New York in the future, missile defense will become the highest priority program for the U.S. Universities should start teaching missile defense technology now to expose engineering students to missile defense.

A searching subsystem is needed to detect the launch of one or more ballistic missiles to provide early warning. The missile's on-board tracking beacon is of primary interest to a missile defense system. A beacon tracking system is used at the launch site to track and keep the missile on the proper flight path during powered flight. During the 1970's, beacon signals were in the 2-6 Giga Hertz (GHz) frequency range. A ground based 2-6 GHz signal will normally propagate straight off the earth. However, experience has shown the signal will bend over the horizon during the missile's powered flight. The searching subsystem at the missile defense site will detect the beacon signal before the missile breaks the horizon to be acquired by radar.

A beacon tracking subsystem at the missile defense site could be already locked on to the missile's tracking beacon and providing early warning of the missile launch prior to the missile's horizon break point. The tracking radar subsystem could operate in synchronism with the bacon tracking system and take over tracking the missile at the horizon break point. Also, the targeting system could be calculating the antimissile intercept point before the missile breaks the horizon. In this scenario, the missile could be destroyed in powered flight or just when it enters ballistic flight. A search window around the point where the missile is predicted to break the horizon will allow the beacon tracking system to locate the beacon signal.

A suitable location (fixed land site or aboard ship site) for the missile defense is required to accomplish the scenario described above. This paper presents a statistical method and a LabVIEW® modeling software program for choosing missile defense system locations to be included in the missile defense course. This statistical method and a LabVIEW® modeling software program would be installed in the search subsystem described above.

A missile defense system location is referred to as the observation site in this paper. The statistical method predicts the minimum energy trajectory of a ballistic missile when launch, impact and observation sites are defined. The portion of the missile's trajectory seen by the observation site is calculated and plotted by the modeling software. The reference frame of this paper is earth-fixed. Missile trajectory end points are converted to geometric parameters using a terrestrial sphere. These parameters are used to step through a mission to determine where the missile breaks the horizon (HBRK); the missile's maximum elevation (MXEL); where the missile disappears over the horizon (LOS); and time that the observation site has to target the missile. Also, data useful for a missile defense system to destroy the missile during the re-entry phase is provided when observation and impact sites are at the same location. Azimuth and elevation calculations values are used to plot the trajectory of the missile as seen from the observation site. These plots provide azimuth and elevation values at any point on the missile's trajectory. Several plots of missile trajectories from a launch site to several impact sites can be made to determine a search window for all missiles from this launch site. These statistics determine if the observation site is a good location for a missile defense system to cover a particular launch site.

Before statistical aspects of the prediction method for choosing a missile defense location are presented, a dynamical description of a trajectory will be given. The trajectory is assumed to be a point particle (missile) whose motion is governed by Newton's force law. The force on the missile will be taken to result solely from the presence of an spherically homogenous Earth, in a vacuum, which is fixed in an inertial frame. When the missile is launched from and returns to the surface of the Earth, its intervening motion will describe an elliptical path. The missile's trajectory will be specified by six parameters of the ellipse. Many geometric parameter calculations are required to determine these six parameters. Kepler's equation relating the eccentric anomaly and true anomaly will be used to determine the time at or after launch of the missile. An iterative solution is developed to calculate each time and eccentric anomaly for each new azimuth and elevation on the trajectory. The trajectory of the missile as seen by the observation site is plotted.

Trajectory
From a military point of view, a ballistic missile has the sole objective of carrying an explosive warhead from the launch point to the impact point or target. Both points are on the trajectory plane and surface of the Earth. A typical trajectory of a ballistic missile is shown in Figure 1. The trajectory is divided into powered, ballistic, and re-entry flights. During powered flight, the missile goes from a static position to dynamic flight and is propelled beyond the appreciable atmosphere. A propulsion system accelerates the missile to a velocity where it will enter ballistic flight, a predetermined flight path beyond the appreciable atmosphere. The missile moves along without further expenditure of energy until it re-enters the atmosphere near the impact point. The location of an assumed observation site is shown in Figure1.[1]

The center line (zero azimuth) of the antenna at the observation site shown in Figure 1 points to the North Pole. The antenna pedestal is assumed to be able to turn 360° counterclockwise or clockwise without damage to the antenna. In this paper, azimuth values are positive in the counterclockwise direction. For the aboard ship site, the antenna's zero azimuth always rotates

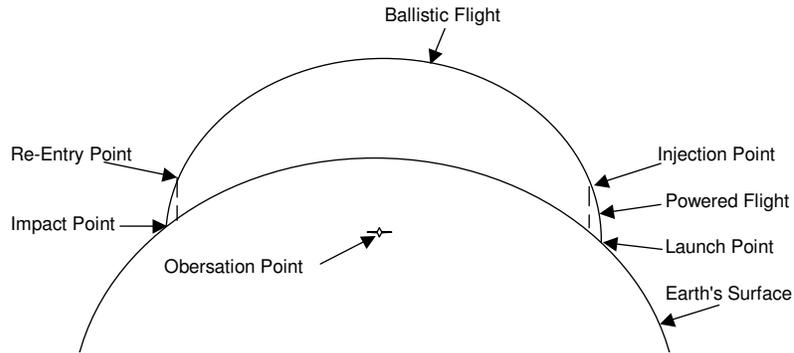so that it always points to the north pole and the ship's latitude and longitude is constantly updated.



Figure 1   Typical trajectory of a ballistic missile

Terrestrial Sphere
The earth is not a perfect sphere. The terrestrial sphere shown in Figure 2 is used as a simpler model that approximates the earth with sufficient accuracy.  The terrestrial sphere is considered to rotate about an *axis* which is a diameter joining the north and the south poles through the center (C) of the sphere. A *great circle* on the terrestrial sphere is a geodesic or the shortest distance between two points on the surface of the sphere, analogous to a straight line on a plane surface. The *equator* is a great circle midway between poles.  The plane of the equator is perpendicular to the axis of the sphere.[2]
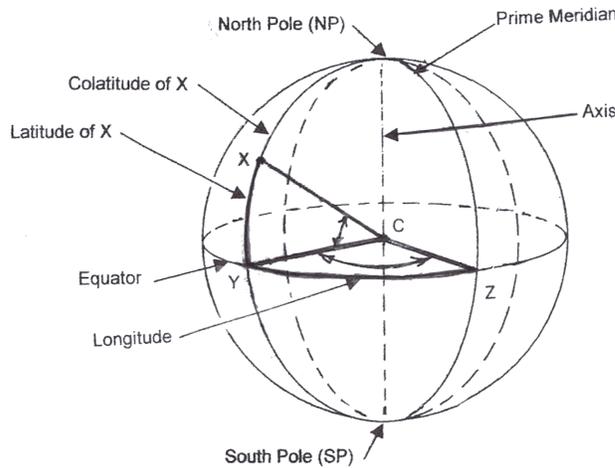


Figure 2   Terrestrial Sphere

The *latitude* of point X is the angular distance (XCY) of the point from the equator (the arc YX) and is measured from 0° to 90 °, north or south of the equator.  The *co-latitude* of point X is the angular distance (XCNP) of the point from the North Pole (the arc XNP) or 90° - (Latitude X)°. If X were south of the equator, the co-latitude of point X would be 90° + (Latitude X)°.  In this paper, latitude will be reckoned positively to the north and negatively to the south of the equator. Therefore, the co-latitude of X would always be 90° - (Latitude X)°.  Parallel of latitudes are small circles parallel to the equator.[2]

A *meridian* is a semicircle of a great circle joining the north and south poles. The prime meridian is the meridian through Greenwich, England. For historical reasons, the zero-point for longitude is the prime meridian. Therefore, the *longitude* of point X is the angular distance (YCZ) along the equator from the prime meridian to the meridian through X (the arc YZ). It may be measured east or west 0° to 360°, or both ways 0° to 180°. In this paper, longitude will be reckoned positively to the east of the prime meridian and negatively to the west of the prime meridian.[2]

Units

A system of units is used in which the mean equatorial radius (R) of the terrestrial sphere is equal to *one earth radius unit* (ERU). Nautical miles are used in calculating an ERU. A nautical mile (NM) is 6076 feet or the length of 1 minute of arc along any great circle of the terrestrial sphere. Thus, a great circle arc of 2° is easily converted into 120 nautical miles. R is defined as follows:[3]

$$R = [(360°) (60 \text{ NM} / °)] / (2\pi) = 3437.74677 \text{ NM} \qquad (1)$$

An ERU is defined as follows:[3]

$$ERU = 3437.74677 \text{ NM} / 3437.74677 \text{ NM} = 1 \qquad (2)$$

Also, a *time unit* (TU) on this terrestrial sphere is the time necessary for a zero drag missile to travel one radian in a circular orbit along the sphere's surface. One TU is defined by the following equation:[3]

$$TU = (R/G)^{½} = 805.94 \text{ seconds} \qquad (3)$$

Where,

   R = 20887750 feet (the mean equatorial radius in feet)
   G = earth's gravitational attraction (32.1578 feet / second$^2$)

Geometric Parameters

Launch, impact and observation sites form an oblique spherical triangle. Sides of this oblique spherical triangle for launch, impact and observation sites are shown in Figure 3.
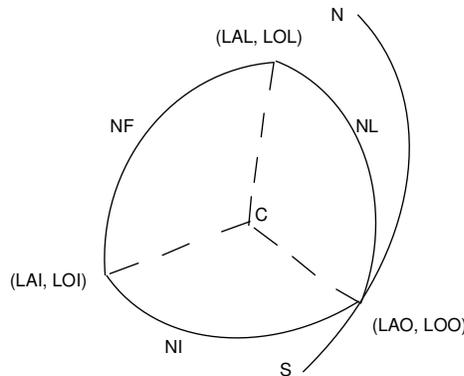


Figure 3   Sides NF, NL, and NI

Parameters shown in Figure 3 are as follows:

   NF ≡ Spherical triangle side from launch site to impact site
   NL ≡ Spherical triangle side from observation site to the launch site
   NI ≡ spherical triangle side from the observation site to the impact site
   LAL ≡ Latitude of the launch site

LOL ≡ Longitude of the launch site
LAI ≡ Latitude of the impact site
LOI ≡ Longitude of the impact site
LAO ≡ Latitude of the observation site
LOO ≡ Longitude of the observation site
C ≡ Center of the terrestrial sphere
NS ≡ North-south pole plane through the observation site

Co-latitudes
Co-latitudes of launch, impact and observation sites are used in calculating spherical triangle sides NF, NI, and NL shown in Figures 4, 5 and 6. Colatitudes are calculated as follows:[2]

$$QO = (\pi / 2) - LAO \tag{4}$$
$$PL = (\pi / 2) - LAL \tag{5}$$
$$PI = (\pi / 2) - LAI \tag{6}$$

Where,

QO ≡ Co-latitude of the observation site
PL ≡ Co-latitude of the launch site
PI ≡ Co-latitude of the impact site

Central Angles
Central angles or difference in longitude angles are (LOI – LOL), (LOI – LOO), and (LOL – LOO). Central angles of the oblique spherical triangle are shown in Figure 4, 5 and 6. For example, these angles are calculated as follows:[2]

   a. If both longitudes are east or both are west of the prime meridian through Greenwich, England, then use (LOI – LOL) or (LOL – LOI) whichever is positive.
   b. If the longitudes are on opposite sides of prime meridian, use (LOI + LOL) or $360^{\circ}$ - (LOI + LOL) whichever is less than $180^{\circ}$.
   c. Repeat a. and b above to calculate angles (LOI-LOO) and (LOL – LOO).

Spherical triangle side NF is shown in Figure 4. NF is calculated as follows:[2]

$$Cos (NF) = ((Cos (PI) Cos (PL)) + (Sin (PI) Sin (PL) Cos (A))) \tag{7}$$
$$NF = Acos ((Cos (PI) Cos (PL)) + (Sin (PI) Sin (PL) Cos (A))) \tag{8}$$

Where,

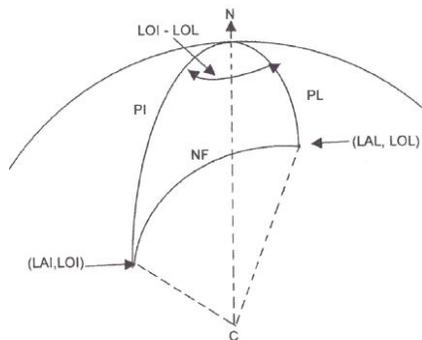A = Central angle (LOI-LOL)



Figure 4   Spherical triangle side NF

Spherical triangle side NI is shown in Figure 5. NI is calculated as follows:[2]

$$\text{Cos (NI)} = ((\text{Cos (PI) Cos (QO)}) + (\text{Sin (PI) Sin (QO) Cos (B)})) \qquad (9)$$

$$\text{NI} = \text{Acos} ((\text{Cos (PI) Cos (QO)}) + (\text{Sin (PI) Sin (QO) Cos (B)})) \qquad (10)$$

Where,

B = Central angle (LOI-LOO)
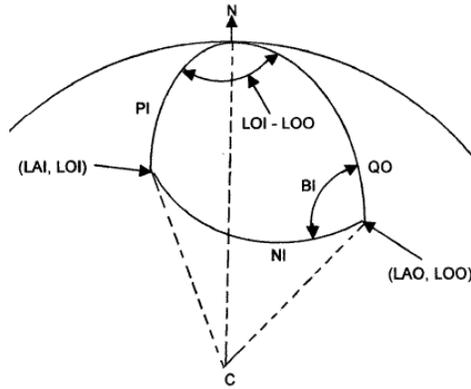


Figure 5   Spherical triangle side NI and azimuth angle BI.

The azimuth from the observation site to the impact site is the angle BI shown in Figure 5. Angle BI is calculated as follows:[2]

$$\text{Cos (PI)} = (\text{Cos (QO) Cos (NI)}) + (\text{Sin (QO) Sin (NI) Cos (BI)}) \qquad (11)$$

$$\text{BI} = \text{Acos} (((\text{Cos (PI)}) - (\text{Cos (QO) Cos (NI)}) / (\text{Sin (QO) Sin (NI)})) \qquad (12)$$

If LOI is less than LOO,

$$\text{BI} = -\text{Acos} (((\text{Cos (PI)}) - (\text{Cos (QO) Cos (NI)}) / (\text{Sin (QO) Sin (NI)})) \qquad (13)$$

Spherical triangle side NL is shown in Figure 6. NL is calculated as follows:[2]

$$\text{Cos (NL)} = ((\text{Cos (PL) Cos (QO)}) + (\text{Sin (PL) Sin (QO) Cos (C)})) \qquad (14)$$

$$\text{NL} = \text{Acos} ((\text{Cos (PL) Cos (QO)}) + (\text{Sin (PL) Sin (QO) Cos (C)})) \qquad (15)$$

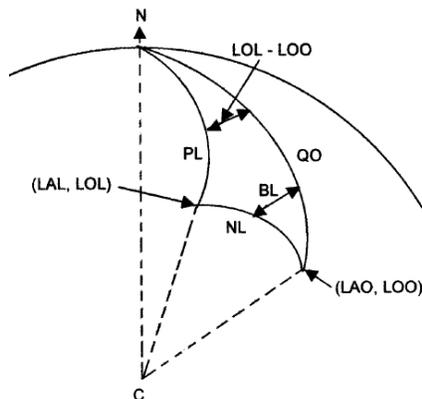Where,

C = Central angle (LOL-LOO)



Figure 6   Spherical triangle side NL and azimuth angle BL.

The azimuth from the observation site to the launch site is the angle BL shown in Figure 6.
Angle BL is calculated as follows[2]:

$$\text{Cos (PL)} = (\text{Cos (QO) Cos (NL))} + \text{Sin (QO) Sin (NL) Cos (BL))} \qquad (16)$$
$$\text{BL} = \text{Acos (((Cos (PL)) - (Cos (QO) Cos (NL)) / (Sin (QO) Sin (NL))} \qquad (17)$$

If LOL is less than LOO,

$$\text{BL} = \text{-Acos (((Cos (PL)) - (Cos (QO) Cos (NL)) / (Sin (QO) Sin (NL)))} \qquad (18)$$

Side DO goes from the observation site to the nearest point (D) on the missile trajectory as shown in Figure 7. Side DO is calculated as follows:[2]

$$(\text{Sin (NL) / Sin } (\alpha)) = (\text{Sin(NF) / (Sin(|BI – BL|))} \qquad (19)$$
$$(\text{Sin (NI) / Sin } (90^{\circ})) = (\text{Sin(DO) / (Sin}(\alpha)) \qquad (20)$$
$$\text{Sin } (\alpha) = (\text{Sin(DO) / Sin(NI))} \qquad (21)$$

From equations 19 and 21,

$$(\text{Sin (NL) Sin (NI) / Sin (DO))} = (\text{Sin(NF) / (Sin(|BI – BL|))} \qquad (22)$$

Thus,

$$\text{DO} = \text{ASIN ((Sin (NL) Sin (NI) Sin (|BI – BL|))/ Sin (NF))} \qquad (23)$$



Figure 7   Side DO

Azimuth Angle BO
Azimuth angle BO goes from the North-South plane to side DO as shown in Figure 7.
First, the right spherical triangle that consists of angle B, side DO and side NL is examined.[2]

$$\text{Cos B} = \text{Tan (DO) / Tan (NL)} \qquad (24)$$
$$\text{B} = \text{Acos (Tan (DO) / Tan (NL))} \qquad (25)$$

The location of North-South pole Plane through the observation site, the location of the observation site, and the travel direction of the missile will determine if angle B is added to or subtracted from BL to calculate BO. Table 1 was developed taking these parameters into consideration.

|  | BL > BI | BL < BI |
|---|---|---|
| Cos (NI) < Cos (NL) Cos (NF) | Add B | Subtract B |
| Cos (NI) > Cos (NL) Cos (NF) | Subtract B | Add B |

Table1   Decision table for B.

Thus, Angle BO is calculated as follows:
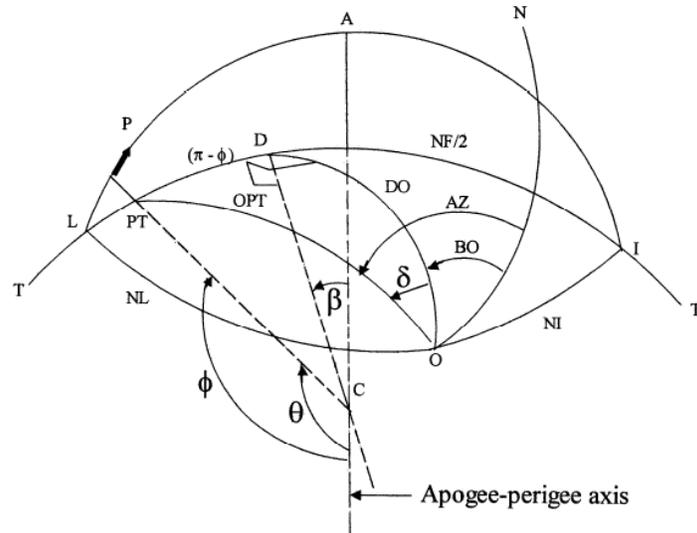
$$BO = BL \pm B \qquad\qquad (26)$$



Figure 8   Minimum energy trajectory of a ballistic missile

Minimum Energy Trajectory
Figure 8 shows the minimum energy trajectory of a ballistic missile.  Parameters shown on Figure 8 are as follow:

> TT' ≡ Intersection of the trajectory plane with the earth's surface
> P ≡ Missile position (arrow indicates missile's direction of travel)
> PT ≡ Intersection of line CP with the earth's surface
> A ≡ Apogee of the trajectory
> L ≡ Launch site
> I ≡ Impact site
> O ≡ Observation site.
> β ≡ Describes the orientation of the ellipse on its plane and
>     is the central angle between the nearest point D and the
>     apogee-perigee axis of the trajectory ellipse.
> θ ≡ Polar coordinate angle (true anomaly) of point on the ellipse
> φ ≡ Angle between reference line DO and line CP
> (π - φ) ≡ DPT
> AZ ≡ Azimuth angle between the North-South plane at the observation sit and
>     OPT (missile)
> δ ≡ Delta angle (∠D-O-PT)

Angle β
Angle β (Beta) is the angle between the nearest point D and the perigee-apogee axis of the trajectory ellipse as shown in Figure 8.  This angle describes the orientation of the ellipse on its plane.  Angle Beta is calculated as follows:[2]

$$\text{Cos (NL)} = (\text{Cos (DO) Cos (LD)}) \tag{27}$$
$$\text{Cos (LD)} = (\text{Cos (NL) / Cos (DO)}) \tag{28}$$
$$\text{LD} = \text{Acos (Cos (NL) / Cos (DO))} \tag{29}$$

Logical decisions are required to determine if LD is added to or subtracted from NF/2.
If Cos (NI) is greater than Cos (NL) Cos (NF), (30)
$$\text{Beta} = (\text{NF/2}) - \text{LD} \tag{31}$$
If Cos (NI) is less than Cos (NL) Cos (NF),
$$\text{Beta} = (\text{NF/2}) + \text{LD} \tag{32}$$
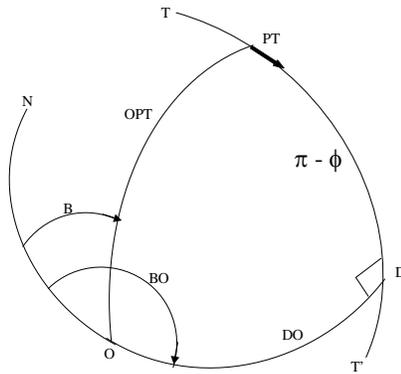


Figure 9   View of trajectory on the surface of the earth

Angle $\phi$
Angle $\phi$ (PHI), shown in Figure 8 and 9, is required for Kepler's Equation (47) and DPT is needed to calculate angle PHI.  DPT is calculated as follows:
$$\text{Sin (DO)} = \text{Tan (DPT) Ctn (BO-B)} \tag{33}$$
$$\text{Tan (DPT)} = \text{Sin (DO) Tan (BO-B)} \tag{34}$$
$$\text{DPT} = \text{Atan (Sin (DO) Tan (BO-B))} \tag{35}$$
The location of North-South pole Plane through the observation site, the location of the observation site, and the travel direction of the missile will determine if DPT is added to or subtracted from $\pi$ to calculate PHI.  Table 2 was developed taking these parameters into consideration.

|  | BO > BL | BO < BL |
|---|---|---|
| Cos (NI) < Cos (NL) Cos (NF) | Subtract DPT | ADD DPT |
| Cos (NI) > Cos (NL) Ccos (NF) | Add DPT | Subtract DPT |

Table 2   Decision table for DPT

Thus, PHI is calculated as follows:[3]
$$\text{PHI} = \pi \pm \text{DPT} \tag{36}$$

Trajectory Ellipse
The trajectory ellipse is shown in Figure 10.  The shape of the trajectory ellipse is described by "a" and "e".  The semi-major axis is the parameter "a".  The eccentricity of the ellipse is "e".

The apogee height above the earth surface is "d". The focus distance is "2ae". Basic relationships for the ellipse are as follows:

$$G1 + H1 = 2ae \tag{37}$$
$$(H2)^2 = (G2)^2 + (H1)^2 \tag{38}$$
$$1 + H2 = 2a \tag{39}$$
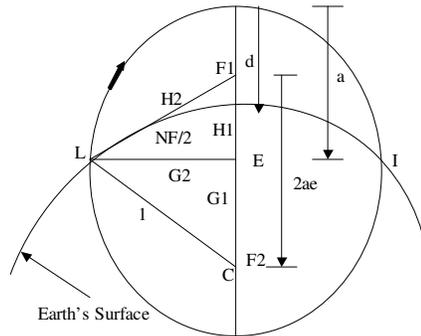$$G2 = Sin (NF / 2) \tag{40}$$
$$G1 = Cos (NF / 2) \tag{41}$$



Figure 10   Trajectory ellipse of the missile

Near Parabolic Trajectory

All ballistic trajectories of missiles launched at different speeds from the launch site (L) shown in Figure 11 are tangent at this initial point on the Earth's surface. If the initial velocity of the missile is less than the velocity needed for a circular orbit, the trajectory will be the portion (L to I) of an ellipse above ground with one focus at the center of the Earth. The portion of the ellipse above ground is approximately a parabola (minimum energy ballistic trajectory).[4]   The missile is actually in orbit but the Earth gets in the way producing a minimum energy ballistic trajectory or a near parabolic trajectory. The value of H1 in Equation 37 is set equal to zero for a minimum energy ballistic trajectory and both focal points (C and F2) are retained. Equations for the minimum energy ballistic trajectory are as follows:

$$G2 = H2 = Sin (NF / 2) \tag{42}$$
$$G1 = 2Cos (NF / 2) = 2ae \tag{43}$$
$$1 + G2 = 2a \tag{44}$$

Therefore,

$$a = (1 + Sin (NF /2)) / 2 \tag{45}$$
$$e = (Cos (NF / 2)) / a \tag{46}$$



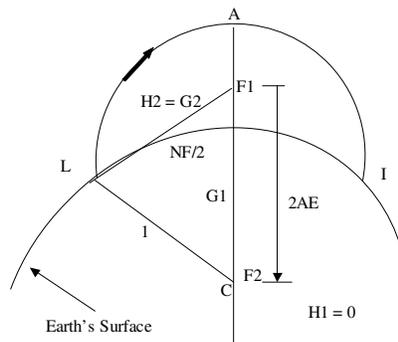Figure 11    Near Parabolic Trajectory

Time

The time (T) is determined using Kepler's equations relating the eccentric anomaly (U) and the polar coordinate angle (PHI) on the trajectory. If T produces a positive elevation, the value of T is decremented through the trajectory to find HRBK then incremented to find MXEL and LOS. If T produces a negative elevation, the value of T is incremented to find HRBK, MAXEL, and LOS. The missile will not break the horizon if MAXEL is negative and the observation site will not see the missile. Each new T is used to compute a new U. First values of U and T are calculated as following:[3]

$$U = \pi - \text{ASIN} ((1 – e^2) \text{ Sin (PHI-BETA)} / (1 + e \text{ Cos (PHI – BETA)}) \qquad (47)$$
$$T = ((a^{3/2}) (U – e \text{ Sin (U)}) + T_0 \qquad (48)$$

Trajectory parameter ($T_0$) is the time of the (theoretical) last passage of the missile through perigee and is given a value of zero.

Iterative Solution

Figure 12 is a pictorial definition of the U. An iterative solution is required to calculate each U for new points on the trajectory. The iterative solution is developed as follows:[3]

$$U – e \text{ Sin (U)} - \Delta\tau \equiv f(U) = 0 \qquad (49)$$

Where,

$$\Delta\tau = (T - T_0) a^{-3/2} \qquad (50)$$

Then $f(U) = 0$ implies that,

$$U = U – (f(U) / f'(U)); \quad f'(U) \neq 0 \qquad (51)$$

Since,

$$f'(U) = 1 – e \text{ Cos (U)} \qquad (52)$$

Then $f'(U)$ can never vanish and Equation (51) can be used as a basis for an iteration. From the definition,

$$\Psi_1(U) = U – (f(U) / f'(U)) \qquad (53)$$

It follows that,

$$\Psi'_1(U) = (f(U) (f'(U)] / \{f'(U))^2 \qquad (54)$$
$$\Psi'_1(U) = - ((U – e \text{ Sin (U)} – \Delta\tau) (e \text{ Sin (U)}) / (1 – e \text{ Cos (U)})^2) \qquad (55)$$

Therefore, the iteration method in which $U^{(n+1)}$ is determined follows:

$$U^{(n+1)} = \Psi_1(U^{(n)}); \quad U^{(1)} = \Delta\tau \qquad (56)$$
$$U^{(n+1)} = U^{(n+1)} – (( U^{(n+1)} – e \text{ Sin}(U^{(n+1)}) – \Delta\tau)(e \text{ Sin}(U^{(n+1)}))/(1 – e \text{ Cos}(U^{(n+1)}))^2) \qquad (57)$$

Where,

$$U^{(1)} = \Delta\tau – ((\Delta\tau – e \text{ Sin}(\Delta\tau) – \Delta\tau)(e \text{ Sin}(\Delta\tau)) / (1 – e \text{ Cos}(\Delta\tau))^2) \qquad (58)$$
$$U^{(2)} = U^{(1)} – (( U^{(1)} – e \text{ Sin}(U^{(1)}) – \Delta\tau)(e \text{ Sin}(U^{(1)}))/(1 – e \text{ Cos}(U^{(1)}))^2) \qquad (59)$$

.
.                                          .
$$U^{(15)} = U^{(14)} – (( U^{(14)} – e \text{ Sin}(U^{(14)}) – \Delta\tau)(e \text{ Sin}(U^{(14)}))/(1 – e \text{ Cos}(U^{(14)}))^2) \qquad (60)$$

This method provides convergence within 15 iterations for any value of e (0 to1) with accuracy greater than 0.00001 radians.[3] $U^{(15)}$ is used to compute the new azimuth and elevation for the next point on the trajectory.

Referring to Figure 12, azimuth and elevation for the next point on the trajectory can be found by determining the relationship between U and the polar $\theta$ (true anomaly shown in Figure 10 and 14) of a point on the ellipse are shown pictorially. The analytical relations are:

$$\text{Sin (U)} = ((1 – (e^2))^{1/2} (\text{Sin }(\theta))) / (1 + (e \text{ Cos }(\theta))) \qquad (61)$$

$$\text{Sin } (\theta) = ((1 - (e^2))^{1/2} \text{ (Sin } (U))) / (1 - (e \text{ Cos } (\theta))) \tag{62}$$

$$\text{Cos } (U) = (\text{Cos } (\theta) + (e)) / (1 - (\text{Cos } (U))) \tag{63}$$

$$\text{Cos } (\theta) = (\text{Cos } (U) - (e)) / (1 - (e \text{ Cos } (U))) \tag{64}$$

From Equations (58) through (61),

$$\text{Tan } (U/2) = ((1 - e) / (1 + e))^{1/2} \text{ (Tan } (\theta/2)) \tag{65}$$

Therefore angle $\theta$ (THTA),

$$\text{THTA} = 2 \text{ Atan } ((1 + e) / (1 - e))^{1/2} \text{ (Tan } (U/2))) \tag{66}$$

Referring to Figure 8, angle $\phi$ for convergence (PHIC) is calculated as follows:

$$\text{PHIC} = \text{THTA} + \text{BETA} \tag{67}$$



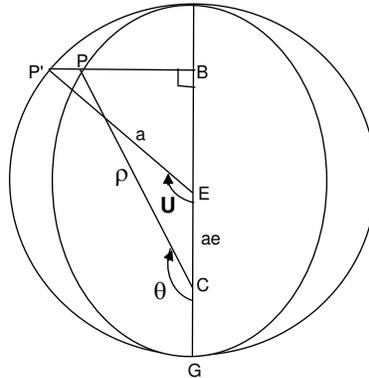Figure 12   Pictorial Definition of Eccentric Anomaly (U)

Parameters shown on Figure 12 are as follows:

C ≡ focus of the ellipse

E ≡ center of the major axis of the ellipse

P ≡ point on the periphery of the ellipse

PB ≡ line through P and perpendicular to EC

P' ≡ intersection of line PB with the circumference of circle of radius a, centered at E.

CP ≡ ρ

CE ≡ ae

EP ≡ a

U ≡ ∠GEP' ≡ eccentric anomaly

θ ≡ ∠GCP ≡ polar coordinate angle (true anomaly) of point on ellipse

Azimuth (AZ)

Figure 13 shows angle AZ.  Angle δ (DELTA) is needed to calculate Angle AZ.  DELTA and BO are also shown in Figure 8.  Figure 8 is used to calculate DELTA with PHIC used in place of PHI as follows:

$$\text{Sin } (DO) = (\text{Tan } (\pi - \text{PDIC}))(\text{Ctn } (\text{DELTA})) \tag{68}$$

$$\text{DELTA} = \text{ATAN } ((\text{Tan } (\pi - \text{PHIC})) / (\text{Sin}(DO))) \tag{69}$$

Also,

$$\text{DELTA} = -\text{ATAN } ((\text{Tan}(\text{PHIC})) / (\text{Sin}(DO))) \tag{70}$$

The location of North-South pole Plane through the observation site, the location of the observation site, and the travel direction of the missile will determine if DELTA is added to or subtracted from BO to calculate AZ. Table 3 was developed taking these parameters into consideration.

|  | BO > BI | BO < BI |
|---|---|---|
| Cos (NI) < Cos (NL) Cos (NF) | Add DELTA | Subtract DELTA |
| Cos (NI) > Cos (NL) Cos (NF) | Subtract DELTA | Add DELTA |

Table 3   Decision table for DELTA

Thus,

$$AZ = BO \pm DELTA \qquad (71)$$



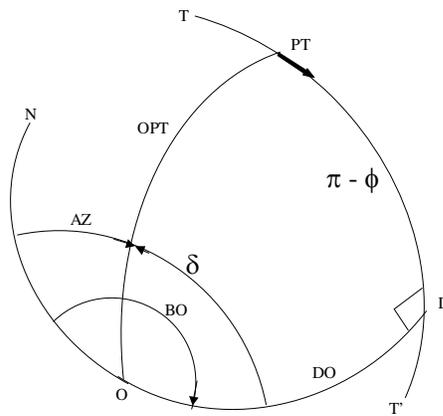Figure 13   Missile's trajectory on the surface of the earth

Elevation (EL)
Angle OPT shown in Figure 13 is calculated as follows:
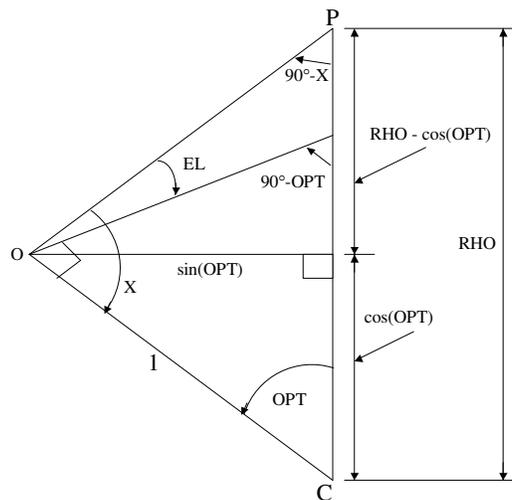
$$OPT = Acos((Cos(DO))(Cos(\pi - PHIC))) \qquad (72)$$



Figure 14   Elevation plane

Referring to Figure 14, ρ (RHO) extends from C to P.  The geometrical equation of the trajectory in polar coordinates is given by:[3]

$$\text{Rho} = (a(1 - (e)^2)) / (1 + (e \, \text{Cos}(\text{PHIC} - \text{BETA}))) \tag{73}$$

Figure 14 shows the elevation plane.  The equation of the elevation angle is as follows:

$$X = \text{ATAN} ((\text{RHO} - \text{Cos}(\text{OPT})) / \text{Sin}(\text{OPT})) \tag{74}$$

$$EL = \pi - (((\pi/2) + \text{OPT})) + (((\pi/2) - X)) \tag{75}$$

$$EL = X - \text{OPT} \tag{76}$$

Thus,

$$EL = \text{ATAN} ((\text{RHO} - \text{Cos} (\text{OPT})) / \text{Sin} (\text{OPT})) - \text{OPT} \tag{77}$$

Modeling Program

LabVIEW® 7.1 Express[5] (Student Version) software was used to build the modeling program. LabVIEW® is ideal software for science and engineering applications.  This graphical software has built-in functionality for simulation, data acquisition, instrument control, measurement analysis, and data presentation.  Table 4 shows the latitudes and longitudes of locations used in the software program.[6]

| Obervation Site | Latitude | Longitude |
|---|---|---|
| Annapolis, Maryland | 38° 58' 00" N | 76° 30' 00" W |
| Chicago, Illinois | 41° 50' 00" N | 87° 40' 00" W |
| Colorado Springs, Colorado | 38° 50' 00" N | 104° 50' 32"W |
| Iraklion, Greece | 35° 12' 00" N | 25° 06' 00" E |
| Kerman, Iran | 30° 00' 00" N | 57° 00' 00" E |
| Miami, Flordia | 25° 46' 00" N | 80° 12' 00" W |
| Pyongyang, Korea | 39° 00' 00" N | 126° 00' 00"E |
| San Antonio, Texas | 29° 26' 48" N | 98° 28' 34" W |
| San Francisco, California | 37° 45' 00" N | 122° 27' 00"W |
| Shemya Island, Alaska | 52° 45' 00" N | 174° 00' 00"W |

Table 4   Latitude and Longitude Table

The Front Panel of the LabVIEW® Modeling Software Program is shown in Figures 15, 16, 17 and 18.  The example shown in these figures is a simulated missile launched from a theoretical missile site near Pyongyang, Korea.  The impact site is San Francisco, California.  The observation site is Shemya Island, Alaska.  In Figure 15, the operator inters the latitude and longitude of launch, impact and observation sites.[6]   Green lights indicate that Latitude and Longitude data was interred correctly.  A red light will indicate where data was not interred correctly and stops the program.  Degrees of latitude cannot be greater than 90 degrees and the total sum of degrees, minutes and seconds cannot be greater than 90 degrees.  Longitude cannot be greater than 180 degrees.  Minutes or seconds cannot be greater than 59.
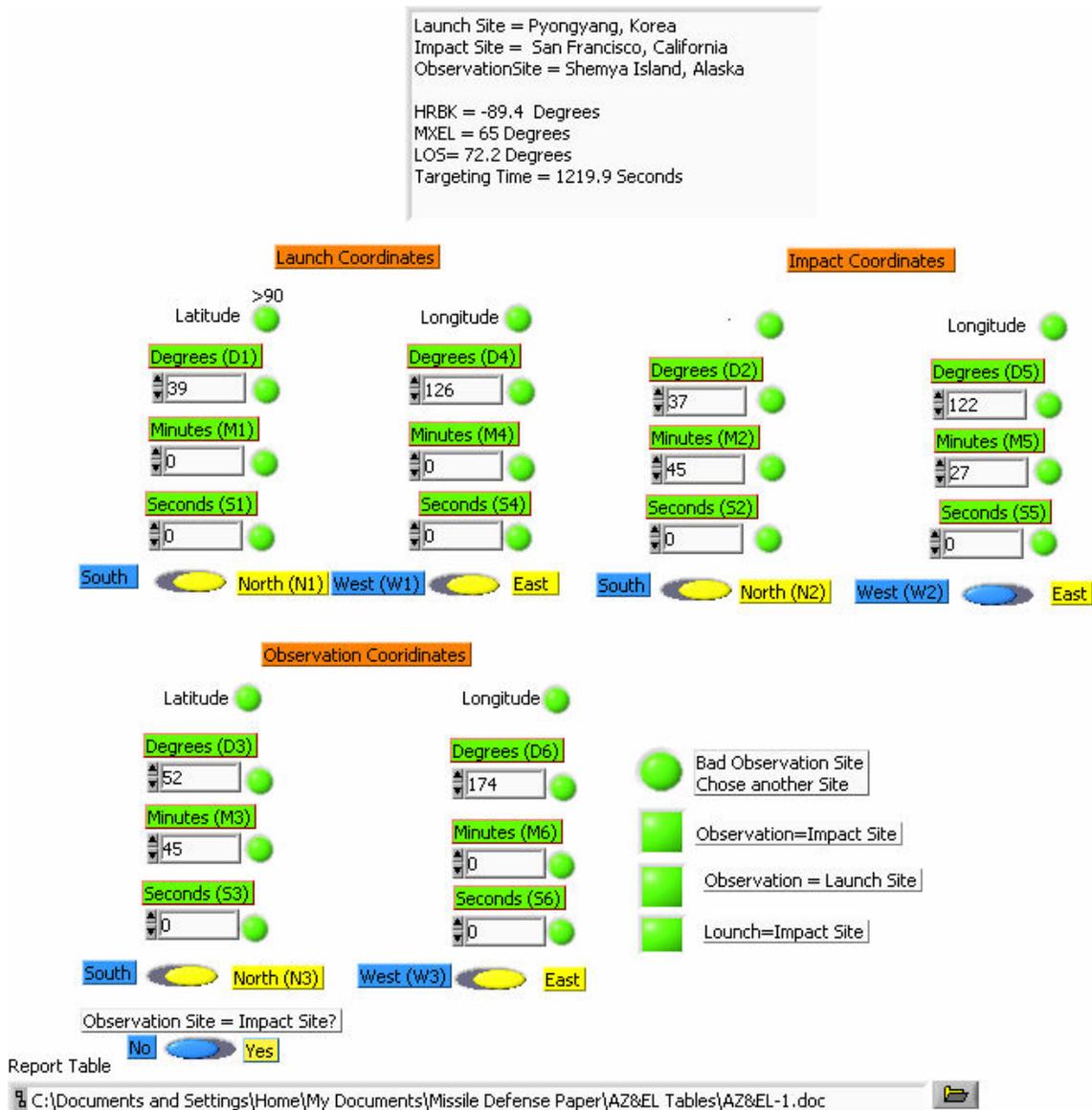
Figure 15   Program Front panel

The operator runs the program after latitude and longitude information is interred.  The first T, AZ1 and EL1 on the missile's trajectory is calculated and displayed on the left side of the operation panel shown in Figure 16.  T, AZ1 and EL1 represent a point on the trajectory.  Time is subtracted from T and entered into the T1 control.  The program is run again to calculate the next point on the trajectory.  This process is continued until HRBK (zero elevation) is obtained.  Time is added to T and entered into the control T1 until MXEL and LOS (zero elevation) are obtained.  New values of azimuth and elevation values are display in indicators below T1.  Time, azimuth, and elevation values shown in these indicators are manually interred into the controls shown on the right side of Figure 16.  A 1 is placed in the Plot control and the program is run again to automatically produce the graph.  Different times can be entered into control T1 to obtain a desirable plot of the missile's trajectory.  The Azimuth and elevation scales on the graph are changed to get a desirable plot of the missile's trajectory as shown in Figure 17.

| Seconds | Degrees | Degreea |
|---|---|---|
| TP1 | AZP1 | ELP1 |
| 1217.5 | -89.3668 | 0 |
| TP2 | AZP2 | ELP2 |
| 1300 | -87.1861 | 7.7242855 |
| TP3 | AZP3 | ELP3 |
| 1400 | -83.4078 | 18.63672 |
| TP4 | AZP4 | ELP4 |
| 1500 | -77.2178 | 31.781881 |
| TP5 | AZP5 | ELP5 |
| 1600 | -65.3564 | 47.127182 |
| TP6 | AZP6 | ELP6 |
| 1700 | -38.1639 | 61.411782 |
| TP7 | AZP7 | ELP7 |
| 1750 | -14.704 | 64.851064 |
| TP8 | AZP8 | ELP8 |
| 1800 | 10.3883 | 63.80705 |
| TP9 | AZP9 | ELP9 |
| 1900 | 42.2232 | 52.913234 |
| TP10 | AZP10 | ELP10 |
| 2000 | 55.9183 | 39.923032 |
| TP11 | AZP11 | ELP11 |
| 2100 | 62.8092 | 28.458637 |
| TP12 | AZP12 | ELP12 |
| 2150 | 65.0864 | 23.384256 |
| TP13 | AZP13 | ELP13 |
| 2200 | 66.9026 | 18.68715 |
| TP14 | AZP14 | ELP14 |
| 2300 | 69.6303 | 10.209524 |
| TP15 | AZP15 | ELP15 |
| 2437.4 | 72.2115 | 0 |

Plot Control
0

Print Control
0

T (Seconds)
2657.4005

T1
1750

AZ1 (Degrees)
74.949294

AZ2 (Degrees)
-14.704

EL1 (Degrees)
-14.362876

EL2 (Degrees)
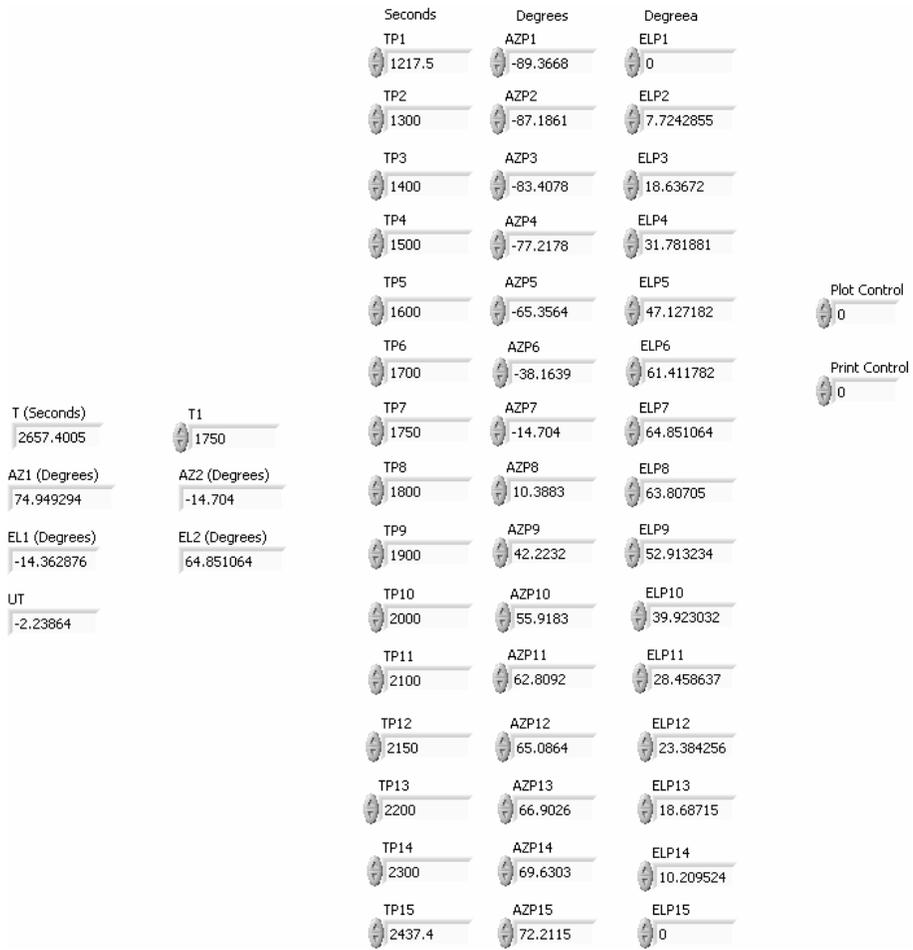64.851064

UT
-2.23864

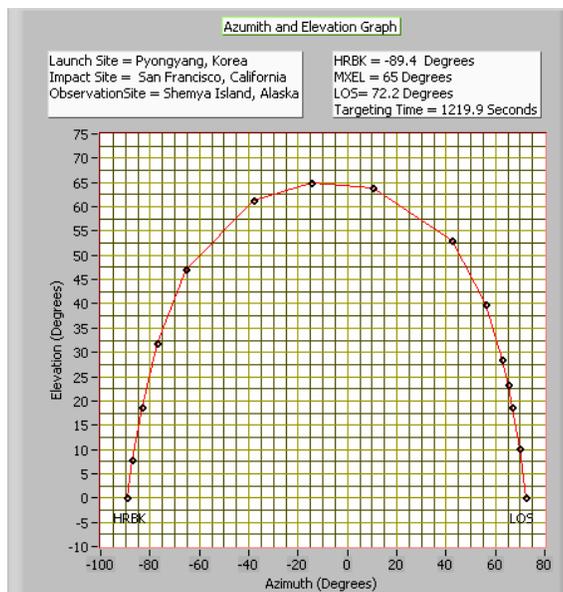Figure 16   Operation Panel



Figure 17   Azimuth and Elevation Graph

The file name shown in Figure 15 must be renamed. Reports will be written in this file one after another if the file name is not changed. A 1 is placed in the Print control and the program is run again to produce the record of this simulation as shown in Figure 18.

```
8/19/2006    10:18 PM
Launch Site = Pyongyang, Korea
Impact Site =  San Francisco, California
ObservationSite = Shemya Island, Alaska

HRBK = -89.3  Degrees
MXEL = 65 Degrees
LOS= 72.2 Degrees
Targeting Time = 1219.2 Seconds

 Time(Sec)      AZ(Deg)      EL(Deg)
1217.500000 -89.366800   0.000000
1300.000000 -87.186100   7.724285
1400.000000 -83.407800  18.636720
1500.000000 -77.217800  31.781881
1600.000000 -65.356400  47.127182
1700.000000 -38.163900  61.411782
1750.000000 -14.704000  64.851064
1800.000000 10.388300  63.807050
1900.000000 42.223200  52.913234
2000.000000 55.918300  39.923032
2100.000000 62.809200  28.458637
2150.000000 65.086400  23.384256
2200.000000 66.902600  18.687150
2300.000000 69.630300  10.209524
2437.400000 72.211500   0.000000
```

Figure 18   Program Generated Report

Results

Shemya Island, Alaska is an excellent observation site as shown in Table 5. All missiles launched from the Pyongyang, Korea site to mainland U.S. breaks the horizon within a 21.6° window. The search window will depend on the antenna's bandwidth. For example, experience has shown that an 8 foot diameter antenna in the 2 to 6 Giga Hertz (GHz) frequency range will have about 7 degrees of bandwidth. This antenna is normally used to track the guidance beacon on missiles. Two of these antennas spaced properly and synchronized would be required for the ideal situation of no holes in the search window.   The targeting time in Table 5 is the time that radar antennas have to skin-track the missile. The Early warning time in Table 5 is the time from the missile's launch to HRBK. Experience has shown that the beacon search antenna will pickup the beacon signal before it breaks the horizon and can give early warning of a missile launch. The beacon signal will not bend over the horizon when the missile in on the launch pad. After the missile is launched, the beacon signal will start bending over the horizon.   The software program could also drive the antenna through the search window by adding an antenna drive routine, installing a LabVIEW® data acquisition card in the computer and an external antenna interface box to provide the drive voltages.

| Site | Location | Values At Observation Site | | | | |
|------|----------|------|------|------|------|------|
| Launch | Pyongyang, Korea | | | | | Early Warning Time |
| Observation | Shemya Island, Alaska | HRBK | MXEL | LOS | Trageting Time | Before HRBK |
| Impact | San Francisco, California | (-)89.3° | 65° | 72.2° | 1219.9 s | 1217.5 s |
| Impact | Colorado Springs, Colorado | (-)82.7° | 37.2° | 51.1° | 1160 s | 1264.5 s |
| Impact | San Antonio, Texas | (-)81.9° | 36° | 53.8° | 1122.1 s | 1312.9 s |
| Impact | Chicago, Illinois | (-)74.2° | 19.1° | 29.4° | 1009.6 s | 1312.9 s |
| Impact | Miami, Flordia | (-)70.1° | 15.1° | 28.6° | 886.5 s | 1395.5 s |
| Impact | Annapolis, Maryland | (-)67.7° | 11.6° | 16.3° | 858.5 s | 1363.5 s |

Table 5   Shemya Island, Alaska coverage of missiles launched from
Pyongyang, Korea to USA impact sites

The West coast of the U.S. can not be defended by the observation site located at Iraklion, Greece as shown in Table 6.  Most of the U.S. mainland can be defended.

| Site | Location | Values At Observation Site | | | | |
|------|----------|------|------|------|------|------|
| Launch | Kerman, Iran | | | | | Early Warning Time |
| Observation | Iraklion, Greece | HRBK | MXEL | LOS | Trageting Time | Before HRBK |
| Impact | San Francisco, California | None | None | None | None | None |
| Impact | Colorado Springs, Colorado | 52.5° | 2.1° | 15.2° | 365 s | 1400 s |
| Impact | San Antonio, Texas | 57.9° | 4.7° | (-)0.45° | 511 s | 1382 s |
| Impact | Chicago, Illinois | 66.7° | 9.4° | (-)10° | 738 s | 1282 s |
| Impact | Annapolis, Maryland | 72.6° | 15.9° | (-)25.1° | 893 s | 1246 s |
| Impact | Miami, Flordia | 73.1° | 17.5° | (-)33.5° | 859 s | 1296 s |

Table 6   Iraklion, Greece coverage of missiles launched from
Kerman, Iran to USA impact sites

Table 7 shows data for a simulated missile launched from a theoretical missile site at Kerman, Iran with impact and observation site located at Chicago, Illinois.  This data will be useful for a missile defense system to destroy the missile during the re-entry phase.

| Site | Location | |
|------|----------|---|
| Launch | Kerman, Iran | |
| Impact | Chicago, Illinois | |
| Observation | Chicago, Illinois | |
| HRBK | | |
| AZ | | (-)30.7° |
| Launch to HRBK Time | | 2558 s |
| MAXEL | | |
| EL | | 19.2° |
| Launch to MAXEL Time | | 3140 s |
| IMPACT | | |
| Launch to Impact Time | | 3146.68 s |

Table 7   Observation and Impact at Chicago, Illinois and
Launch site at Kerman, Iran

Recommendations
A text book for an elective course covering missile defense technology should be written. This paper could be a chapter in the textbook. Other authors could provide chapters. The book could be used to teach an elective course at universities. We should start now to develop this course because of the time needed to establish a course at a University. Graduates with this type of knowledge will be in great demand if the U.S. is attacked by North Korean or Iranian ballistic missiles bearing nuclear or biological weapons.

**References**
1. Burgess E., *Long-Range Ballistic Missiles*, The Macmillan Company, New York, NY, p 58, 1961.
2. Richardson M., *Plane and Spherical Trigonometry*, The MACMILLIAN Company, New York, NY, pp 310-337, 1950
3. Shapiro I. I., *The Prediction Of Ballistic Missile Trajectories From Radar Observations*, McGraw-Hill Company, Inc., New York, NY, 1958.
4. Butikov E. I., *Families of Keplerian Orbits*, European Journal of Physics, pp 175 – 183, February 10, 2003.
5. Bishop, R.H., *LabVIEW® 7 Express*, Prentice Hall, Upper Saddle River, NJ, 2006.
6  Payne, M.M., *National Geographic Atlas of the World*, Fourth Edition, National Geographic Society, Washington, DC, 1975.